Student Activity Sheet: Forces in the Pairs Death Spiral

Diana Cheng and Asli Sezen-Barrie, Towson University, Towson, MD
Alexander Barrie, University of Colorado at Boulder, Boulder, CO
Timothy Akers and Kevin Peters, Morgan State University, Baltimore, MD

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Part 1: Qualitative Questions

1. What force holds the man in place?
2. What prevents the man from tipping over towards the lady during the death spiral?
3. If the man squats down to lower his height, how will this motion affect the tension in the arms?
4. If the man suddenly lets go, in what direction path will the lady go?

Part 2: Developing relationships between quantities

1. Draw a visual diagram representing the relationship between three forces that impact the woman as she performs the death spiral: 1) The straight-arms tension connecting her hand with the man's hand (T), 2) The lady's net weight (Y) going vertically downwards, and 3) Centrifugal force going horizontally outwards from the man.
2. What geometric shape is formed by the three forces, T, Y and C? Write an equation relating the magnitudes of these forces.
3. Use the following information to develop an equation for the lady's angular velocity (ω) as a function of the lady's mass (m), radius between her skating blade and the man's pivoting skate (r), gravity (g), and T.
    - The lady's net weight (Y) is the difference of two opposite forces: Her weight (calculated by $m \times g$), and the weight she supports on her skating foot (we will assume the lady supports ¾ of her weight on her foot)
    - Newton's second law, Force = Mass × Acceleration, applies in this situation. In circular motion, the force involved is centrifugal force, C. The acceleration in circular motion can be represented as $\omega^2 \times r$.
4. Using your existing equation for ω, write equivalent equations to express the following:
    - m as a function of ω, T, r, and g.
    - T as a function of ω, m, r, and g.
    - r as a function of ω, T, m, and g.

Part 3: Quantitative Questions

Problem A: A female skater with mass 60 kg is performing a death spiral with her partner. The centrifugal force on the woman is 261.52 N. As she lowers her body, her skating boot travels from 1.5 meters away from his pivoting leg to 2.0 meters away. Will this increase in radius cause an increase or decrease in angular velocity, and why? By how many radians per second does her angular velocity change? What is the percentage change of her angular velocity (round to nearest tenth)?

Problem B: A female skater with mass 65 kg is performing a death spiral with her partner. When her skating foot is 4.92 ft away from his pivoting leg, the partners' hands maintain 400 N of tension. They then increase the amount of tension they have to 500 N but keep the same radius. How does this affect the angular velocity, and why? By how many revolutions per second does the
angular velocity change?

Problem C: A female skater with mass 65 kg is performing a death spiral with her partner. When she is 150 centimeters away from the man's pivoting leg, they begin rotating at 2.078 radians per second. They stay the same distance apart but they decrease the angular velocity by 0.138 radians per second.

a) Will the tension in their hands need to increase or decrease in order to accomplish this change in angular velocity? By how much is the tension changed?
b) By how much does their centrifugal force change?

Problem D: A man is comfortable with providing 500 N of tension for his partner in a death spiral. His coach measured that he and his partner rotated at 0.374 revolutions per second. If he maintains the same radius and tension for a woman who has 8 kg less mass, how will his angular velocity be impacted? Will he rotate at a faster or slower angular velocity with the woman with less mass?

Problem E: If a man is able to use his hand to create tension between 300 N for a death spiral with radius 1.5 meters and 350 N for a death spiral with radius 2 meters, what is the range of masses of female skaters who he could support in a death spiral at an angular velocity of 1.629 radians per second?

Problem F: A man and a woman whose mass is 60 kg are rotating at 1.910 radians per second and they have straight-arms tension 400 N. The same man then performs the death spiral with the same angular velocity and tension, but with a different skater whose mass is 2.1 kg higher. Would the death spiral with the second skater be at a smaller or larger radius? What is the change in radius length in centimeters?

Problem G: If a man and a woman of mass 70 kg want to rotate in the death spiral at an angular velocity between 0.217 revolutions per second and 0.241 revolutions per second with radius 190 cm, what is the range of tensions they need to create?

Problem H: A female skater who weighs 132.28 pounds is performing a death spiral that has a maximum angular velocity of 2.1 radians per second.

a) Rounded to the nearest tenth, what is the maximum radius between a female skater's skate boot and the man's pivoting foot?
b) Rounded to the nearest hundredth, what is the centrifugal force this woman feels during this death spiral?

Sample Solutions to Student Activity: Forces in the Pairs Death Spiral

Part 1: Qualitative Problems

1. Friction from the man's pivoting foot holds the man in place: his toe pick is planted at the center of the circular path formed.
2. The man leans back so that his weight creates a rotational moment in order to balance him in a vertical position.
3. The tension will go down because will be a smaller component of the woman's weight along the arm vector.
4. In the horizontal direction, she will begin to go in a straight line tangent to the circle they were spinning on (i.e., keep moving the same direction she was already traveling), and in the vertical direction, she will fall down onto the ice.

Part 2: Developing relationships between quantities

1. See Figure 2, left hand side.
2. A right triangle is formed, and $T^2 = Y^2 + C^2$

3. $\omega = \sqrt{\frac{\sqrt{T^2 - 0.25^2 m^2 g^2}}{r \times m}}$

4. $m = \frac{T}{\sqrt{\omega^4 r^2 + 0.25^2 m^2 g^2}}$; $T = \sqrt{\omega^4 r^2 m^2 + 0.25^2 m^2 g^2}$; $r = \frac{\sqrt{T^2 - 0.25^2 m^2 g^2}}{\omega^2 m}$

Part 3: Quantitative Problems

In order to solve the quantitative problems, we created the following table of data:

Variable	T	m	r	ω	
Units	Newton	kg	meters	Radians / sec	Revolutions / sec
Problem A	300	60	1.5	1.705	0.271
	300	60	2	1.476	0.235
Problem B	400	65	1.5	1.940	0.309
	500	65	1.5	2.205	0.351
Problem C	400	65	1.5	1.940	0.309
	450	65	1.5	2.078	0.331
Problem D	500	50	1.5	2.542	0.405
	500	58	1.5	2.347	0.374
Problem E	300	64.2	1.5	1.629	0.259
	350	59.9	2	1.629	0.259
Problem F	400	60	1.7	1.910	0.304
	400	57.9	1.77	1.910	0.304
Problem G	350	70	1.9	1.515	0.241
	300	70	1.9	1.360	0.217
Problem H	500	60	1.9	2.047	0.326
	500	60	1.8	2.104	0.335

Problem A: The radius is inversely related to angular velocity, so an increase in radius causes a decrease in radius. There is a 0.229 radians per second decrease, which corresponds to a 13.4% decrease; students must first find that T = 300 N.

Problem B: Tension is directly related to angular velocity, so an increase in tension results in an increase in angular velocity. The angular velocity begins at 0.309 revolutions per second and is increased by 0.042 radians per second to 0.351 radians per second; must first convert 4.92 feet to 1.5 meters, and then convert 1.940 radians per second to 0.309 revolutions per second as well as 2.205 radians per second to 0.351 revolutions per second.

Problem C:

a) The tension decreases by 50 N, from 450 N to 400 N; students must first convert the radius to 1.5 m
b) The centrifugal force decreases by 53.95. (420.88 N – 366.93 N = 53.95 N)

Problem D: The angular velocity will be faster by 0.185 radians per second or 0.031 revolutions per second; the angular velocities of interest are 2.347 and 2.542 radians per second, or 0.374 and 0.405 revolutions per second.

Problem E: The masses of ladies range from 59.9 kg to 64.2 kg.

Problem F: The radius would become smaller; 1.77 m - 1.7 m = 0.07 m = 7 cm.

Problem G: The range of tensions is between 300 N and 350 N; students must first find that the angular velocity in radians per second of interest are 1.360 and 1.515 respectively.

Problem H:

a) The maximum radius is 1.9 meters; students must first convert 132.28 pounds to 60 kg.
b) The centrifugal force is 477.90 N.