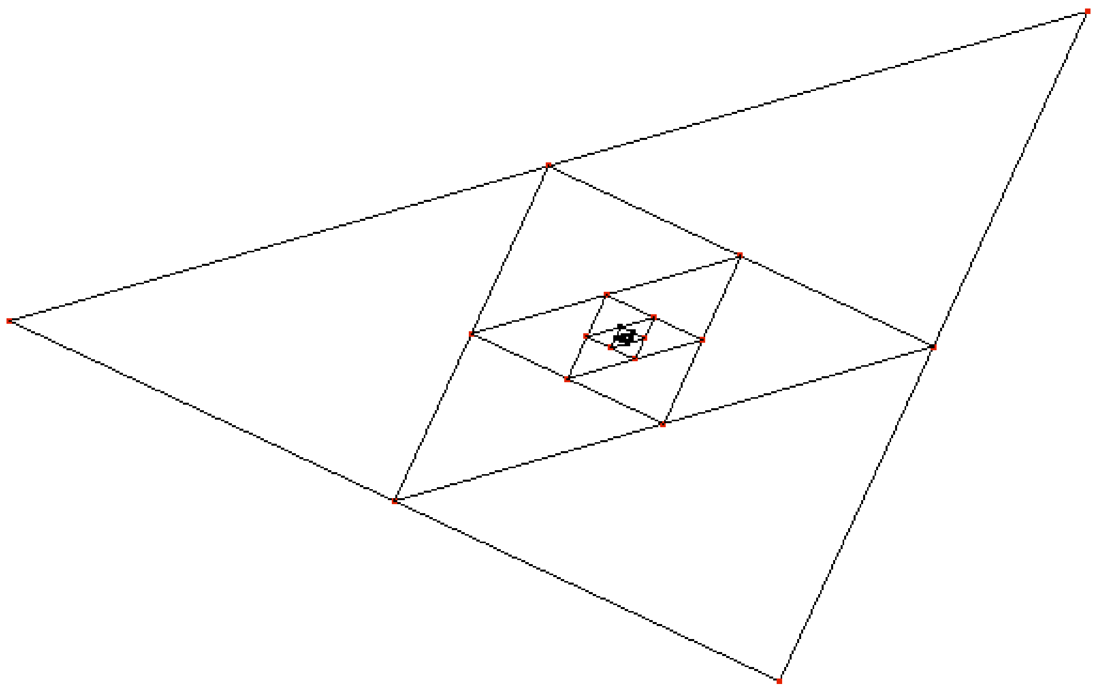
The Centroid



In This Issue:

- **Five Years of NCCTM Trust Fund Scholarships**
- **Closed-Form, Recursion, and Mind-Reading: Defining Sequences by Various Means**
- **Florence Nightingale, the Passionate Statistician**
- **Mathematics: How Sweet it is**
- **Discovering Mathematics**
- **2004 Math Fair Winners**
- **2004 Logo Contest Winners**

Official Journal of the North Carolina Council of Teachers of Mathematics
Volume 30 • Number 2 • Fall 2004

The Centroid

OFFICIAL JOURNAL OF THE NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS
VOLUME 30 • NUMBER 2 • FALL 2004

Articles

News & Information

From the Editor

The New North Carolina Curriculum

Holly Hirst
Appalachian State University
Boone, North Carolina

Another school year is upon us, and many teachers are facing new challenges in the classroom brought on by preparing for the new North Carolina mathematics curriculum. In particular, the grades 9-12 courses in Discrete Mathematics and Advanced Functions and Modeling are ramping up in preparation for the new college admission requirements that start effective fall 2005: All students applying for admission to UNC colleges and universities will be required to have four course units of math, instead of three.

The Centroid would like to provide a venue for exchange of ideas among these teachers, and so we are asking those of you with class-tested ideas to send us a write up! We will help you prepare it for the 2005 issues of the journal. Contact us for more information at the e-mail address provided.

We hope you enjoy the articles in this issue. Two of the articles (*Discovering Mathematics* and *Closed-Form, Recursion, and Mindreading*) deal with issues related to sequences and recursion – topics in Goal 3 of the discrete mathematics course. The Women and Minorities column highlights nurse Florence Nightingale's contributions to statistics. *Mathematics: How Sweet it is* examines children's literature that uses candy to assist with learning math.

In honor of NCCTM's 35th year in 2005, we have included an article recapping the history of the NCCTM Trust Fund Scholarship program, which celebrates five years of operation in 2004.

As always, we encourage you to consider assisting with *The Centroid* by:

- **submitting a manuscript** – general articles are welcome, as are the following special categories of articles: *A Teacher's Story*, *History Corner*, *Teaching with Technology, It's Elementary!*, *Math in the Middle*, and *Algebra for Everyone*.
- **becoming a reviewer** – please send e-mail to me if you are interested in helping in this way.
- **serving on the editorial board** – we are interested in adding several K-12 teachers to the board. If you are interested in serving on the board, send me e-mail.

Contact information. Feel free to contact us at any time with submissions, news items, questions, or concerns.

Holly Hirst <HirstHP@appstate.edu>
Brian Felkel <FelkelBH@appstate.edu>
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
828-262-3050

Mini-Grants from NCCTM

The mini-grant program is designed to promote excellence in mathematics education. There are no preconceived criteria for projects except that students should benefit from the grant. Possible projects for consideration include math clubs, field days, contests, workshops for parents, math activities, math laboratories, and research topics. A total of $6,000 will be awarded in each of the three NCCTM regions. Applications will be accepted only from persons who are NCCTM members as of 1 September 2004.

Completed applications must be received by 15 September 2004 to be considered. For more information and submission guidelines, contact Phyllis W. Johnson by email at <pwjohnson210@earthlink.net> or by phone at 252-752-1796.

Presidents' Messages

Jan Wessell
State President

Welcome to a new school year! New mathematics programs along with a revised mathematics curriculum will bring challenges to all of us. I am confident that with appropriate staff development, discussion groups, lesson study groups, and trial and error, each of us will cultivate a repertoire of good instructional techniques to enhance the teaching and learning of mathematics.

I believe that more than anything, it is important to work within collegial groups. Just as our students learn from each other, so do we. Recently there has been research and emphasis on collegial groups also known as lesson study groups. One such group is the Lesson Study Research Group (LSRG) housed at the Columbia University Teachers College <http://www.teacherscollege.edu/lessonstudy/>. The activities of the LSRG include sharing information about how lesson study is conducted, exploring how lesson study can be adapted for the U.S. educational context, developing tools and materials for supporting lesson study practice in the U.S., and gathering information about the impact of lesson study on teachers, students, schools, and the U.S. education system.

Lesson study is more than looking at a variety of lessons. The purpose of lesson study is to dissect how students learn. The Lesson Study Group at Mills College <http://www.lessonresearch.net/> has been working with teachers for several years and has developed specific elements of lesson study.

These are just two of the many research projects taking place across the United States. My challenge to you is to do your *own* research and to develop your *own* lesson study group. A new mathematics curriculum and new mathematics programs offer us a platform for change and a starting point for lesson study. Once you are immersed in the process, tell us about it. Write an article for *The Centroid* about your experiences or share your experiences at a conference. Start the new school year off right! Take the challenge.

Vickie Moss
Central Region President

The Central Region held its spring conference and Math Fair at Greensboro College on March 27. The target population of the conference was lateral entry and first year teachers. We had 75 teachers registered for the conference, and more than 25 students from Greensboro College assisted with the Math Fair. Opening sessions were presented by the NCDPI staff, who gave an overview of the newly revised Standard Course of Study (SCS). Grade specific sessions followed that focused on strategies for implementing the SCS and best classroom practices.

At the conclusion of the conference participants visited the math fair exhibits and attended the awards ceremony. It was a great opportunity to partner with teachers, parents, and students and to support quality math education in our region. We appreciate the use of the facilities at Greensboro College and the help its staff and students provided to make the conference a success.

We congratulate Jeane Joyner, our incoming state president, and Emogene Kernodle, who was recently elected as incoming Central Regional president. Next spring the Central Region plans to join the Eastern Region for the spring conference.

Kathryn Hill
Eastern Region President

Greetings members of the Eastern region. I hope you had a wonderful summer and are ready to begin another exciting school year.

We had a very successful conference this past spring at Meredith College. Approximately 120 teachers attended the sessions led by representatives from DPI on upcoming changes in the curriculum across the grade bands. Other topic specific workshops gave teachers that extra boost to finish out the school year with some new ideas to use in the classroom. Many thanks go to Lucy Kay and Jane Gleason and the members of their committees for all the hard work that went into making the 2003 Spring Conference such a great success.

The Eastern and Central region officers are currently planning a joint 2005 Spring conference. The conference will be held in the Chapel-Hill/Raleigh area and the theme will be "Algebra for All." If you are interested in helping out with the planning or with presenting at the spring conference, please let us know. We are excited about the opportunity to hold a joint conference for the members of these two regions.

As the new school year begins, please keep in mind that there are many opportunities for our students to participate in math contests around the state. Check the North Carolina Department of Public Instruction web site to find out how your school can register a team for competition. Even though the contests will not be held until the spring of 2005, now is the time to get your teams together for practice sessions throughout the year.

The State Math Conference will be held October 7-8 at the Koury Convention Center in Greensboro. What a great time to exchange ideas with old friends, to meet new educators, and to get new ideas for the classroom. I hope to see many of you there! Have a great fall.

Betty Long
Western Region President

I hope each of you had a wonderful summer with time to relax and enjoy your favorite things. I want to take this opportunity to thank everyone who helped to make the Western region spring math conferences and math fair successes. This includes the Western region officers and others who helped with these events as well as all those who attended. A total of 336 in-service and pre-service teachers attended the math conferences, and 136 students entered projects in the math fair. I am delighted that so many people participated, and I hope each of you benefited from the experience.

Now our thoughts turn to the new school year. Be sure to put the State Mathematics Conference on your calendar. The conference will be held on October 7-8 at the Koury Convention Center in Greensboro. Check the NCCTM website <http://www.ncctm.org> for more details. There will be lots of workshops and sessions on a wide variety of topics.

Another NCCTM opportunity for teachers is the mini-grant program. In October, NCCTM will award $5000 per region in grants to promote excellence in mathematics education. We usually receive only a few applications. I encourage you to submit a proposal if you have planned a class project or activity which might not otherwise get funded. The application deadline is September 15. E-mail Phyllis Johnson <pwjohnson210@earthlink.net> or visit the NCCTM website for more information.

The Western region officers are planning the 2005 Western Region Mathematics Conference for late February. The focus will be on classroom assessment in the context of the recent revision of the North Carolina Standard Course of Study for Mathematics. In addition to the sessions on assessment, we plan to offer other sessions and workshops dealing with topics of high interest to pre-service and first-year teachers. More specific details will appear in the next issue of *The Centroid*. In the meantime, if you have questions about the spring conference, please contact one of the Western region officers.

The 2005 Western Region Math Fair will be on Saturday, March 19 in the Plemmons Student Union at Appalachian State University. Any student attending school in North Carolina can enter a project in one of the following categories: K-2, 3-5, 6-8, 9-12. The registration form with more details will appear in the next issue of *The Centroid* and will be mailed to all NCCTM members no later than December.

In addition to the above-mentioned activities, NCCTM sponsors other events such as the logo contest and the math contest. I want to encourage you to consider getting involved in one or more of these very worthwhile activities. I think you will find it very rewarding. If you would like to serve on an NCCTM committee or have suggestions, questions, or concerns, please contact me at (828) 262-2372 or <longbb@appstate.edu>. I want and need your input. Only by working together can we continue to improve the mathematics education of our students.

It's Elementary!

Mathematics: How Sweet it is

Leslie Anne Perry[1]
East Tennessee State University
Johnson City, Tennessee

Math is everywhere—even in candy! And children's books about candy are interesting tools for teaching and reinforcing essential mathematics skills and concepts. Below is an annotated bibliography with 16 children's books that are appropriate for a range of age and grade levels. These books are all available in paperback and can be obtained from children's book clubs, bookstores, or school and community libraries.

Classroom Use

When using these books in a classroom, it is not necessary for the children to have candy while the books are being read. A teacher may prefer to read an entire book and then distribute candy for the children to use as manipulatives when selected pages of the book are re-read. An alternate approach would be for the teacher to use pieces of candy on an overhead projector to demonstrate math concepts as selected pages are re-read. The children could then be given a piece of candy, such as a Hershey's Kiss or a piece of Twizzlers candy, at the conclusion of the lesson.

Mathematics activities for these 16 books are not restricted to the ones described within the books themselves. Teachers are free to use their imaginations and incorporate additional activities appropriate for their students. Remember: candy + literature + math = LEARNING. Math never tasted better.

Editor's Note: Some schools prohibit the use of candy in teaching. The lessons in these books may still be used quite effectively with other manipulatives, such as tiles, coins, and fraction bars. Just make sure the children don't eat the "candy" in this case!

Annotated Bibliography

Hirschmann, K. (2002). *Math Magic*. New York: Scholastic.
Pieces of NECCO Sweetheart candy are used to demonstrate concepts related to addition, multiplication, and division. The candy is first sorted by color, and then a variety of equations are created.

Hutchings, A. and R. Hutchings. (1997). *The Gummy Candy Counting Book*. New York: Scholastic (Cartwheel).
Using a rhyming text, and pictures of various types of gummy candy, this book illustrates sets from 1 to 12. A set of 12 is then divided into various smaller sets.

McCourt, L. (1999). *Candy Counting: Delicious Ways to Add and Subtract*. Mahwah, NJ: BridgeWater Books.
This book contains 14 addition and subtraction word problems that are demonstrated with various types of candy. Illustrated solutions to the problems are provided in the back of the book.

[1] *Dr. Perry is Associate Professor of Curriculum and Instruction at East Tennessee State University. Her specialty is reading in elementary education, and she has published more than 20 articles on elementary education and teacher preparation.*

McGrath, B. B. (1994). *The M & M's Counting Book*. Watertown, MA: Charlesbridge.

In this book, sets from 1 to 12 are created, then the set of 12 M & M's is grouped into various smaller sets. Next, the 12 M & M's are used to form a square, circle, and triangle. Subtraction facts are also demonstrated.

McGrath, B. B. (1998). *More M & M's Math*. Watertown, MA: Charlesbridge.

M & M characters guide the reader as he/she estimates the number of pieces of candy in a bag of M & M's, sorts them by colors, and places the candy in the appropriate spaces on a prepared graph. The reader is instructed to add the number in each row and determine how close his/her original estimate was. Then the pieces of candy are used as manipulatives for various division and multiplication problems. Ordinal numbers 1st through 6th are also illustrated.

McGrath, B. B. (2000). *Skittles Riddles Math*. Watertown, MA: Charlesbridge.

Number sentences are shown as various sets of Skittles are added or subtracted. A number line is used for subtracting a larger number from a smaller number, and pie graphs are used to illustrate various fractions. A numerator and denominator are demonstrated, as are equivalent fractions when fractions are reduced to their lowest terms.

McGrath, B. B. (2002). *The M & M's Color Pattern Book*. Watertown, MA: Charlesbridge.

The M & M's are sorted by colors, then several simple patterns are presented. Addition is demonstrated when 4 pieces of each of the 6 colors are added together to get 24. Two additional patterns are shown, then pieces of candy are used to form a circle. Multiplication is demonstrated with sets of M & M's, as is skip counting by two and even and odd numbers. New patterns are also created.

Pallotta, J. (1999). *The Hershey's Milk Chocolate Fractions Book*. New York: Scholastic (Cartwheel).

This book features a 12-square chocolate bar and a painting theme and teaches concepts related to the fractions 1/12 through 13/12. Twelve additional fractions are demonstrated. The book also includes explanations for these terms and concepts: equivalent fractions, numerator, denominator, simplifying a fraction to its lowest terms, and improper fraction.

Pallotta, J. (2000). *Reese's Pieces: Count by Fives*. New York: Scholastic (Cartwheel).

Using Reese's Pieces candy and a construction site for the setting, this book first has the child reader count from 1 to 10. Then the reader is guided as he/she counts by 5's to 100. On each page, some type of vehicle is used to help count the pieces of candy.

Pallotta, J. (2001). *The Hershey's Kisses Addition Book*. New York: Scholastic (Cartwheel).

This book uses Hershey's Kisses and a clown theme to teach concepts related to 21 addition equations. Seventy-two other equations are presented in list form. The book also includes explanations for these terms and concepts: plus sign, minus sign, equals sign, equation, addition, sum, arithmetic, reversing the equation, double-digit number, addend, vertical equation, integer, and subtraction.

Pallotta, J. (2001). *Twizzlers Percentages Book*. New York: Scholastic (Cartwheel).

Twizzlers candy, along with mini spaceships, illustrates concepts related to fraction, decimal, and percentage. These symbols are demonstrated: division sign, decimal point, percentage symbol, equal sign, and multiplication sign. The base 10 system is explained, as is place value. Instructions are also provided for computing a percentage and candy is used to illustrate various percents. Batting averages are explained as is *rounding off*.

Pallotta, J. (2002). *The Hershey's Kisses Subtraction Book*. New York: Scholastic (Cartwheel).
Using Hershey's Kisses and a clown theme, this book illustrates 12 subtraction equations and 1 division equation. One hundred additional subtraction facts are presented in list form. Explanations are given for these terms and concepts: minus sign, equal sign, equation, difference, counting down, addition, minuend, subtrahend, comparison subtraction, zero, negative number.

Pallotta, J. (2002). *The Hershey's Milk Chocolate Multiplication Book*. New York: Scholastic (Cartwheel).
A 12-square milk chocolate bar, along with a painting theme, is used to teach a variety of concepts. Twenty-two multiplication equations are demonstrated, and 121 additional multiplication facts are included in list form. Within the text are explanations for these terms and concepts: grid, array, multiplication sign, equals sign, equation, factor, product, horizontal equation, vertical equation, commutative property of multiplication, square number, multiplicand, multiplier, and associative property of multiplication.

Pallotta, J. (2002). *Hershey's Milk Chocolate Weights and Measures*. New York: Scholastic (Cartwheel).
A variety of concepts and terms are presented in this book through the use of Hershey's products and pictures of clowns. Included are: inch, foot, yard, mile, centimeter, decimeter, meter, ounce, pound, ton, milligram, gram, kilogram, metric ton, fluid ounce, cup, pint, quart, gallon, milliliter, centiliter, liter, kiloliter, second, minute, hour, day, week, year.

Pallotta, J. (2002). *Twizzlers Shapes and Patterns*. New York: Scholastic (Cartwheel).
In this book, an architect and child construction workers/artists are used to present a variety of geometry concepts. Twizzlers candy is used to demonstrate a line, angle, triangle and circle. Candy is also used to form a right angle, parallel lines, a square, oval, rectangle, pentagon, hexagon, heptagon, octagon, nonagon, decagon, and several patterns.

Pallotta, J. (2003). *Hershey's Kisses Multiplication and Division*. New York: Scholastic (Cartwheel).
For this book, Hershey's Kisses and a sports theme are used to present various concepts related to multiplication and division. Twenty-one equations are demonstrated and the multiplication tables (101 facts) and division tables (102 facts) are presented in list form. Also, included within the text, are explanations for these terms and concepts: multiplication sign, equal sign, equation, factors, product, square number, vertical equation, division sign, dividend, divisor, quotient, square root, factor family.

Did you know?

Using the Library of Congress to Find Book Citations

Looking for books by a particular author, on a particular topic, with a particular title? The United States Library of Congress is a great place to start your search. The website for the catalog search is very easy to use, and virtually all published books have been recorded in the Library.

For example, using the basic search feature and specifying "Author/Creator Browse" as the type of search allowed us to find many books by Jerry Pallotta (typed into the text box as "Pallotta, J"). In fact, listed were 50 children's books, with many involving math concepts – and not just about candy! Books are easier to find in stores if you are armed with title and publisher information.

<www.catalog.loc.gov>

News From DPI

New Mathematics Standard Course of Study

Bill Scott
North Carolina Department of Public Instruction
Raleigh, North Carolina

The revised *Mathematics Standard Course of Study (SCS),* adopted by the State Board of Education in March 2003, will continue to be the focus of the NCDPI Mathematics staff.

- The NCDPI Mathematics staff will have information sessions at the State Mathematics Conference in October and at all of the spring NCCTM Regional Mathematics Conferences to share the revised *SCS*, highlight changes, and identify NCDPI resources that support the revised *SCS*.
- It is extremely important that teachers and administrators become familiar with the revised *SCS* since grades K-8 will be implemented in the 2004-2005 school year.
- In the 2004-2005 school year, the state testing program will be field testing in grades 3-8 and the revised National Assessment of Educational Progress (NAEP) will be administered.
- The *Mathematics Transition Document for Grades 3-8* is available for use in classrooms during the 2004-2005 school year. It includes all of the objectives from the 2003 Mathematics *SCS* as well as skills and concepts from the 1998 Mathematics *SCS* that need to be addressed in each teacher's instructional program for 2004-2005.
- The grades 9-12 *SCS* will be implemented in the 2005-2006 school year and the state's testing program will field test Algebra 1, Geometry, and Algebra 2 that year.
- *Indicators for K-12 Mathematics* is the new support document that will accompany the revised *SCS*. The *Indicators* document includes concepts, skills, vocabulary, and sample problems that illustrate each objective.
- Existing NCDPI mathematics resources will be updated as time allows in order to reflect changes in the *Mathematics SCS*.

Please visit the LearnNC web site <www.learnnc.org/dpi/instserv.nsf/Category7> regularly. The site is organized around the following categories:

Announcements/Calendar: the latest information about NCDPI professional development opportunities.

Instructional Resources: downloadable pdf files of NCDPI mathematics resources aligned with the 1998 *SCS*.

Mathematics Curriculum: 1998 *SCS*, 2003 *SCS*, *Mathematics Transition Documents* and working drafts of the *Indicators for K-12 Mathematics* in several downloadable formats.

Resources for the 2003 Standard Course of Study: new and revised resources aligned with the 2003 *SCS*, along with *Observation Profiles*, *Resources for Mathematics K-8* (formerly the Strategies books) and *Indicators for K-12 Mathematics* as they are completed.

NCTM 2005

The National Council of Teachers of Mathematics 2005 Annual Meeting and Exposition will be in Anaheim, California, Wednesday, April 6 through Saturday, April 9, at the Anaheim Convention Center, the Anaheim Marriott Hotel, and the Hilton Anaheim Hotel. The conference theme is *Embracing Mathematical Diversity*. < www.nctm.org/meetings/anaheim/>

Five Years of NCCTM Trust Fund Scholarships

Bill Paul[1]
Sugar Grove, North Carolina

For the past 34 years, the North Carolina Council of Teachers of Mathematics has been instrumental in providing services to North Carolina teachers. Whenever possible, new and innovative ideas have been considered and initiated. Several projects involving considerable financing are currently in effect, but none has been as extensive as the establishment of the NCCTM Permanent Trust Fund five years ago.

The History of the Fund

During the late summer of 1999, informal discussions were begun to consider how accumulated funds might best be utilized to assist teachers. Officers of the organization had often dreamed of a Trust Fund to support teacher projects in mathematics education. The Board of Directors had this topic as an agenda item on several occasions during the past 15 years; however, there was simply not enough money in the NCCTM coffers to fund such a venture. Now, for the first time, with higher than expected income from several conferences and an accumulation of gifts made to the organization as memorials or to honor members, NCCTM found itself in the unique (and satisfying) position to bring this long-standing desire to fruition.

The next step was to present a proposal to the Board. At the October 1999 Board Meeting a recommendation was made to set aside $110,000 into a separate account with income from the investment to be used to support mathematics education projects in North Carolina. After considerable discussion and a thorough examination, the recommendation was unanimously approved. The NCCTM president was to appoint six Trustees who were to establish guidelines for the Trust, determine how the funds were to be invested, and develop procedures for awarding grants. This was no small task. The Trustees appointed were Diane Frost, Bill Scott, Ron Hann, Bob Joyner, Sue Sams, and Bill Paul. All of these members had many years of service to the organization and needed to meet as soon as possible in order to report their activities to the Board in April.

On November 6, 1999 the Trustees met in Asheboro to hash out the details as charged and worked that day until a plan took shape. With funds available to be invested immediately, the committee was excited when they realized that income would be sufficient by early the next spring to make the initial awards. At that time, fall of 1999, investment rates were still at a high level and our conservative estimate of 6% would provide enough funds for six $500 scholarships semiannually. The funds were conservatively invested, but at a fairly high rate of interest. This was to be the first step in what the Trustees hoped would some day lead to many types of grants, if our Trust Fund continued to grow.

The trust committee created two documents. The first and most important was a set of guidelines named the *NCCTM Permanent Trust Fund*, which established procedures and terms of office for Trustees, determined investment and spending policies, and created a process for awarding grants. The second document was an application form. Teachers could apply for a scholarship to do graduate work in Mathematics Education. This form was to be distributed quickly to all NCCTM members. One key element was that applications would be reviewed after March 1 and October 1 each year although submissions could be made at any time, and the course(s) had to be in mathematics or mathematics education.

These two documents were presented to the Board of Directors in April. The Trust Fund Chair was to be an ex-officio member of the Board, and the Trust was now fully established. The first group of applications was reviewed in April and six scholarships were awarded.

Even through turbulent investment times, which began shortly after the fund was

[1] *Bill Paul is Professor Emeritus of Mathematical Sciences at Appalachian State University. He has been active in NCCTM since its inception and currently serves as the Chair of the NCCTM Trust Fund.*

created, sufficient funds have always been available to finance scholarships. In order to continue these scholarships in perpetuity, a key restriction was placed on the Trust, which was that only the interest income could be used and thus the original investment is preserved. Also the NCCTM membership form has been revised so that members are given the opportunity to donate to the Trust.

The Trust Fund Today

Through March 2004, 51 scholarships have been awarded. A teacher may receive this scholarship twice, and several have. Recipients are indicated on the North Carolina map below, and from that we can see that this Trust is indeed available statewide.

In order to address the effectiveness or success of this Trust, previous winners were asked to comment whether or not this was a viable Trust. All of the responses were very positive; a few comments are printed below. Many of the replies cited the invigoration of learning through reflecting on experience, and many noted the financial impossibility of graduate school without help from the trust. Clearly, through this Trust, NCCTM is making a positive impact on North Carolina educators and North Carolina's schools.

If you are considering graduate school in mathematics, please note that Fund scholarship application forms are available through the NCCTM website, on the next page of this issue of *The Centroid* or by contacting the Trust Chair, Bill Paul <bnpaul@skybest.com> or (828) 297-3839.

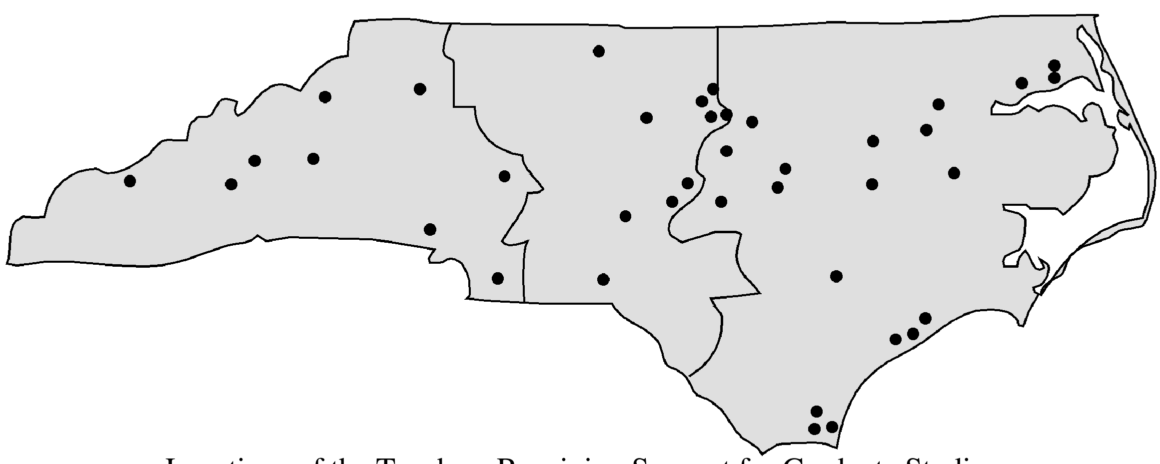
It was a very interesting and useful experience to be both student and teacher at the same time. I found myself examining both roles. – Virginia Johnson

Helping educators receive a degree or learn something new in mathematics helps restore the love of learning mathematics, which then gets passed on naturally through example to our students.
– Christine Kreider

Not only does this scholarship opportunity promote teachers' interest in selecting math courses within their graduate career, but it produces lasting effects in our classroom. – Heather Neal

With the help of this trust, I was able to obtain my master's degree, take classes to enhance my teaching skills, and learn more ways to better prepare my mathematics students for their future.
– Betty Jo Roby

I am so thankful that NCCTM is committed to higher education for classroom teachers. We should never stop learning! – Tina Starling



Locations of the Teachers Receiving Support for Graduate Studies
Through NCCTM Trust Fund Scholarships

NCCTM Trust Fund Scholarship

North Carolina Council of Teachers of Mathematics

$500 scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction.

Applicants must be:
- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting this application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Currently enrolled in a mathematics or mathematics education course, or have completed a mathematics or mathematics education course within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are:
- **March 1**
- **October 1**

Send completed applications to:
NCCTM Trust Fund Chairperson
P.O. Box 121
Sugar Grove, NC 28679

Direct inquiries to:
Bill Paul, Chairperson
Phone: (828) 297-3839
E-mail: bnpaul@skybest.com

(Please print all information.)

PERSONAL INFORMATION:

Name:____

Home address:____

Home phone:____________________ Home e-mail:____________________

NCCTM membership number:____

EMPLOYMENT INFORMATION:

How many years of teaching experience?____

Currently employed in what school system?____

School name:____

School address:____

School phone:____________________ School e-mail:____________________

Current teaching assignment:____

Principal's name:____

COURSE INFORMATION:

Institution of higher education:________________________________

Graduate degree program in which you are currently enrolled:____________________

Course name:________________________ Course number:________

Dates of enrollment: *(circle one)* Fall semester Spring semester Summer session Year:______

Name of course instructor:__________________________________

PROFESSIONAL ACTIVITIES WITHIN PAST 5 YEARS:

BRIEF STATEMENT OF FUTURE PROFESSIONAL GOALS:

REQUIRED SIGNATURES:

Applicant's Signature:________________________Date:_________

Principal's Signature:________________________Date:_________

Instructor's Signature (if currently enrolled):________________Date:_________

REQUIRED ATTACHMENTS:
Please attach a copy of verification of acceptance and enrollment in accredited graduate program in North Carolina.

NOTE: Applications must be complete to be considered. If your application is approved, an official course grade report must be submitted to verify successful completion of the course before scholarship funds will be issued.

Women and Minorities in Mathematics

Incorporating Their Mathematical Achievements Into School Classrooms

Florence Nightingale, the Passionate Statistician

Jill T. Richie[1] and
Sarah J. Greenwald[2]
Appalachian State University
Boone, North Carolina

Most American schoolchildren are familiar with the legend of the "Lady with the Lamp." Florence Nightingale's contributions to statistics are less well known than her nursing career, but they are still highly noteworthy (Spiegelhalter, 1999), since much of her celebrated success in nursing reform would not have been possible without them. In addition, not until the middle of the twentieth century would there be another woman statistician who achieved a similar level of recognition and influence as Florence (Salsburg, 2001). Incorporating her mathematical achievements into the classroom can be especially rewarding because, unlike many other women mathematicians, students are already familiar with Florence. Examining her mathematics in the context of her other achievements also provides a rich arena to highlight connections to other subject areas.

During the Crimean War, Florence Nightingale revolutionized the profession of nursing. She and her nurses dramatically raised standards of patient care, including sanitation and nutrition, which drastically reduced deaths from infection and disease in military hospitals. Her efforts inspired noted author Henry Wadsworth Longfellow to create a poem that cemented her image as "the lady with the lamp" (Dossey, 2000).

Florence realized that the problems she found in Crimea were not limited to wartime hospitals. Armed with statistical data she had collected, Florence turned her attention to large-scale reforms of public health. It was Florence's devotion to the systematic collection and vivid presentation of data to further social change that led her biographer Sir Edward Cook to dub her the "Passionate Statistician" (Cook, 1913).

Florence's Early Life

Florence Nightingale was born in 1820 in Florence, Italy, to wealthy British parents. The Nightingales were among the "upper ten thousand"; the economically and socially advantaged segment of society that ruled England. Both she and her sister Parthenope were thoroughly schooled in reading and writing by their governess. At seven years of age Florence was already a prolific and keenly observant letter-writer; by the age of ten she had written her own autobiography in French. She was also noted in her family for compiling very detailed lists of nearly everything around her, many of which appeared in her journals and letters.

Beginning at age twelve, Florence was educated by her father William Nightingale. As a devout Unitarian, her father rejected the philosophy that women were too fragile for the rigors of higher education. Florence studied history, politics, ethics, mathematics, philosophy, and composition. She also had a particular aptitude for languages, becoming fluent in Latin, Greek, French, German, and Italian. At the same time, young Florence learned all the manners and household skills expected of privileged Victorian girls. She was formally presented to Queen Victoria in May 1839 (Dossey, 2000).

---

[1] *Dr. Richie is Assistant Professor of Statistics at Appalachian State. Her interests include statistics education and applications of statistics in science.*

[2] *Dr. Greenwald is Associate Professor of Mathematics at Appalachian State. A member of the Association for Women in Mathematics, she has long been interested in the contributions of women and minorities.*

Florence's studies under her father were equivalent to a Cambridge college education (Dossey, 2000). At the time, only men were allowed to attend. When she was twenty, she had reached the limits of William Nightingale's mathematical ability (Diamond and Stone, 1981). Her insistence on further instruction from an outside source was met with resistance from her mother. Here Florence got support from her Aunt Mai, who located willing, educated tutors for her niece. One was James Sylvester, who is noted in mathematics for developing the theory of invariants with Arthur Cayley (Stinnett, 1990). Florence learned algebra, geometry, and arithmetic, which she in turn taught to several children before starting training as a nurse (Lipsey, 1993).

Along with her mathematics, Florence did a self-guided course in social and health statistics. The creation of the first register of births, deaths, and marriages (when Florence was seventeen) by the General Registry Office was an unprecedented event that made social statistics a popular topic for conversation (Florence Nightingale Museum Trust, 2003). Prior to this, such records (if they existed) were usually in the hands of local church parishes. Florence also studied hospital blue books and any other data she could obtain. One of her most ambitious projects done in 1853, was to gather her own data by sending questionnaires to hospitals regarding health administration, which she then laboriously analyzed (Diamond and Stone, 1981).

Florence the Nurse

In nineteenth century England, nursing was not considered a suitable profession for respectable women. Nonetheless, feeling called by God, Florence pursued it. When she was 29, she rejected a marriage proposal from her longtime suitor Richard Monckton Milnes. Unfortunately, despite progressive attitudes about education, her parents still felt marriage was necessary for a woman of their class, and they turned deaf ears to her request to study nursing. Two years later, a meeting with Elizabeth Blackwell, the first woman to earn a medical degree in the United States, caused Florence to reassert her position. She was finally permitted to go to Germany for training (Dossey, 2000).

Florence became the head of a charitable hospital for sick gentlewomen. Women of Florence's class were expected to perform volunteer service, and the clients of the home were women from good families. During this period her social connections let her meet and interact with doctors who were on the forefront of medical innovation. Among them was John Snow, who used the new Registry Office data and map plots of cholera deaths to establish that cholera is a water-borne illness (Dossey, 2000). Though it is not documented, one can only imagine that this effort impressed Florence, who was extremely interested in this kind of data use.

Florence was also deeply involved in the treatment of cholera patients, taking a leave of absence from her position to do so. Then, in 1854, Florence was appointed Superintendent of the Female Nursing Establishment in the English hospitals in Turkey by the Secretary of War. This was an unprecedented honor for a woman. With 40 volunteer nurses, she went to the military hospital at Scutari and from there into the public eye and history books.

Florence the Statistician

Florence's fascination with statistics and her desire to nurse were both present from her childhood. In the Crimea she found the opportunity to meld her two passions into a single pursuit. One of her first acts was to institute uniform statistical record keeping to replace haphazard and contradictory military journals (Kopf, 1916). These data would form the foundation of her later work when she returned home. The Royal Commission, charged with formalizing Florence's reforms, would come to rely heavily on her statistical analyses, and it had as one of its goals the establishment of a "statistical department for the army" (Dossey, 2000). She also pressed for more uniform data collection on the civilian front, both in hospitals and in the national census. Both of these proposals were "too advanced for their time" (Dossey, 2000; Kopf, 1916) since hospitals were not used to the rigorous collection of data, and so her proposals were either implemented briefly in some hospitals or not at all.

In an era where even statisticians believed that statistics should be "the driest of all readings," (Heyde and Seneta, 2001), Florence drew upon her extensive training and became an innovator in standardized data collection, tabulation, and graphical displays (Stinnett, 1990). Florence thought that both visual and auditory communication was necessary to understand the real meaning of data: "the diagram which is to affect thro' the Eyes what we may fail to convey to the brains of the public through their word-proof ears" (Diamond and Stone, 1981). Her closest associate and collaborator for twenty years was medical statistician William Farr, who shared her practical view of applied statistics, though he remained rather critical of what was at the time an unconventional reliance on graphs (Diamond and Stone, 1981). She was also influenced by the Belgian statistician Adolphe Quetelet, considered to be the founder of social statistics (Dossey, 2000).

Florence's most well-known invention was the polar area chart (erroneously referred to as "coxcomb" charts in some texts), an example of which is shown in Figure 1. Very few significant precedents existed for this kind of display (Kopf, 1916).

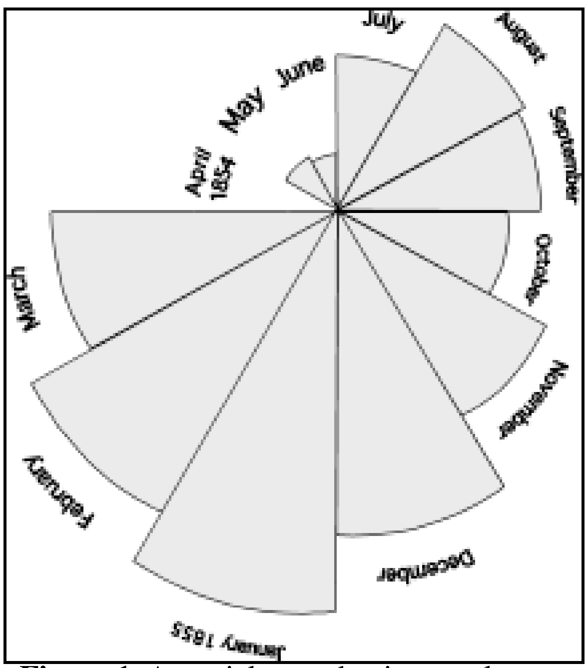


Figure 1. A partial reproduction a polar area chart showing military fatalities due to disease in the Crimean War

Polar Area Charts

Polar area charts are variations on the pie charts first used by William Playfair in 1801 (Small, 1998). In a pie chart, the circle or "pie" has a constant radius. The total area is proportionally divided among categories to show their relative frequencies. For example, if one category has twice as many items in it as the others, then its piece in the pie chart is twice as large as any other piece.

In a polar area chart, the circle is divided into angles or "wedges" of the same size for each category. The radii of the wedges vary. Each radius is equal to the square root of the frequency for that category. The square root is used for the radius because the area of a circle is πr^2; using the square root of the frequency for radius means that the *areas* of wedges in the polar area chart are still proportional to one another, like in the pie chart.

Table 1 shows the numbers of gold, silver, and bronze medals the United States won in the first four modern summer Olympic games.

Table 1. US Olympic Medals 1896 – 1908

Olympics	Gold	Silver	Bronze
1896	11	6	2
1900	20	14	19
1904	80	86	72
1908	23	12	12

Figure 2 shows a standard pie chart for the percentages of each the three medal types the United States won in the 1896 Olympics. We would need a separate pie chart for each year if we wanted to compare all four years.

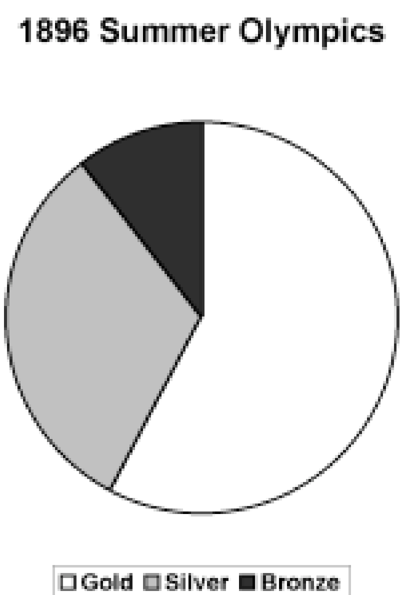


Figure 2. Pie Chart for US Medals in the 1896 Summer Olympics

Figure 3 shows a polar area chart for the same data for all four years. Each quadrant or quarter of the circle represents one Olympics. The radius of each colored slice or wedge is the square root of the gold, silver, or bronze medal count, and the three slices for each year are stacked on top of each other. As in the pie chart, gold medals are shown in white, silver medals in gray and bronze medals in black.

The advantage of the polar area chart for this data is that it displays not only the relative percentages within each year, but it also gives an indication of the total number of medals won, something the pie charts do not do. In 1904, for example, the very large wedges show that far more medals were won in that year than in any of the other three years. The relative size of the wedges also shows that the U.S. won nearly equal percentages of gold, silver, and bronze that year. In 1908, the totals for silver and bronze were the same, and this is represented by the striped wedge.

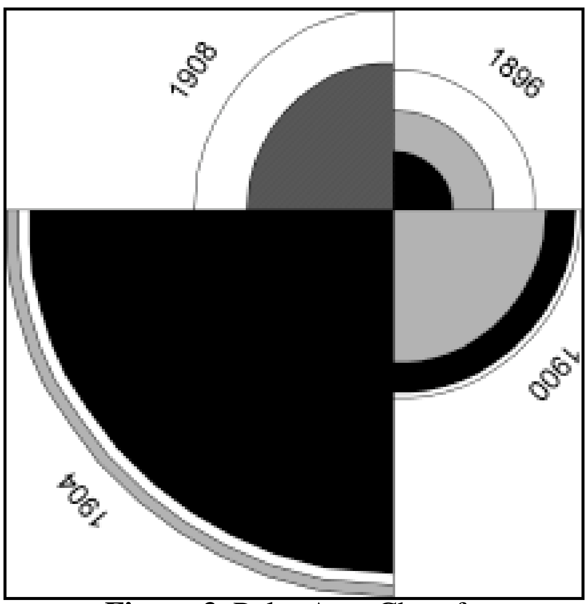


Figure 3. Polar Area Chart for US Medals in the 1896, 1900, 1904, and 1908 Summer Olympics

Though they were innovative and dramatic, polar area charts were subsequently replaced by pie or bar charts in common usage because of the laborious hand calculations that were required to find square roots in the days before calculators. Figure 3 requires 12 square root calculations. Many of Florence's diagrams representing yearly data about military deaths due to disease, wounds and other causes had 36 square roots, three for each month. Then, the charts themselves had to be drawn by hand with rulers and curved templates, making pie and bar charts more attractive options in terms of time and effort.

Technology was a common problem for statisticians of the late 19th and early 20th centuries: Ideas about how to summarize and visualize large amounts of data effectively often preceded the ability to actually implement those ideas. Modern computers allow people to make a wide range of graphs with a few clicks of a mouse, so statisticians are now revisiting many of the ideas first proposed a century ago.

Florence's Legacy

Florence Nightingale was the first woman elected as a Fellow of the Royal Statistical Society, and she was also made an honorary member of the American Statistical Association. She received the St. George's Cross from Queen Victoria for her *Notes on Matters Affecting Health, Efficiency, and Hospital Administration of the British Army*, one of the first published documents that made use of her statistical data and charts (Stinnett, 1990). Karl Pearson, the developer of the Pearson product moment correlation coefficient and one of the grandfathers of modern statistics, said that Florence was a "prophetess" in the area of applied statistics (Stinnett, 1990).

In the later years of her career, Florence tried unsuccessfully to establish an Oxford professorship in applied statistics, which would have been the first position of its kind. Unfortunately, like many of her ideas, this was not fully embraced in her lifetime. Her methods are now standard practice in many fields, especially in health services auditing, quality control, epidemiology, (Spiegelhalter, 1999) and the national census of countries such as the United States (Kopf, 1916).

NCTM Standards

The activities presented address many points in the NCTM *Principles and Standards for School Mathematics*, especially with regard to the middle school curriculum. Students in sixth through eighth grades are expected to acquire a thorough grounding in algebra and geometry. Pie, bar, and polar area charts all rely on geometric proportionality to display data, and comparisons of pie and polar area charts in particular can generate interesting discussions about area. For example, why do we use the square root of counts in the polar area chart rather than the counts themselves?

The NCTM middle grades standards also emphasize the ability to manipulate real-life quantitative data, including looking at how data is collected and interpreted. In grades three through five, students learn the basics of data collection and how to make simple representations such as bar charts. In the middle grades, there is increased emphasis on comparison, integration, and explanation. The students must know how to manipulate not only whole numbers but fractions, percentages, exponential relationships, and ratios to represent and explain quantitative relationships, all of which are necessary for the construction of pie, bar, and polar area charts. As in the worksheet presented below, students need to be able to create the graphs and also explain what those displays mean, answer questions based on the data, and form conjectures about causes and relationships.

Activity Sheet: Florence Nightingale's Polar Area Diagrams

The following activity sheet gives students a chance to practice making pie and bar charts and construct Florence's polar area chart firsthand. The data provided looks at male and female athlete participation in the last eight Summer Olympics. The numbers are of a similar magnitude to those found in Florence's Scutari data. There are interesting trends in the data that require the students to draw conclusions using the related history. Participation by men dips markedly in 1980, the year that the United States boycotted the Summer Games in Moscow. Participation by women, on the other hand, shows steady increase, probably thanks to the enactment of Title IX in 1972 in the U.S. as well as the women's movements in many countries during the same era.

Historical information about the Summer Olympics, including the number of events, pictures of the posters and medals, and competition highlights can be found at the web site of the Olympic Movement (International Olympic Committee, 2004). The questions given at the end of the worksheet can be assigned or used for class discussion. These questions are designed to encourage the critical thinking and interpretation skills needed to explain the data, which is as important as the presentation.

Square Root Activity

Have your students search the internet for information about methods of finding square roots by hand, which were taught in American schools until about World War II. Ask them to use these methods to try to find the square root of a small number or a perfect square such as four, then for a larger number. Discuss with students how the invention of hand calculators and personal computers has changed the teaching, learning, and practice of mathematics.

Florence Teaches Math Activity

When teaching mathematics, Florence posed the following types of questions to her students: How tall is the reindeer? Are you as tall? How tall are you? How far is the highest point of Europe from the equator? How far do you come to school? If you walked the same number of miles each day that you come to school, how long would it take you to walk to the equator? (Lipsey, 1993). Have your students solve some of these problems, either individually or in small groups. Then ask them to create other problems of their own that are similar.

Women Statisticians Activity

Have your students research the lives of other prominent female statisticians such as Gertrude Cox (who helped create the statistics department at North Carolina State University) and the very prolific Florence Nightingale David (whose parents were close friends of Florence Nightingale). Discuss their contributions to the field of statistics.

Census Activity

Suppose your students were asked to conduct a census of the students or classes at your school, similar to the U.S. decennial census. Have students research how the U.S. Census Bureau performs the census and create a standard data collection sheet for your school census that anyone can use. Discuss as a class what their census should look like and what kind of questions should be asked so that they would be able to analyze the data to make summaries and comparisons? Information and a number of free lesson plans that include large, full-color maps are available from the Census Bureau (United States Census Bureau, 2000).

Invent Your Own Graph Activity

Ask students to try to invent a new type of graph or picture-based method of displaying data. Compare these to the graphs they already know and discuss the similarities and differences.

References

American Statistical Association: Journal of Computational and Graphical Statistics (JCGS) [On-line]. Available: https://www.amstat.org/publications/jcgs/index.cfm?fuseaction=main

Cook, E. (1919). *The Life of Florence Nightingale 1820 – 1910.* London: MacMillan and Company.

Diamond, M. and Stone, M. (1981). Nightingale on Quetelet. *Journal of the Royal Statistical Society, Series A (Statistics in Society)*, 144(1), 66-79.

Dossey, B.M. (2000). *Florence Nightingale: Mystic, Visionary, Healer*. Springhouse, Pennsylvania: Springhouse Corporation.

Florence Nightingale Museum Trust (2003). Florence Nightingale Museum: The Passionate Statistician [On-line]. Available: http://www.florence-nightingale.co.uk/stats.htm

Heyde, C.C and Seneta, E. (editors). (2001). *Statisticians of the Centuries*. New York: Springer-Verlag.

International Olympic Committee (2004). Olympic Games [On-line]. Available: http://www.olympic.org/uk/games/index_uk.asp

Kopf, E.W. (1916). Florence Nightingale as statistician. *Publications of the American Statistical Association*, 15(116), 388-404.

Lipsey, S. (1993). Mathematical Education in the Life of Florence Nightingale. *Newsletter of the Association for Women in Mathematics*, 23(4), 11-12 [Reprinted On-line]. Available: http://www.agnesscott.edu/lriddle/women/night_educ.htm

The MacTutor History of Mathematics Archive*: Biography of James Joseph Sylvest*er [On-line]. Available: http://www-history.mcs.st-andrews.ac.uk/Mathematicians/Sylvester.html

Salzburg, D. (2001). *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century*. New York: W.H. Freeman and Company.

Small, H. (1998). Florence Nightingale's Statistical Diagrams. Presented at the Stats & Lamps Research Conference, Florence Nightingale Museum at St. Thomas' Hospital, 18 March 1998 [On-line]. Available: http://www.florence-nightingale.co.uk/small.htm

Spiegelhalter, D.J. (1999). Surgical audit: statistical lessons from Nightingale and Codman. *Journal of the Royal Statistical Society, Series A (Statistics in Society)*, 162(1), 45-58.

Stinnett, S. (1990). Women in statistics: sesquicentennial activities. *The American Statistician*, 44(2), 74-80.

United States Census Bureau (2000). Census in Schools Program [On-line]. Available: http://www.census.gov/dmd/www/teachers.html

Math History at St. Andrews MacTutor

Looking for more stories like Florence Nightingale's to use to bring the history of mathematics and statistics alive for your students? Check out the University of St. Andrew's Scotland MacTutor Archives <www-gap.dcs.st-and.ac.uk/~history/>. There are searchable indices of biographies, history topics, famous curves, and mathematicians of the day (a list of people who were born or died on the current date).

Here is a list of a few mathematicians with September birthdays:

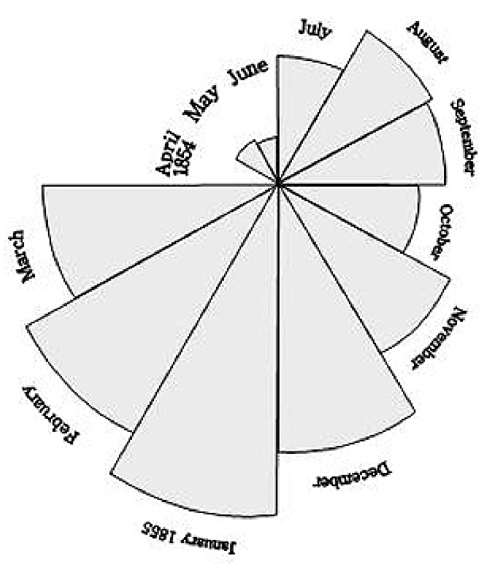
- September 3: Lev Semenovich Pontryagin
- September 7: Georges Buffon
- September 13: Constantin Caratheodory
- September 18: Adrien Legendre
- September 24: Girolamo Cardano
- September 28: Julian Coolidge

Activity Sheet: Florence Nightingale's Polar Area Diagrams

Florence Nightingale is known by legend as the "Lady with the Lamp," an innovator in the field of nursing. Born at a time when most colleges accepted only men, she insisted on learning mathematics from the time she was twelve years old, and then put those skills to good use analyzing data she collected herself in military hospitals. She wanted to clearly show that disease, not wounds, caused most of the British Army's deaths during the Crimean War (1854-1856). Most other statisticians at the time thought statistics should be very plain (and maybe even boring), but Florence decided to use interesting graphs to make her points stand out and to show people what was going on so that she could convince them to make changes in the way things were run.

Florence Nightingale was the first woman elected as a Fellow of the Royal Statistical Society, and she was also made an honorary member of the American Statistical Association. Her methods are widely used today, especially in health care and the census.


Portrait of Florence Nightingale, circa 1858, from the Library of Congress Prints and Photographs Division [#LC-USZ62-5877].



One of Florence's inventions was called a polar area chart, which is very similar to a pie chart. In a polar area chart, the circle is divided into angles or "wedges" of the same size for each category. The radius of each wedge is then equal to the square root of the count (or frequency) for that category. The square root is used for the radius because the area of a circle is πr^2, and using the square root of the frequency for radius means that the wedges in the polar area chart are still proportional to one another, like in the pie chart. An example is shown to the left.

Each wedge represents the number of deaths due to disease for the British Army in Crimea. There are no fatalities at the start of the war in April 1854, but as the months pass the numbers increase and the radii of the wedges get bigger until January 1855. She explained that this was due to the fact that, as fighting went on, the hospitals became more crowded and dirty, so there was more disease. The winter was also very cold – notice that January is the largest piece of the graph.

The data given below show the number of men and women athletes from all countries who have competed in each Summer Olympics since 1972. How many athletes participate? How has their participation changed over time? Are there more participants or fewer? Are the numbers of men and women similar? What are the causes of these trends or changes? These questions are similar to the kind Florence was trying to answer with her data.

Year	Host City	Host Country	Women		Men	
1972	Munich	West Germany	1058	15%	6065	85%
1976	Montreal	Canada	1247	21%	4781	79%
1980	Moscow	USSR	1124	22%	4093	78%
1984	Los Angeles	USA	1567	23%	5230	77%
1988	Seoul	South Korea	2186	26%	6279	74%
1992	Barcelona	Spain	2708	29%	6659	71%
1996	Atlanta	USA	3523	34%	6797	66%
2000	Sydney	Australia	4069	38%	6582	62%

(1) On a separate sheet of paper, make a pie chart for each year to show the percentage of women and men athletes.

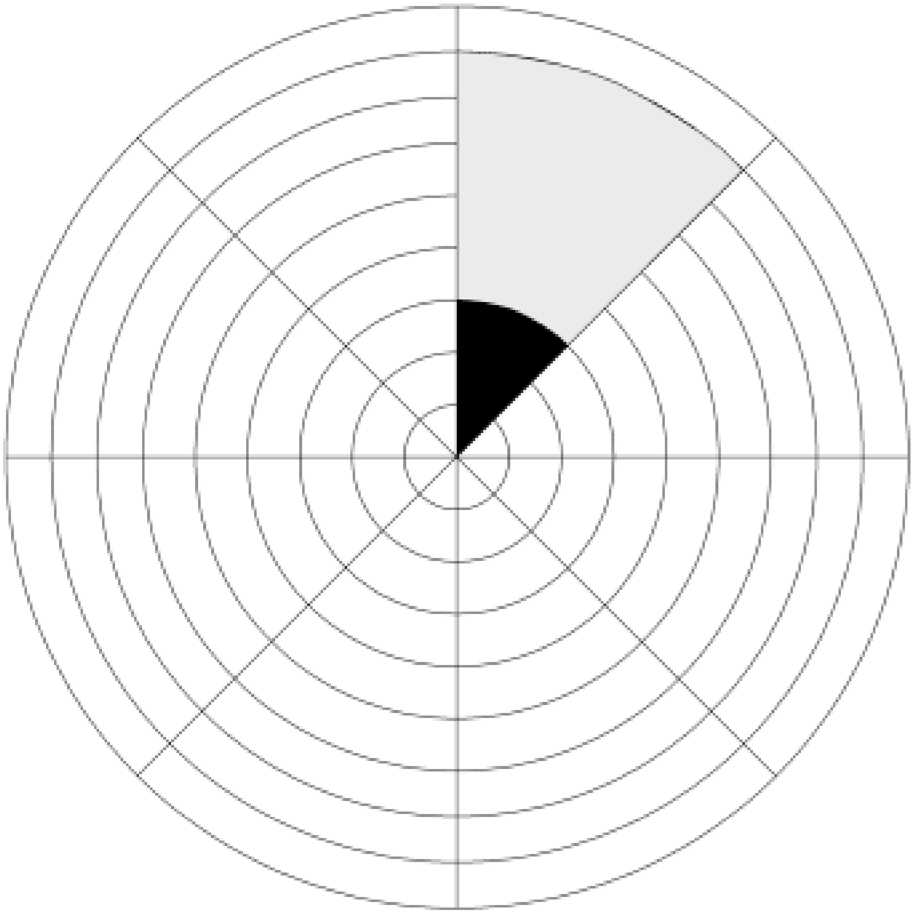
(2) On a separate sheet of graph paper or using a computer program, make a comparative bar chart showing the number of women and men athletes for each year.

(3) Create a polar area chart for the data using the table and the grid on the following page.

		1972	1976	1980	1984	1988	1992	1996	2000
Women	Count	1058	1247	1124	1567	2186	2708	3523	4069
	Square Root	33							
Men	Count	6065	4781	4093	5230	6279	6659	6797	6582
	Square Root	78							

- Find the square roots for all the counts in the table.
- For each year, draw the radius for the women's count and shade in the wedge.
- For each year, find the radius for the men's count (which is larger). Shade in the extra area covered by the men's wedge with a different color. When you are done it should look like the women's wedge is on top of the men's wedge for each year. The year 1972 has already been done for you. Label each section with the appropriate year.

There are eight equal sections on the grid, one for each year. The circles on the grid represent the tens digit (e.g., 10, 20, 30). For convenience, round all your square roots to the nearest tens place when drawing the graph.



(4) Answer the questions below using the tables and your graphs.

a) Describe participation by men athletes over time. Are there more men participating now than in the past, fewer men participating now, or are the numbers staying the about same? Explain your answer.
b) What year had the lowest participation by men? Can you find a historical reason why this is true?
c) Describe participation by women athletes over time. Are there more women participating now than in the past, fewer women participating now, or are the numbers staying the about same? Explain your answer.
d) In 1972, the United States Congress passed a law called Title IX. One of things it did was to create more opportunities for women athletes in high school and college. Can you use this to help explain the reason for your answer to part (c)?
e) What can you say about the total number of participating athletes for each year?

To quote Florence Nightingale, she believed that statistics is "...the most important science in the whole world; for upon it depends the practical application of every other science and of every art; the one science essential to all political and social administration, all education, all organization based on experience, for it only gives results of our experience."

INNOVATOR AWARD NOMINATIONS

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The purpose of this award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state. Further, this program must have been sustained for a period of at least three years. Any NCCTM member may submit nominations by sending in the form below. Nominations will be retained in the active file for at least three years.

NOMINATION FORM

Name of Nominee:____

Present Position:____

Outstanding contributions to mathematics education in North Carolina that serve as the basis for this nomination (a separate attached document may be used):

Additional information that would be of value to the selection committee (a separate attached document may be used):

Signature:______________________________________Date:___________________

Name (print/type):____

Position:____

Business or Institution:____

Address:____

Phone: Business:__________ Home: _________________Email:___________________

Send to: Phillip Johnson, Math and Science Education Center, ASU Box 32091 Boone, NC 28608-2091

Closed-Form, Recursion, and Mind-Reading: Defining Sequences by Various Means

Harold Reiter[1]
University of North Carolina at Charlotte
Charlotte, North Carolina

My friend said to me, 'I'm thinking of a sequence of positive integers the first four terms of which are 1, 2, 4, 8. Can you guess the next one?' I said, 'How about 16?' He replied, 'right, and the next one?' I replied '32.' 'Wrong,' he said, 'it's 31, and the next term after that is 57.' Hum...

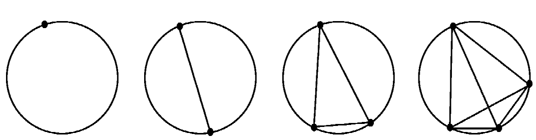
Recently a preliminary version of a mathematics contest contained the following problem:

> *Consider the sequence 2, 4, 8, 14, 22,... What is the sum of the next two members of the sequence?*

The test editor pointed out that including a definition of the sequence might improve the question. The author claimed that all good students would see that the differences of successive terms is an arithmetic sequence, so the next two terms are obviously $22+10=32$ and $32+12=44$.

These two sequences highlight a fundamental question we need to ask when we give problems like this to our students: Is there a need to define the sequence, or is it sufficient to give just the first five terms?

Consider the first sequence, 1, 2, 4, 8, 16, 31, 57,... It is definitely not clear from the first four terms that the sequence intended here is the maximum number of regions into which a circle can be cut using the chords joining n points on the circumference (assuming that we disallow three chords to intersect at the same point). Let R_n denote the number of such regions. Then, as can be seen from the pictures below, $R_1=1$, $R_2=2$, $R_3=4$, and $R_4=8$.



$R_1, R_2, R_3,$ and R_4

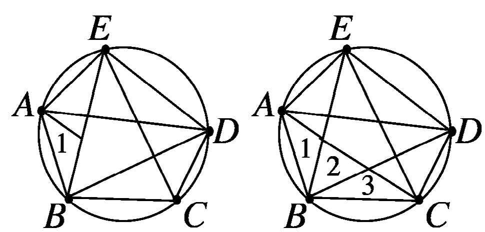
In general $R_n = 1+\binom{n}{2}+\binom{n}{4}$, where $\binom{n}{k}$ denotes the calculation for combinations:

$$\frac{n!}{k!(n-k)!}$$

Let's examine the case $n = 5$ to illustrate the formula.

$$R_5 = 1+\binom{5}{2}+\binom{5}{4}$$

Consider the figure below, and note the new regions that are formed when the line from A to C is drawn. The region marked with a 1 gets added when we reach the point of intersection of $\overline{AC}$ and $\overline{BE}$. This is one of the five regions counted by $\binom{5}{4}$ in the formula.



Constructing R_5

Extending this segment to the point where $\overline{AC}$ meets $\overline{BD}$ creates region 2. Finally, extending to the point C creates region 3. This region can be thought of as being created by the pair of vertices (A,C). It is one of the 10 pairs counted by $\binom{5}{2}$.

Clearly, we need to define this sequence if we wish students to arrive at the intended answer.

[1] *Dr. Reiter is Associate Professor of Mathematics and has just completed a term as President of the national mathematics honor society, Mu Alpha Theta.*

Defining Sequences

There are three ways to define sequences of real or complex numbers: By English description, with a closed form, and recursively.

The *closed form method* enables the student to compute the n^{th} term as a function of n. For example, using the formula $a_n = \frac{1}{2n-1}$ we can find the 10^{th} term of the sequence:

$$a_{10} = \frac{1}{20-1} = \frac{1}{19}$$

On the other hand a sequence like

$$a_1 = \sqrt{2},\ a_2 = \sqrt{2+\sqrt{2}},$$
$$a_3 = \sqrt{2+\sqrt{2+\sqrt{2}}},...$$

is much more difficult to define in this way. Yet it is fairly easily to define *recursively*. To define a sequence recursively, we need an anchor, and a method for obtaining new members of the sequence in terms of those at hand. In the case above what is needed is the first term, $a_1 = \sqrt{2}$, and the recognition that each term can be obtained from the previous term by adding two and then taking the square root of the sum:

$$a_{n+1} = \sqrt{2+a_n}$$

That this method is a legitimate way to define a sequence of numbers is not obvious. It depends on the *Principle of Mathematical Induction*:

If $P(n)$ is a statement about the integer n for which

- $P(1)$ *is true, and*
- *for every positive integer n, the truth of $P(n)$ implies the truth of $P(n+1)$,*

then it follows that $P(n)$ is true for all positive integers.

This principle also provides a framework for proving such things as the combinatorial formula for R_n.

One method we could use to overcome the difficulty that students might have the wrong sequence in mind when we write only the first few terms is the idea of *simplicity*. What the test-question author had in mind for a solution to her proposed problem was the sequence starting with

$$a_1 = 2,\ a_2 = 4,\ a_3 = 8,$$
$$a_4 = 14,\ a_5 = 22...$$

that she felt was the simplest. But what does simplest mean? Of course, arithmetic sequences are simple, and so are geometric sequences. What about reciprocals of such sequences, or sums of several of these sequences? Simplicity is at best vaguely defined.

Using Sequences in Class

In conclusion, sequences defined by listing the first several terms have a rightful place in the mathematics classroom, but perhaps not on math contests. One of my most enjoyable classes includes the problem of finding the next few terms of the sequence

$$0, 1, 10, 2, 100, 11, 1000, 3, 20, 101,...$$

Picking out patterns and making conjectures are two highly enjoyable parts of the exercise. I invite you to try this one and the following exercises from my course in discrete math at UNC-Charlotte. You can access the entire course from my website (Reiter, 2004).

The On-Line Encyclopedia of Integer Sequences (Stone, 2004) is an excellent website for sequence puzzlers.[1] People can use the website to find the next terms of the sequences below. Of course, that would defeat the purpose of the puzzle!

For more information on sequences, induction and recursion, see (Graham, 1994) and (Rosen, 2003).

Editor's Note: A rationale for the general case of the circle cutting formula may be

[1] Thanks to Richard Askey for the reference for this site.

found in the Ask Dr. Math archives on the Math Forum web site: <mathforum.org/library/drmath/view/55262.html>.

Exercises.

Find both a closed form and a recursive definition of each of the following sequences.

1. 1, 4,9,16,25, ...
2. 0, 0.1, 0.11, 0.111, 0.1111, ...
3. 0,1,3,6,10,15,21, ...
4. 0,4,16,36, 64,100 ...
5. 1, 3/2, 11/6, 50/24, 274/120, ...
6. $a_1=\sqrt{2},\ a_2=\sqrt{2+\sqrt{2}},\ a_3=\sqrt{2+\sqrt{2+\sqrt{2}}},...$
7. 1,2,3,5,8,13,21, ...
8. 1,3,4,9,10,12,13, 27,28,30,31, ...
9. 0,1,10,2,100,11,1000,3,20,101,...

References.

Graham, R. L., D. E. Knuth and O. Patashnik. (1994). *Concrete Mathematics* (2nd Ed.). Reading, MA: Addison-Wesley.

Reiter, H. B. (2004). *Math 1165 -Discrete Mathematics* [On-line]. Available: http://www.math.uncc.edu/~hbreiter/m1165/.

Rosen, K. (2003). *Discrete Math and its applications* (5th Ed.). New York: McGraw Hill.

Stone, N. J. A. (2004). *On-Line Encyclopedia of Integer Sequences* [On-line]. Available: http://www.research.att.com/~njas/sequences/.

NCCTM FALL CONFERENCE

34th Annual Conference of the North Carolina Teachers of Mathematics

Transitioning into the New Mathematics Curriculum

October 7 & 8, 2004

Joseph S. Koury Convention Center
Sheraton Greensboro Hotel at Four Seasons

Registration Fees: $55 ($45 for members, $5 for students)

Registration: Forms available at www.ncctm.org or at the conference Wednesday evening, or Thursday and Friday all day.

Hope to see you there!

2004 NCCTM Annual Meeting Preview

Holly Hirst
Appalachian State University
Boone, North Carolina

Not planning on attending NCCTM's 34th Annual Conference in Greensboro in October? I urge you to change your mind! The meeting schedule is finalized, and I was lucky enough to get a preview. This year's theme is *Transitioning into the New Mathematics Curriculum*. Below are some teasers highlighting a wonderful program.

Set aside some time to come to the Koury Convention Center in Greensboro on Thursday and Friday, October 7 and 8, 2004. Information on travel, housing, and registration may be found on the NCCTM website <www.ncctm.org>.

Keynote Speakers

We are lucky to have two excellent keynote speakers in attendance at the conference.

Stuart J. Murphy, an expert in visual learning – how people learn from images – is the author of *MathStart*, a series of children's books designed to teach math concepts within the context of stories and with visual support. He has helped develop several curricula at the elementary and high school levels, and has contributed articles to many journals.

A popular speaker, Mr. Murphy is a frequent lecturer at conferences and meetings, and in K-12 classrooms. He will be at the Conference all day Thursday and will present two sessions on *Using Reading to Open Doors to Math*.

Carol Newman, the NCTM Affiliate Services Committee Representative for the Southeast Region, will be speaking Thursday on *NCTM affiliate services, NCTM membership, and NCTM resources*.

Ms. Newman is a mathematics curriculum specialist for the Broward County, Florida, schools. She is a member of the board for the Florida Association of Instructional Supervisors & Administrators and past president of the Florida Association of Mathematics Supervisors.

Materials Marketplace

This year, NCCTM is proud to sponsor the Materials Marketplace. At the conference, new teachers, pre-service teachers, and ILT teachers will have the opportunity to purchase at discount prices resources to help them start building their resource base.

Experienced teachers: We are looking for donations of new or gently used supplies such as manipulatives, posters, books, professional development materials. Please contact Kim Tyson and Shana Runge via e-mail <ncctmmarketplace@hotmail.com>.

Minicourses

In addition to 45-minute sessions and 90-minute workshops, there are several minicourses scheduled during the conference. Minicourses are typically three hours long and provide a hands-on opportunity for exploration of a topic for a small fee ($2). Here's a list of titles to tempt you:

- Building the Concepts of Geometry
- Concrete to Abstract: The Montessori Method
- Statistics is for Lovers!
- Teaching Advanced Functions and Modeling with Technology
- Seminar for Mathematics Supervisors

Exhibitors

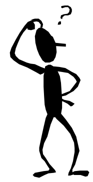
The exhibit hall at the convention center will be a busy place during the conference. Exhibitors include many of the major textbook publishers, marketers of teaching resources, and others with items of interest to teachers.

The Exhibits will be open during the conference, giving attendees opportunities for browsing through the latest and greatest texts, manipulatives, and other resources for the PreK-12 mathematics classrooms.

Never Been???

If you haven't been to the conference before, there is a session just for you! Past President Theresa Early will host an informal orientation session at 8:30 a.m. on Thursday. Come and learn about the Conference format, the convention center layout, and what NCCTM has to offer its members.

Problems to Ponder



Fall 2004 Problems

Gregory S. Rhoads
Appalachian State University
Boone, North Carolina

Grades K–2 Sally's mother is giving Sally and her two brothers their weekly allowance. Sally's mother will divide 18 pennies, 12 nickels, and 6 dimes between the 3 children. If each child is to receive the same amount, what is the total amount of Sally's allowance?

Grades 3–5 Curtis is making bags of jellybeans for the other students in his class. Jellybeans cost $5.00 per pound and there are approximately 56 jellybeans in a pound. If Curtis wants to put 7 jellybeans in a bag for each student, and there are 32 other students in his class, how many pounds of jellybeans should he buy and how much money will he need?

Grades 6–8 A running track at a school is 1/4 mile long. It has two straight side portions of 330 feet each and a semicircle at each end. What is the diameter of the semicircle?

Grades 9–12 If $a + b = x$ and $a^2 + b^2 = 2x$, then write $a^3 + b^3$ as a polynomial in x.

Directions for submitting solutions

1. Neatly print the following at the top of each solution page:
    - Your full name (first and last)
    - Your teacher's name
    - Your grade
    - Your school
2. Submit one problem per page.

Students who submit correct solutions will be recognized in the next issue of *The Centroid*. We wish to publish creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission. Keep in mind that proper acknowledgement is contingent on legible information and solutions.

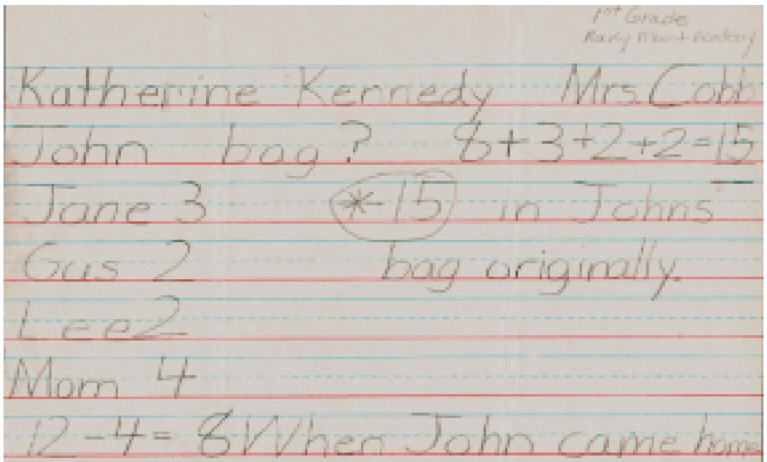
Send solutions by 15 October 2004 to:

Problems to Ponder
c/o Dr. Greg Rhoads
Dept. of Mathematical Sciences
Appalachian State University
Boone, NC 28608

As these problems are intended to stimulate independent thinking, it is expected that a submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

Grades K-2 Spring 2004 issue

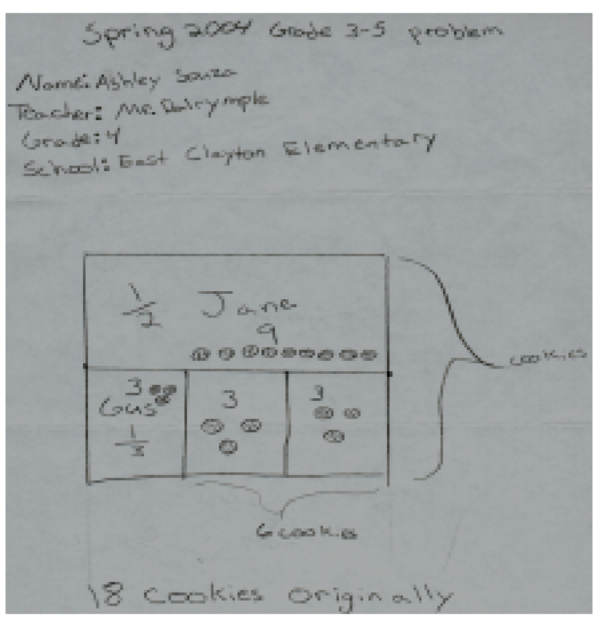
John's mother gave him a bag with cookies in it. John offered his sister Jane some cookies and Jane took 3 from the bag. John saw his friends Gus and Lee and gave them each 2 cookies from the bag. When John came home, his mother baked some more and put 4 more cookies in John's bag. John went to his room, dumped the cookies on his desk and counted 12. How many did John have in his bag originally?
Solution: Katherine Kennedy, 1st grade, Rocky Mount Academy (Teacher: Mrs. Cobb)



Correct Solutions were received by Michael Armstrong, Reid Smith, and Tyler Sudbrink of **Greensboro Day,** Katherine Kennedy of **Rocky Mount Academy.**

Grades 3-5 Spring 2004 issue

John's mother game him another bag of cookies. John gave half of the cookies to his sister Jane, then gave one-third of the remaining cookies to his friend Gus. If there were 6 cookies left in the bag, how many cookies were there originally?
Solution: Ashley Souza, 4th grade, East Clayton Elementary (Teacher: Mr. Dalrymple)

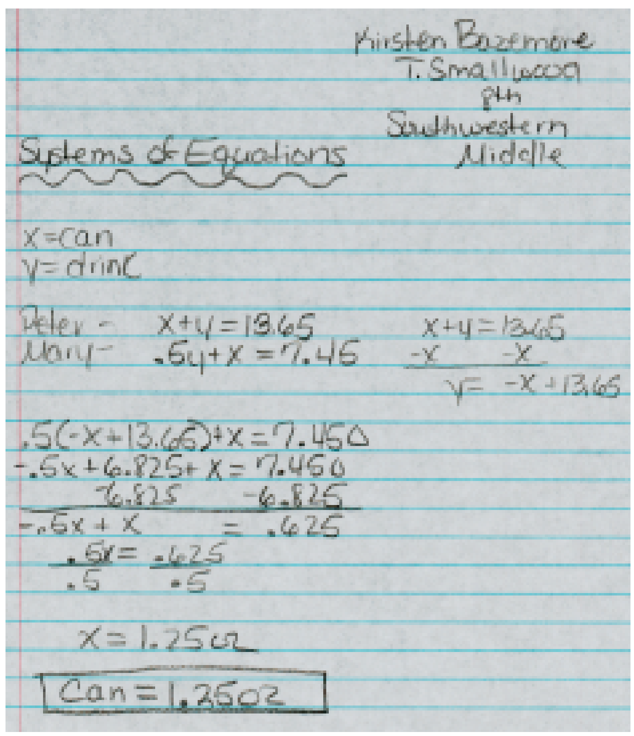


Correct Solutions were received by Hayle Austin, Tanner Morrison, Brian Petrich, Drew Sparks and Jasmine Thornton of **Burnsville Elementary,** Ashley Souza of **East Clayton Elementary,** Will Vannoy of **Ellendale Elementary,** Christian Alba, Clarissa Albano, Celyna Albano, Naomi Barbee, Lyndsay Barnes, Victoria Bird, Justin Blake, Michael Brewer, Jasmine Brodie, John Danford, Leanna Doty, Alison Gerepka, Samantha Grinstead, Mackensie Hicks, Gideon Lance, Erin Lile, Jaron McClain, Morgan McKinney, Jennifer Reeves, Kristen Terry, Lauren Thompson, Nick Tomso, Alec Van Orsoun, Tara Wagner, and Blake Williams of **Eno Valley Elementary,** Taylor McCoy of **James W. Smith Elementary,** Emily Manning, Andrew Reed, William Shuler, Shawn Wilds of **Maysville Elementary,** Kasey Barnhill, Blake Byrum, Jeffrey Carr, Stephanie Gioia, Sarah Rozier, Shakell Spence, Samuel Stretar of **Moyock Elementary,** Brianna McNeil, Keanna McNeil of **W.H. Knuckles Elementary.**

Grades 6-8 Spring 2004 issue

Peter gets a soft drink from a machine and notices the can and drink together weigh 13.65 ounces unopened. Mary drank exactly half of the soda in her can and her can and remaining soda weigh 7.45 ounces. If both Peter's and Mary's cans were identical, how much does an empty can weigh?

Solution: Kirsten Bazemore, 8th grade, Southwestern Middle School (teacher: Mrs. Smallwood)

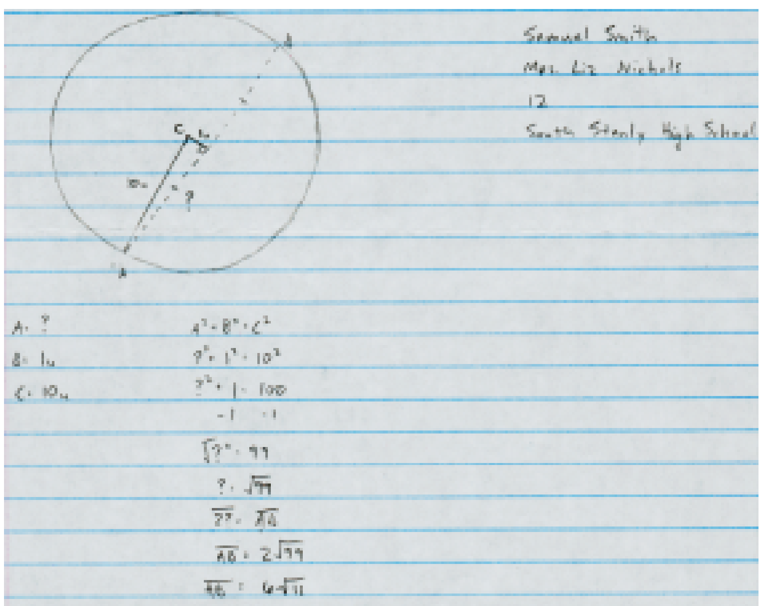


Correct Solutions were received by Christina Faidas, Matthew Green, Chelsey Harshman, Cameron Jones, and Stephen Tipton of **Arendell Parrott Academy**, Meg Gibson, Lizze Gillikin, and Rhett Smith of **Centennial Campus Middle**, Mike Daniels and Emily Johnson of **East Alexander Middle,** Katelyn Baucom, Jennifer Beane, David Black, Brooke Corley, Juan Cortes, Tammy Dean, Justin Freeman, Luis Gomez, Tyler Griffin, Bliss Hamilton, Drew Kelly, Lisa Marie Killough, Amanda Mills, Randell Richardson, Jackie Riggins, Williams Sandoval, Brooke Smith, Ashton Wheeler, Alanna Williams, and Cam Wright of **East Union Middle**, Eyad Al-Kobri, Courtney Alvey, Dionicio Atherley, Pedro Bulluch, Summer Dari, Lolita Doward, Adam Elhammoumi, Alexis Farmer, Sarah Frick, Justin Hartney, Jessica Hill, Zaina Hroub, Brandon Johnson, Macy Jones, Garrett Landrum, Erin McAuliffe, R.J. Patel, Joshua Pender, Safa Qassim, Daquain Richardson, Erica Rubin, Shanae Savage, Yasmin Shabaneh, Bryn Shurbutt, Meghan Smith, Isaiah Stamper, Shakiyla Taylor, Shakeita Thorne, Alex Trull, Kristopher Washington, Alexia Williams, and A.J. Woddard of **J.W. Parker Middle**, Tierra Greer and Paul McNeil of **Lumberton Junior High**, Theresa Chester of **New Bridge Middle**, Sarah Allred, Amy Beasley, Chris Cimino, Amanda Clifton, Kyle Collins, Kaitlin Crepps, Nick Decker, Christina Faison, Ulylisa Frazelle, Emily Garrett, Katelyn Harris, Hannah Hemphill, Alex Hogue, Thad Johnson, Ayana Littlejohn, Taylor McClean, Erin McMurtrie, Chelsea Parker, Lauren Peeler, Tyler Profitt, Jamie Robinette, Hannah Scott, Layne Spencer, Ginny Talley, Brandon Terrel, Ethan Thompson, Charmisa Wesley, and Rachel Wilkinson of **Southeast Middle**, Krystal Bailey, Jasmine Barnes, Amanda Barnhill, Cadie Bartely, Kirsten Bazemore, Ethan Boller, Yashonti Bridgers, Geoffrey Brown, Aylonnah Browning, Charles Castelloe, Brittney Cherry, LaShay Cherry, Kelly Collins, Celeste Cuttino, Jalissa Dempsey, Michael Drake, Paul Faison, Keion Fogg, Jonelle Freeman, Althea Hill, Mary Anne Hughes, Chris Jernigan, Cameron Johnson, Randy Johnson, Marissa Johnson, Allen Jones, Christian Lilley, Shoyka McDowall, Lonnekqua Monterro, Shantara Outlaw, Brittney Payton, Michelle Pugh, Paul Schub, Bruce Taylor, Chanel Tucker, Tevin Veale, Jeffrey Veale, Brandon Williams, and Desmond Williams of **Southwestern Middle**, Ethan Brooks, Rose Buchanan, Cara Chandler, Jim Frazier, Brooke Guthrie, Abby Haithcock, Sloane Hayes, Seth Hill, John Hollander, Abby Moser, Ross Powell, Hannah Rhodes, and Tiesa Smith of **Turrentine Middle**, Alex Vannoy of **West Alexander Middle.**

Grades 9-12 Spring 2004 issue

The closest distance between a chord in a circle of radius 10 and the center is 1 unit. What is the length of the chord?

Solution: Samuel Smith, 12th grade, South Stanly High School (Teacher: Mrs. Liz Nichols).



Correct Solutions were received by Allen Souza of **Clayton High,** David Chester of **Jacksonville High,** Catherine Cheng, Jermaine Franks, Mark Frisch, Zach Hohns, Stacey Holladay, Brandon Hubbard, and Summer Price of **Middle Creek High,** Samuel Smith of **South Stanly High,** Ross Powell and Hannah Rhodes of **Turrentine Middle,** and Tasha Pearson of **West Caldwell High.**

Discovering Mathematics

Laura Davis[1]
Southeastern Community College
Whiteville, North Carolina

Do you remember the excitement you felt the first time you walked into a classroom of your very own? Would you like to rekindle the passion you felt as a beginning teacher? If there was a way to get your students actively involved in their learning, communicating with each other in the language of mathematics, seeing the usefulness of mathematics, and engaging in higher level thinking, would you try it? Discovery learning can help!

Many of us have strayed away from our first love—teaching children mathematics. Our creativity has suffered due to increasing pressure to prepare our youngsters for multiple-choice end-of-class (EOC) tests. I have heard too many times the lament, "If it can't be answered a, b, c, or d, I can't spend time on it." Although I certainly understand the necessity of training students for standardized tests, our first and foremost goal should be to make mathematics meaningful. We should strive to lead our students to a mathematical epiphany—what I call a "light bulb moment." Isn't that why we went into teaching?

I believe strongly in having standards and will argue in favor of a standardized curriculum, but we don't need cookie cutter teachers. As professionals we each make decisions daily regarding both content and teaching practices. NCTM recognizes that it is imperative for mathematics teachers today to redirect their efforts toward teaching critical thinking and mathematical communication. Not only should our students be able to compute fluently, and choose correct answers on the EOC tests, but they must also have a developing sense of number that they can apply appropriately outside the classroom. They should recognize and apply mathematics to physical, social, and mathematical situations. Many students master the manipulation of symbols and technology, but they must also be able to judge the meaning, utility, and reasonableness of the results. We teachers need to restructure our class time so that students are allowed opportunity to investigate conjectures and explore relationships to gain mathematical insight. They should be expected to communicate mathematically in written and oral form. They should be forced to reflect on their experiences and use the language of mathematics to express their ideas precisely. The ability to draw inferences and evaluate conclusions is an important life skill, but many of us claim we don't have time to spend on this. How often in "real life" are mathematical problems posed in the form of a multiple choice EOC equation to be solved? We *must* show excitement over the tackling of a complex problem and structure our lessons so that students can succeed in doing multifaceted and meaningful mathematics. I believe one way we can achieve these ideals is by using what I call "discovery lessons" in our classrooms.

Discovery Learning

Ormrod defines discovery learning as

> *an approach to instruction through which students interact with their environment by exploring and manipulating objects, wrestling with questions and controversies, or performing experiments* (Ormrod, 1995, p. 442).

This approach initially grew out of Piaget's theory of constructivism, which states that students will learn more willingly and more enduringly if they create the knowledge themselves (Bencze, 2002). Rather than our demonstrating rote procedures then letting students practice 20 problems similar to the ones we demonstrated, we should intentionally design mathematics lessons that arouse the students' curiosity, have them gather information, then engage them in

[1] *Laura Davis is a mathematics instructor at Southeastern Community College in Whiteville, North Carolina. This article is based on a series of professional development workshops she has conducted through the years. Ms. Davis has experience teaching mathematics at grades five through college.*

significant exploration as well as the formulation of rules. The amount of teacher intervention recommended could be an indicator of the philosophy of education of the teacher/facilitator.

Discovery learning may take place in many forms on a continuum. At one end we find discovery learning in its purest sense, where only the tools and information needed to solve a problem or learn a concept are provided and the learners make sense of them without outside intrusion. In the middle of the continuum we find discovery learning as experimentation (even trial and error) with some teacher intervention – clues, coaching, a written framework to help learners get to a reasonable conclusion. At the far end of the continuum is the expository teaching model of discovery learning where the learners "discover" what the teacher intends by using a process prescribed by the teacher and following the teacher's example.

Whether you use the discovery method as part of a whole class lecture, a take-home worksheet, or a cooperative group experience will be a professional decision you make based on your circumstances. I most often create a directive worksheet that students can use in groups while I act as a facilitator. This uses valuable class time, but I have found that it makes my students stronger critical thinkers and they actually perform better on standardized tests because they are more mathematically sophisticated and confident.

Remember, the driving force in using this approach is our interest in teaching not only mathematics content, but also thinking skills through direct encounters with materials and problems. In discovery learning we teachers construct or select interesting problems, questions, or theorems and our students construct explanations and discover underlying principles through direct interaction with the lesson materials we provide. Therefore, it is imperative that our prompts be well phrased and that our problems are relevant.

My recommended steps for creating a successful discovery experience are listed below and are based upon Bloom's cognitive taxonomy (Huitt, 2000). When properly structured, a discovery lesson should lead students through all levels of critical thinking. Don't make the mistake of thinking of student discovery as a lucky occurrence. Students will need background preparation both in prerequisite knowledge and in classroom procedure to be successful with this approach. Following this article I have included a model discovery lesson I have used successfully.

Creating a Discovery Experience

Following are a few guidelines I recommend for creating solid discovery lessons. An example worksheet-style lesson, "Algebra Discovery: Sequences for Middle Grades" is provided after this article as an example to illustrates these steps.

1. Choose appropriate *objective(s)* from the curriculum upon which you will build your lesson. I have chosen for the example to work with sequences. The model lesson correlates to North Carolina *Standard Course of Study* (SCS) for Grade 7 Objective 5.01:

    > Identify, analyze, and create linear relations, sequences, and functions using symbols, graphs, tables, diagrams, and written descriptions.

    or the SCS Algebra I Objective 1.02:

    > Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.

    It is also important when appropriate to give students historical background (in this lesson a snippet about Fibonacci) so that they can appreciate mathematics as an important human achievement.
2. Establish a *motivation* for the learning. This may mean introducing an interesting problem the students do not currently possess the skills to solve. It could mean posing an evocative question to encourage further investigation. The example starts by asking Fibonacci's famous rabbit population growth question to interest students in learning about sequences.
3. *Lead* the students through the method *from concrete to abstract*. Begin with something they do know and expand upon that (Bloom's *knowledge* and *comprehension* levels of learning). Through probing questions, clues, and partially completed examples, your worksheet or verbal presentation should lead students to discover without being

directly told the details needed to solve the types of problems in your objective.

I often lead my students through the derivation of a necessary formula or proof in this way. We are hoping for a “light bulb moment” more than anything (Bloom’s *analysis* level of thinking). In the Sequences lesson I have the students begin by making a concrete representation of the problem. They are then asked to look for a pattern and make a generalization.

4. Allow students to *practice* their new learning by applying the techniques to slightly different problems, hopefully in various content areas, and establish new procedures in their memory (Bloom’s *application* level). On the second page of the Sequences lesson I define recursive, arithmetic, and geometric sequences and ask students to apply their learning to new sequences.
5. Have the students *write conjectures* for the theorems or formulas they discovered (Bloom’s *synthesis* level). In the example discovery lesson, students are led to create recursive, arithmetic, geometric, and non-mathematical sequences of their own.
6. Ask each group to discuss their results verbally, and then ask each student to write on paper, an *evaluation* of what he or she learned in the activity using complete sentences, and correct terminology (Bloom’s *evaluation* level). In my example, students are asked to write a short paragraph summarizing what they learned about sequences and explaining why this knowledge is useful.

Conclusion

The concept of discovery learning is not new. It has appeared numerous times throughout history as a part of the educational philosophies of Rousseau, Pestalozzi, Dewey, and others. I am not trying to reinvent the wheel with this article. I would like, however, to rekindle your interest in this exciting approach to teaching mathematics. Since discovery learning by definition takes place in problem solving situations where the learner draws on his own experience and prior understanding to determine the truths that are to be learned, it is a deep, individual, constructivist, learning environment.

“Emphasis on discovery in learning has precisely the effect on the learner of leading him to be a constructionist, to organize what he is encountering in a manner not only designed to discover regularity and relatedness, but also to avoid the kind of information drift that fails to keep account of the uses to which information might have to be put” (Bruner, 1962).

In other words, use this method to discover mathematics and help your students find new meaning in what many find to be an intimidating subject.

References

Bencze, J.L. (2002). *Constructivist Learning Theory* [On-line]. Available: http://www.oise.utoronto.ca.

Bruner, J.S. (1962). *The Process of Education*. Cambridge, MA: Harvard University Press.

Bruner, J.S. (1966). *Toward a Theory of Instruction*. Cambridge, MA: Bleknap Press.

Huitt, W. (2000). *Bloom’s Taxonomy of the Cognitive Domain* [On-line]. Available: http:// chiron.valdosta.edu.

North Carolina Department of Public Instruction. (2003). *North Carolina Standard Course of Study for Mathematics* [On-line]. Available: http://www.ncpublicschools.org.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Ormrod, J. (1995). Educational psychology: Principles and applications. Upper Saddle River, NJ:Prentice Hall.

The LearnNC Education Glossary

Looking for information on education topics, like Bloom’s taxonomy mentioned in the article above? Check out the LearnNC site’s searchable glossary. <vote.learn.unc.edu.glossary>

Algebra Discovery: Sequences for Middle Grades

In 1202, Leonardo da Pisa (better known as Fibonacci), asked an interesting question.

> *How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair gives birth to a new pair and each new pair needs two months to grow up and be able to have babies of their own?*

Complete the diagram below to begin exploring this question. We have labeled the first pair of rabbits with the letters A and a. Since they do not have babies in the first 2 months, there is only one pair at the end of two months. However, on the first day of the third month they give birth to another pair of bunnies, B and b. So there are now two pairs at the end of the third month.

The Fibonacci Rabbit Problem

End of Month	Rabbit Pairs	Number of Pairs
1	Aa	
2	Aa	
3	Aa →Bb	
4	Aa →Cc Bb	
5	Aa →Dd Bb→Ee Cc	
6	Aa →Ff Bb→Gg Cc→Hh Dd Ee	
7		

Exploration Questions

1. Look at the column you filled in "Number of Pairs". Do you see a pattern? ______
2. Can you predict how many pairs of rabbits there will be at the end of month 7? ______
3. Now complete month 7 on the diagram above to confirm your answer.

The numbers in your last column form a "**sequence**" or a pattern.

4. List the first 12 numbers (sometimes called **terms**) in Fibonacci's sequence. (Hint: Look at your last column to get started.)

   1, 1, 2, 3, ____, ____, ____, ____, ____ , ____, ____, ____

5. How many pairs will there be at the end of the year? ______
6. How many rabbits will there be at the end of the year? ______

____

____

Fibonacci's sequence is **recursive**. This means that you can find each number in the pattern if you know one or more of the previous numbers. In this case, each answer was found by adding the two numbers before it.

7. Examine the first few terms of the recursive sequence below, and then fill in the blanks with a numbers you can justify by explaining the pattern:

   1, 3, 7, 15, 31, _____, _____, _____

8. What is the pattern? ____

9. Now make up a recursive sequence of your own. ___________________________________

The following patterns are called **arithmetic** sequences. In these, the same number is either added or subtracted each time to **generate** the next number.

10. Beside each set write what number was added or subtracted to generate the arithmetic sequence.

    5, 10, 15, 20, 25, 30… ______ 6.5, 6.25, 6, 5.75, … ______

    99, 97, 95, 94, 93, … ______ 1, 3/2, 2, 5/2, 3,… ______

11. Find the 15^{th} number in the sequence 16, 20, 24, … ______

12. Make up your own arithmetic sequence. ____

Following patterns are **geometric** sequences. To get the next term you must multiply or divide by a common number.

13. In each blank below write the number that was either multiplied by or divided by in each sequence to generate the next term.

    5, 10, 20, 40, 80, … ______ 640, 160, 40, 10, … ______

    100, 50, 25, … ______ 2, 8, … ______

14. Find the sixth term of this geometric sequence: 2000, 200, 20, 2, 0.2, ______

15. Make up your own geometric sequence. ____

16. Is the Fibonacci sequence arithmetic or geometric (or neither)? Explain. __________________

Some sequences are not mathematical. For example, the initials of the months of the year make a pattern some people would recognize. Complete: J, F, M, A, M, J, J, ___, ___, ___, ___, ___

17. Consider: S, M, T, W, ___, ___, ___.

18. What was the pattern in the previous sequence? ___________________________________

19. Make up your own non-mathematical sequence. ___________________________________

20. Finally, write a short paragraph to summarize what you learned in this lesson. Be sure to answer the question, "How is this useful?"

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in the State. Presented in the fall at the State Mathematics Conference, the award, named in memory of W.W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

If you have nominated someone in the past who has not received the award to date,
or if you would like to nominate someone now,
please submit as much of the following information as possible.
Nominations are accepted at any time.

Nominations should include the following information. Use as many typewritten pages as needed. If possible, attach a vita of the nominee.

Name of the nominee

Current position

Your relationship to the nominee (e.g. principal, co-worker, etc.)

The nominee's contributions to mathematics education, NCTM, NCCTM, etc. (Please include information on specific offices held and honors received by the nominee.)

Any information about contributions to the community, teaching, and education that would be of value to the Rankin Award Committee in its deliberations

Other relevant information

Letters of endorsement from other colleagues may be included.

Date of nomination

Nominator* Name
Current position
Business or educational institution
Preferred mailing address
Preferred telephone number

*The Rankin Award Committee reserves the right to use portions of nomination information in the presentation of the award if the candidate is selected.

Send to: Dr. Ralph DeVane
P. O. Box 1762
Cullowhee, NC 28723

2004 NCCTM State Math Fair Winners

Primary and Elementary Divisions

Congratulations to the students for jobs well done! Watch for details on the 2005 Regional and State Math Fairs.

Primary Division

Grades K–2

1st Place
Nicholas Kowalski
It's Lunch Time: Let's 'Half' a Sandwich
South Greenville Elementary School
Greenville, NC

2nd Place
Cole Rassin and Austin Miller
When Does 1+1=10? The Binary System
Dr. John Codington Elementary School
Wilmington, NC

3rd Place
Ethan Walters
Which AFC Football Team Won the Most Games in 2003?
Dr. John Codington Elementary School
Wilmington, NC

Honorable Mentions

Ashlynn Eddy and Lyndsay Wilcox
Taste vs Kernels
Meadowlark Elementary School
Winston-Salem, NC

Frederick Micheli
Ratios
Rocky River Elementary School
Concord, NC

Elementary Division

Grades 3–5

1st Place:
Jordan Clark-Brown
How Much Does the Davidson Elementary Morning Car Pool Line Cost?
Davidson Elementary School
Davidson, NC

2nd Place:
Le Khoi Chau
How Would You Invest for Higher Education?
Lincoln Heights Elementary School
Charlotte, NC

3rd Place:
Katie Scheuerle and Morgan McMahon
Step Up to the Beat
Mount Mourne Elementary School
Mooresville, NC

Honorable Mentions

Tyler Reinhold
More Shots For Your Money
D.F. Walker Elementary School
Edenton, NC

Hollis Elmore
Coins to Fly By
D.F. Walker Elementary School
Edenton, NC

Maggie Pendergrass
Disney or Bust
Dr. John Codington Elementary School
Wilmington, NC

Molly O'Brien and Dana Guthrie
Life As a Sponge
White Oak Elementary School
Cape Carteret, NC

2004 NCCTM State Math Fair Winners

Middle School and High School Divisions

Congratulations to the students for jobs well done! Watch for details on the 2005 Regional and State Math Fairs.

Middle School Division

Grades 6–8

1st Place
Rebecca Gregory
Swing, Swing
Camden Middle School
Camden, NC

2nd Place
Bobby Schultz
Trapped in Motion
Magellan Charter School
Raleigh, NC

3rd Place
Leah Stephens
Knot Theory and Mathematical Knots
Aycock Middle School
Greensboro, NC

Honorable Mentions

Caitlin Byrd and Alissa Kalinowski
Overtime Math
E.B. Aycock Middle School
Greenville, NC

Chris Wong
Origami
Aycock Middle School
Greensboro, NC

Ben Yuen
Making Sense of Dollars
Brawley Middle School
Mooresville, NC

High School Division

Grades 9–12

1st Place
Kim Canuette
Folding in a Little Math
North Duplin High School, NC
Calypso, NC

2nd Place
Chris Hutchinson and Katie Montgomery
Mass of Saturn
Cape Fear High School
Fayetteville, NC

Honorable Mention

Candace Matthews and Dana Jo Outlaw
Phi: One H of a Lot Cooler Than Pi.
Hertford County High School
Ahoskie, NC

NCTM 2005

The National Council of Teachers of Mathematics 2005 Annual Meeting and Exposition will be in Anaheim, California, Wednesday, April 6 through Saturday, April 9, at the Anaheim Convention Center, the Anaheim Marriott Hotel, and the Hilton Anaheim Hotel. The conference theme is *Embracing Mathematical Diversity*.

2004 NCCTM Math Logo Contest Winners

From a field of approximately 2,000 entries, twelve logos submitted by the following students were judged to be the winners in the 2004 Logo Contest.

State Winner
Joshua Arnold, Grade 6
Weddington Middle School
Waxhaw, NC
Teacher: Ms. Stevens

Eastern Regional Finalists

Jeffrey Smitherman, Grade 1
Rocky Mount Academy
Zebulon, NC
Teacher: Ms. Ann Cobb

Ethan Mairs, Grade 8
Williston Middle School
Wilmington, NC
Teacher: Ms. Katie Woodard

John Thomas Rains, Grade 5
Dublin Primary School
Elizabethtown, NC
Teacher: Mrs. Penny Guyton

Jacob Jasbolka, Grade 12
Sanderson High School
Raleigh, NC
Teacher: Ms. June Blackwell

Central Regional Finalists

Kathy Chan, Grade 1
Glendale Acres Elementary
Fayetteville, NC
Teacher: Mrs. Bender

Sarah Attayek, Grade 8
Aycock Middle School
Oak Ridge, NC
Teacher: Ms. Carolyn Warren

Meredith Mock, Grade 5
Vienna Elementary School
Pfafftown, NC
Teacher: Shane O'Neal

Katy Purgason, Grade 10
Caldwell Academy
Greeensboro, NC
Teacher: Ms. Lynn Church

Western Regional Finalists

Parker Hills, Grade 5
Catawba Spring Elementary School
Stanly, NC
Teacher: Ms. Pat Freeman

Joshua Arnold, Grade 6
Weddington Middle School
Waxhaw, NC
Teacher: Ms. Stevens

Jake Nelson, Grade 5
North Brook Elementary School
Vale, NC
Teacher: Ms. Denise Smith

Sara Lake, Grade 9
Independence High School
Charlotte, NC
Teacher: Ms. Carol Huss

2005 NCCTM Math Logo Contest

The Celebrate Mathematics Committee will once again sponsor a Math Logo Contest. The winning logo will be the mathematics logo for NCCTM for the 2005-2006 school year. This logo will be used on a poster to promote interest in mathematics and as the basic design for NCCTM's 2005 T-shirt.

A professional graphic artist will prepare the final art of the winning entry for printing. Though the illustration is an important part of the logo, entries are judged on idea or concept conveyed. Use of copyrighted work (clip art, cartoon characters, etc.) will automatically disqualify the entry. The words of the slogan should not be on top of or touch the design.

Duplicate the following label and include on the back of each entry. Entries submitted with incomplete information may be disqualified.

Student's Name:________________________________ Grade:______
Home Address:____

____

School Name:________________________________ LEA:_______
School Address:_______________________________________

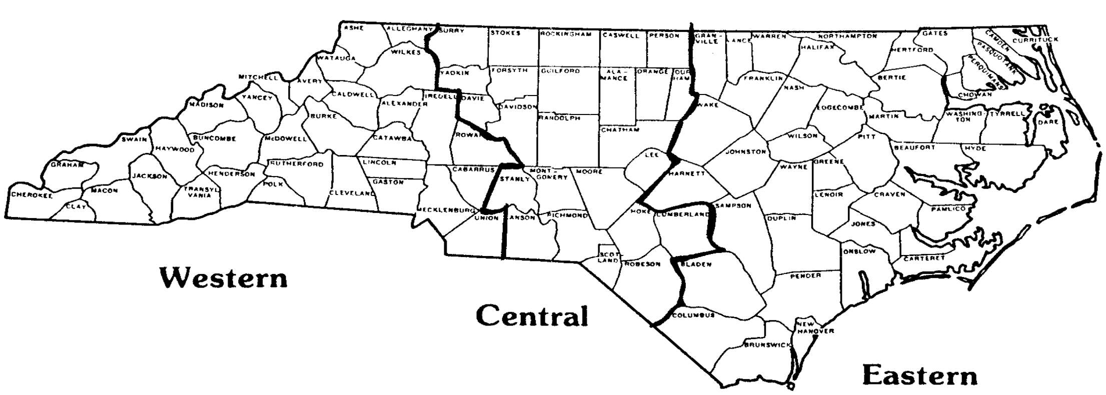
____

Teacher's Name:______________________________
*NCCTM Region: Eastern Central Western
(circle one) *See NCCTM Regional Map Below

Please submit the entries on **8 1/2 x 11 paper in black ink or black marker only.**
Entries must be postmarked by 1 March 2005
Mail to: Rebecca Caison, MATH LOGO CONTEST,
101 E. Laramie Drive, Mebane, NC 27302

North Carolina Council of Teachers of Mathematics

NCCTM Regional Structure



NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

MEMBERSHIP – NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

Name: ______________________________ Home Telephone: (____) - ____________

Address:______________________________ School Telephone: (____) - ____________

City: ________________ State: ________ Zip: _________ E-mail: ____________________

School System: ______________________________

MEMBERSHIP STATUS
❑ New ❑ Former/Renewing Member # _________

POSITION
❑ Teacher
❑ Department Chair
❑ Supervisor/Administrator
❑ Full-time College Student
❑ Retired
❑ Other ___________________

LEVEL
❑ K-3
❑ 4-6
❑ Junior High/Middle School
❑ Senior High
❑ 2-Year College/Technical
❑ 4-Year College/University

MEMBERSHIP DUES

❑ 1 year:	$10.00	____________
❑ 3 years:	$25.00	____________
❑ 10 years:	$75.00	____________
❑ Full-time Student:	$5.00	____________
❑ Contribution to Trust Fund:		____________
Total Payment Enclosed:		____________

Payment by ❑ Check ❑ Visa ❑ MasterCard

Card # ______________________________

Exp. Date ______________________________

Signature ______________________________

Please make your check or money order payable to NCCTM. Send this form and your payment to
NCCTM
PO Box 1783
Salisbury, NC 28145-1783
Payments by credit card may be mailed or faxed to
704-642-0840