## The Centroid

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- Addressing Equity Issues at Multiple Levels: Facilitating Equity in Mathematics Education
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- Reminiscing, Problem Posing, and Problem Solving with Magic Cards
- Evelyn Boyd Granville: Complex Solutions to Real-Life Problems
- 2008 NCCTM Mathematics Logo Contest Winners


The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics-pre-kindergarten through teacher education. The Centroid is published in January and August. Subscribe by joining NCCTM; see the Membership Form on the last page.

## Submission of Manuscripts

We invite the submission of news, announcements, and articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways.

News and announcements (president's messages, award winner announcements, professional development announcements, etc.) must be received by December 1 for the spring issue and by July 1 for the fall issue.

Articles that have not been published before and are not under review elsewhere may be submitted at any time to the address below. Submit one electronic copy via email attachment (preferred) or diskette in Microsoft Word or rich text file format. To allow for blind review, the author's name and contact information should appear only on a separate title page. Manuscripts should not exceed 10 pages double-spaced with one-inch margins. Figures and other pictures should be included in the document in line with the text (not as floating objects). Scannable photos are acceptable and should be large glossy prints mailed to the editor or minimum 300 dpi tiff files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.

Manuscripts should follow APA style guidelines from the most recent edition of the Publication Manual of the American Psychological Association. References should be listed at the end of the article, and should also follow APA style, e.g.,

Bruner, J. S. (1977). The process of education (2nd ed.). Cambridge, MA: Harvard University Press.
National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.
North Carolina Department of Public Instruction. (1999). North Carolina standard course of study: Mathematics, Grade 3. Retrieved October 17, 2005, from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html
Perry, B. K. (2000). Patterns for giving change and using mental mathematics. Teaching Children Mathematics, 7, 196-199.
Ron, P. (1998). My family taught me this way. In L. J. Morrow \& M. J. Kenney (Eds.), The teaching and learning of algorithms in school mathematics: 1998 yearbook (pp. 115-119). Reston, VA: National Council of Teachers of Mathematics.

General articles and teacher activities are welcome, as are the following special categories of articles:

- A Teacher's Story,
- History Corner,
- Teaching with Technology,
- It's Elementary!
- Math in the Middle, and
- Algebra for Everyone.


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## About the Cover

The Centroid logo is based on the following theorem: The limit of the sequence of midtriangles of a triangle is the centroid of the triangle.

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## From the Editor

Holly Hirst<br>Appalachian State University, Boone North Carolina

Welcome to the 2008-2009 school year! This will be my last issue as managing editor; I have enjoyed the work a lot, but after five years and 10 issues, it is time to let someone with fresh ideas take over the direction of the Journal. I will still be involved in the Centroid as the production co-editor and a reviewer, but Debbie Crocker will assume the managing editor role, and will handle submissions and correspondence.

We hope you enjoy this issue; the article on magic cards and on Evelyn Boyd Granville both contain activities ready to use with students. The article on addressing equity issues is a preview of a manuscript on equity currently under preparation.

Please keep those submissions coming! As always we are hoping you will share your good ideas with teachers from around the state.

## NCCTM State Conference: Oct 30-31

## Reported by NCCTM President Randy Harter

Our 2008 State Mathematics Conference, Shaping the Future, scheduled for October 29-31, in Greensboro, is structured in new ways this year to better address the need for leadership during these challenging times. Thanks to Conference Co-Chairs Pat Sickles and Judy Rucker and Program Co-Chairs Marta Johnson and Ron Preston for organizing an excellent professional growth opportunity for NC educators. At the Leadership Pre-session on Wednesday, October 29, Murrel Hoover will lead the morning session entitled "Focusing on Student Discourse: The Key to Effective Teaching and Professional Development." The afternoon session will feature updates on developments and innovations in North Carolina in curriculum and assessment led by Jere Confrey of NC State University, the mathematics leadership team from NCDPI, and others.

The main conference program on Thursday and Friday is organized in four grade level bands: K-2, 3-5, 6-8, and 9-12. Four outstanding keynote speakers, listed below, will launch the conference themes for teachers in each grade level band:

- Grades K-2: Susan Friel, Professor, UNC-Chapel Hill, School of Education, 2007 NCCTM Innovators Award recipient;
- Grades 3-5: Jere Confrey, Joseph D. Moore Distinguished Professor of Mathematics Education, NC State University, Friday Institute for Educational Innovation;
- Grades 6-8: Glenda Lappan, University Distinguished Professor and Acting Director of the Division of Science and Mathematics Education, Michigan State University, Connected Mathematics Project Director, Past President of NCTM;
- Grades 9-12: Murrel Hoover, Teachers Development Group, WVCTM Distinguished Service Award Recipient.
In their opening talks on Thursday, October 30, these keynote speakers will focus on the kind of learning environment and mathematical tasks that promote mathematical reasoning, problem solving, and student discourse. On Friday, to reflect further on these themes, the keynote speakers will be joined in a panel for their grade level band by NC classroom teachers, NCDPI mathematics consultants, and other major conference speakers such as National Math Panel member Doug Clements (K-2), standards-based curriculum developer Susan Jo Russell (3-5), textbook author Randy Charles (6-8), and National University of Singapore HS of Mathematics and Science Principal Hang Kim Hoo.

Another feature for the State Conference program on Thursday, October 30, is a four-part Learn and Reflect series on Equity: All Means All offered by a team of five presenters from our national affiliate, NCTM, Shonda Lemons-Smith of Georgia State University, Audrey Jackson of St. Louis Public Schools, Ruth Casey of University of Kentucky and T3, Harold Asturias of University of California-Berkeley, and John Staley of

Baltimore County Public Schools. The series will kick-off first thing Thursday morning with "Equity in K-12 Mathematics Education: Do We Have the Will?" Later on Thursday the series continues with break-out sessions for PreK-6 and 7-12 teachers, challenging participants to make high level mathematics accessible and engaging for all students, and closes Thursday afternoon with a reflection session focused on discussion questions distributed at the kick-off session.

Help us get the word out on this outstanding program. Register now and invite three colleagues to join you who have never attended a State Mathematics Conference before.
[http://www.ncctm.org](http://www.ncctm.org)

## Presidents' Messages

## State President <br> Randy Harter

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Challenges, Vision, and Leadership
Mathematics educators in North Carolina will be facing important challenges and decisions over the coming months. K-5 schools will be adopting new mathematics textbooks for 2009. The North Carolina Department of Public Instruction (NCDPI) will be seeking input from teachers in finalizing the revision of the Standard Course of Study for grades 6-12 in preparation for new textbooks for 2010. The State Board of Education has established challenging new high school graduation requirements for students entering grade 8 this fall, including the completion of Algebra 2 or Integrated Mathematics 3, plus one more math course, by all but a few anticipated exceptions for whom parents and school administrators agree require an "opt out" plan. The NC Blue Ribbon Commission on Testing and Accountability submitted its recommendations to the State Board of Education earlier this year. The recommendations call for new and different accountability models at all levels, especially in high schools, and curriculum revision focused on power standards and interdisciplinary connections. The commission's findings include concern that the current testing program and accountability system has not positively impacted high school graduation rates (currently $69.9 \%$ graduate in four years, $71.8 \%$ in five years), nor reduced the demand for remediation at community colleges and universities across the state. At the national level, The Center for Mathematics Education at the University of Maryland is hosting a National Leadership Conference in Washington, DC, this month entitled The Future of High School Mathematics: New Priorities and Promising Innovations in response to "concern about the condition of high school mathematics (that) has been expressed with renewed urgency in many recent advisory reports and policy recommendations." So it is an exciting and challenging time for mathematics educators in North Carolina and across the country.

The North Carolina Council of Teachers of Mathematics (NCCTM) seeks to provide leadership for its members and the education enterprise in the state to improve the teaching and learning of mathematics in these challenging times. Effective leadership requires a vision. The NCCTM vision for improving the teaching and learning of mathematics in North Carolina schools is shaped by the NCTM process standards (National Council of Teachers of Mathematics, 2000). We emphasize mathematical reasoning, problem solving, and communication. We want students to experience mathematics as a creative, sense-making venture of solving challenging problems individually and collaboratively. The development of number sense and insight into the many patterns and relationships that lead to computational and symbolic fluency are essential for effective problem solvers. Computational fluency includes accuracy, efficiency, and flexibility. The ultimate goal is mathematical power for all students. Mathematical power is "an individual's ability to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems" (NCTM, 1989).

The learning environment should be focused on worthwhile problematic tasks and meaningful discourse among learners in small and large groups. We are in the learning business, and ultimately what matters most in classrooms is what is going on in the minds of our students relative to the mathematics at hand. Instructional decisions and interventions should be driven by the teacher's insight into students' mathematical thinking.

## Textbook Adoption Decisions

In the months following the State Mathematics Conference, local LEAs will face decisions on K-5 mathematics textbooks that will significantly impact the learning opportunities for ten cohorts of NC students, from next year's fifth graders, the graduating class of 2017, through Kindergarten students of 2013-14, the graduating class of 2026. Teachers bear the responsibility for decisions that will impact the lives of these young students and their future opportunities. To what extent have our mathematics programs produced high school graduates who are (A) empowered as creative, competent, and articulate mathematical thinkers and problems solvers, and (B) equipped to consider, without remediation, university majors that require significant post-secondary mathematics and lead to promising careers in science, technology, engineering, and the mathematical sciences? How will the demands of the workplace change over the next two decades for the students directly impacted by our K-5 mathematics textbooks decisions in February, 2009? Among those students are many whom we hope to inspire to become mathematics educators, entering the field between 2021 and 2030 and beyond. Like many of us, their ideas of what mathematics is and what it means to do and teach mathematics will be shaped in lasting ways by their own school experiences. What an awesome opportunity and responsibility we have as teachers. We shape the future.

At last year's State Math Conference Leadership Pre-session, Barbara Reys, Director of the Center for the Study of Mathematics Curriculum at the University of Missouri, gave research-based guidelines for the selection of textbooks. Those guidelines are available at the very bottom of the NCCTM website. (See parts 2 and 3 in particular.) Among her key points:

- There are more varied options in math textbooks today than ever before.
- Textbooks available today reflect huge differences in philosophies about how students learn mathematics.
- The development of new types of textbooks was prompted by several factors, including: NCTM's vision for school mathematics; NSF's leadership in funding curriculum development; materials used in high performing countries; and documentation of redundancy and low cognitive demand of traditional American textbooks.
- Most publishers carry more than one textbook series (publisher developed and NSF-funded materials).
- Textbooks can serve as a catalyst for improving school mathematics programs. Critical elements include: leadership, shared goals, teacher buy-in, professional development.

Further assistance on preparation for textbook adoption is available at <http://www2.edc.org/mcc/resources/ default.asp>.

Too few NC teachers have had the opportunity to learn about the range of textbooks available today and what determines what goes into those books. Barbara Reys' presentation makes reference to two kinds of textbooks: Publisher-produced textbooks and NSF-funded textbooks. There are now just three big players in the school textbook business, plus a few smaller publishers aiming products for a smaller niche market. Publisherproduced textbooks are generally, of necessity, market-driven. They are developed in response to the demands of the market-place, that is, by the curriculum requirements of a few large states and by the preferences of the great majority of those who will be making decisions about what books will be purchased. Publishers cannot afford to consider what kind of books teachers ought to prefer, nor what kind of books academic research shows are best in promoting student learning, nor the implications for textbooks of the relatively low mathematics achievement of U.S. students in international studies. Their final objective is to maximize sales, and that drives everything. This is not a criticism of publishers. It is the reality of a free enterprise system in which CEO's answer to stockholders. Publishers are good at reading the market. That is why publisher-produced textbooks are used in the great majority of classrooms in NC and the U.S. They know what the majority of their consumers are looking for, and that determines their product. These books can come with a stack of supplemental materials that can be nearly a foot high and are first and foremost, by design, "teacher friendly." How are they working for our students?

Textbooks developed from curriculum projects funded by the National Science Foundation (NSF) are put together under very different conditions. Following the publication of the Curriculum and Evaluation Standards for School Mathematics by NCTM in 1989, NSF issued a national call for proposals for the development of innovative, comprehensive, new mathematics programs for grades $\mathrm{K}-5,6-8$, and $9-12$. They funded thirteen major projects to support these national standards that called for emphasis on mathematics as problem-solving,
reasoning, communication, and connections. Lessons had to be based on research on learning rather than the demands of the textbook market. Lessons were piloted in classrooms and then re-written based on the resulting feedback. Publication began in the mid 90 's. A few of these projects have since been funded by NSF for revision, and new editions have just recently become available. Go to [http://www2.edc.org/mcc/](http://www2.edc.org/mcc/) and click on Curriculum Summaries for a complete listing of these 13 projects. All three of the elementary projects listed there have implementation sites in NC, some since 1994. A few more recently funded NSF curriculum projects are also available now.

Without the four critical elements listed in Barbara Reys' presentation-leadership, shared goals, teacher buy-in, and professional development - schools are unlikely to seriously consider the standards-based NSFfunded materials that are designed to better meet the needs of students when implemented with fidelity. Other than allowing local schools to adopt textbooks not on the state approved list, an option many LEAs would discourage, the tradition-bound system administered by the State Textbook Commission has not been helpful, in past adoptions, in encouraging the use of these research-based NSF-funded textbooks. Any such texts that have made it through that system in past years were likely approved after an initial rejection and appeal. State testing policy has also inhibited such implementation at the high school level, but not at K-8. We are optimistic that current leadership at NCDPI and the State Board of Education will be more progressive. While national standards have certainly impacted the goals of many state-wide professional development initiatives since the publication of the first standards document in 1989, there has not yet been a single state-wide professional development initiative in NC with the explicit goal of supporting participating teachers and schools in a highfidelity implementation of any of these standards-based instructional materials, though there have been a few local projects. The result has been that many of our most informed teachers that have benefited from state initiatives have come to look to supplemental instructional materials offered at the NCDPI website for what they consider state of the art materials, while nearly all of their schools continue to adopt market-driven textbooks as their primary instructional resource, just as the schools of teachers who have not benefited from such projects. Textbook adoption decisions have a long lasting impact on the future of our students. It's time for the leadership in NC to take the next steps in developing shared goals, and promoting teacher buy-in through appropriate professional development so that more NC students can benefit from the major efforts of national leaders over the past two decades to produce research-based instructional materials.

## Eastern Region President Rose Sinicrope

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Greetings from the East!
All of us who attended the 2008 regional conference at Meredith were inspired by Jan Wessell's opening session. Jan shared her teacher's tools. Elizabeth Murray did a fantastic job in putting together the conference program. We, the conference committee-Elizabeth Murray, Tim Hendrix, Holt Wilson, Julie Cazin, Julie Kolb, Katie Stein, Kitty Rutherford, Alan Faulkner, and Rose Sinicrope - want to thank the 50 some teachers who worked so hard to present sessions. With Tim's help, rooms for all sessions were well-equipped and easy to find. Under Alan Faulkner's direction, students from our student chapters at North Carolina State University and East Carolina University and students from Meredith College set-up and cleaned-up for the conference. It was a very pleasant morning. We extend a special thanks to Dean Elizabeth Wolfinger and Meredith's School of Natural and Mathematical Sciences for welcoming us to campus and providing refreshments.

The eastern regional Mathematics Fair was held at East Carolina University on March 14. After many years of commitment, Bobbie Parker retired. Freda Winters, Katie Carbone Lynnly Martin, and Ron Preston-you did it! Thanks for a great Math Fair. Thanks to all the judges, teachers, parents, and especially students. The fair was a tremendous success.

The East was also busy with the High School Mathematics Contest, MathCounts, and the NCCTM Logo contest. There are so many NCCTM members who give of their time and talents to support the teaching and learning of mathematics. You do make a difference.

Plans will soon be underway for our regional conference on the last Saturday in February, February 28, 2009, at North Carolina Wesleyan College in Rocky Mount. Gail Stafford has graciously accepted the role of site coordinator. We hope that you will join us in a relaxed morning to learn something new, to visit with friends, and to get to know some of the wonderful teachers of mathematics in the east!

## Central Region President <br> Rebecca Caison

## rbcaison@mebtel.net

Greetings to the members of the Central Region.
"Go Wild with Math at the North Carolina Zoo" was a huge success! Anne Crawford did an outstanding job of planning and organizing the program. After attending the fantastic sessions in the morning, teachers were treated to an afternoon touring the NC Zoo. Special thanks to the Educational Staff at the NC Zoo; Carolyn Brown, Education Director at the zoo; NCDPI Staff; the NCCTM members who were speakers; the NCCTM Central Region Officers; and Josh Underdonk, Math Teacher at the Asheboro High Zoo School for making the conference a success. The current plans are to have the 2009 Central Region Spring Conference in the WinstonSalem area.

The Central Region Math Fair was outstanding. The enthusiasm for mathematics displayed by the students was exciting to watch. Thanks to Wendy Rich and her committee for a job well done. Congratulations to Barbara McGill who was elected president-elect of the Central Region. She has been active in NCCTM and brings much experience to the Board of Directors. I look forward to working with her as we plan for the upcoming year. I hope to see everyone at the NCCTM annual conference in Greensboro, October 30th and 31st. The Conference and Program Committees have been hard at work to plan an outstanding conference. Check out all the opportunities on the NCCTM website. If you have any questions of suggestions contact one of the regional officers or me. I hope this school year is the best ever for each of you.

## Western Region President <br> Debbie Crocker

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It's hard to believe another school year has begun! I hope all of you are energized from summer and ready to start again! We have some important dates for this year to share already.

The Western Region Conference was held on Saturday, February 23, 2007 at Jacobs Fork Middle School. A special thanks to all of the speakers who volunteered their time to make the conference a success! The participation by pre-service teachers was great, but we would like to increase participation overall. With that in mind, the next Western Region Conference will be on Saturday, February 21, 2009. Contact me if you would like to volunteer to speak to either teachers or pre-service teachers at the Western Region Conference this year. Share your ideas!

The Western Region Math Fair was held at Plemmons Student Union on the campus of Appalachian State University on Saturday, April 12, 2008. Participation was great with 299 students, 245 projects, and 43 schools involved. A huge thank you goes to coordinators Betty Long, Cindy Robinson, and Theresa Compton for all of their hard work! Additionally, a number of parents, teachers, Appalachian students, and other school personnel volunteered time to make the fair a huge success. Thanks to all who volunteered. Mark your calendars! The next Western Region Math Fair will be Saturday, March 28, 2009 at Plemmons Student Union on the campus of Appalachian State University. Get your students started on their projects! Volunteer to judge, help in the student holding rooms, do registration, or be a runner!

Don't forget the NCCTM State Conference 30-31, 2008 at Koury Convention Center in Greensboro. The program promises to be a great one!

Good luck as you begin the new school year! Put NCCTM on your calendar and participate in all of the wonderful mathematics events and activities sponsored and supported by your professional organization. You can contact me at crockerda@appstate.edu with questions, concerns, or ideas. I hope to hear from some of you!

# Addressing Equity Issues at Multiple Levels: Facilitating Equity in Mathematics Education 

Dr. Vincent Snipes<br>Winston-Salem State University, Winston Salem, North Carolina<br>Dr. Judith Reed<br>National Council of Teachers in Mathematics (NCTM), Reston, Virginia<br>Patricia Valdez<br>Pajaro Valley Unified School District, Watsonville, California

## What is Equity?

If you pick up just about any policy publication published in the last ten years, you are bound to find the word "equity" or the phrase "for ALL students" somewhere in the pages. Concern for achievement gaps, teaching students of different achievement levels, different ethnicities, and different languages (to name a few), keeps the issue of equity at the forefront of discussions about mathematics education. However, for every publication with equity mentioned somewhere, there is likely a slightly different definition also assumed by the writers of the document. Even among those who research mathematics education, multiple definitions of equity have emerged and influenced the field. Some definitions focus more on providing opportunities for all students, including having high expectations and support for student success (National Council of Teachers of Mathematics, 2000). Others discuss equity in terms of gaps in opportunities and achievement between students of different racial background and socioeconomic status (National Research Council, 2001).

Before we delve into the main focus of this paper, it is crucial to divulge the definition of equity that the authors of this paper subscribe to. For the purposes of this document, the authors borrow the definition of equity suggested by Allexsaht-Snider and Hart. Allexsaht-Snider and Hart (2001) define equity with regards to two premises: 1) "that all students, regardless of their race, ethnicity, class, gender, or language proficiency, will learn and use mathematics;" 2) "all of the people who are involved with and interested in the education of children must become aware of the social, economic, and political contexts of schooling that can either hinder or facilitate mathematics learning for underrepresented students (Apple, 1992)" (p. 93). Implicit in this definition are requirements that must be satisfied including equitability in (1) access to resources, (2) instruction, and (3) outcomes for ALL students. This definition points to the importance of equitable opportunities for learning but also the importance of considering outcomes as well.

Before moving forward with this definition, it is important to have clarity on what it is that we are actually referring to when we talk about ALL students. While as educators we are interested in the welfare of each and every student in our class no matter who they are, in discussions of equity we are typically concerned with those students who have historically been underserved by mathematics education in the U.S. These students include those listed in the definition by Allexsaht-Snider and Hart, but also those suggested by Bishop and Forgasz (2007), including students from rural areas and "learners with physical and mental impairments" (p. 1146).

Success for ALL students in mathematics education demands equity, which includes high expectations and support for the students. Barriers to equity in mathematics education deny students access to financial and physical resources, an enriched and challenging mathematics curriculum, and knowledgeable and caring teachers. The following are suggestions and examples that can be utilized at the classroom, school, and district levels to facilitate equity in mathematics education.

## Facilitating Equity in Mathematics Education

## Classroom Level

- Believe that all students from diverse backgrounds can be successful in the mathematics classroom.
- Teachers should reflect on their own personal teaching and interaction with students.
- Teachers should use available resources and mathematical tasks involving high cognitive skills throughout the entire high school mathematics curriculum, not just in the upper level courses.
- Encourage all children to take risks and construct mathematical meaning.

Example: During the school year, the school curriculum specialist observed different mathematics teachers several times. After several visits, the specialist noticed that one of the mathematics teachers had different expectations for her two sections of geometry. One section was the college prep section and the other section
was the regular section. Both sections had the same state standards to follow and used the same textbook, but the teacher showed a difference in the resources that she made available to the different sections. The college prep geometry students utilized dynamic geometry software on the computers all school year, but the teacher's regular geometry section never went to the computer lab to use the software. The curriculum specialist at the school also noticed that the college prep section was always more enthusiastic about their geometry class than the regular geometry section. The school curriculum specialist had a meeting with the classroom teacher and discussed her observations, and now the classroom teacher takes all geometry sections regardless of the level to the computer lab to use the geometry software.

## School Level

- Schedule mathematics department meetings to discuss equity issues.
- Encourage strong and experienced teachers to teach both the lower and upper level mathematics courses.
- Provide all mathematics teachers with adequate technology equipment and supplies no matter what courses they teach
- Send mathematics teachers to professional development institutes focusing on equity issues and other mathematics content topics.
Example: At one school, the students kept doing poorly in the Algebra I classes. The principal requested that every mathematics teacher teach at least one Algebra I class to help to improve the failure rate, even the stronger experienced upper-level teachers. The hope was that the experienced teachers would share successful teaching ideas with the less experienced teachers. As a mathematics department, the teachers discussed teaching strategies and what common assessments to utilize. This dialogue between the mathematics teachers resulted in a positive improvement in the performance of the students in Algebra I at the school.


## District Level

- Encourage reasoning, inquiry, and explanation in the classrooms.
- Ensure district policies do not suppress the mathematics education of various student populations.
- Provide incentives to recruit "highly qualified" mathematics teachers to low performing schools
- Continuously review the alignment of the mathematics curriculum, instruction, and assessment used in the mathematics classroom.
Example: All school districts need to examine their policies annually to make sure the policies are equitable. In one school district, teachers expressed their concern about what they felt was an unfair district policy that would deprive students of taking quality mathematics courses. The district policy that caused an uproar was that if a student took Algebra I in the 8th grade with a teacher that was not fully licensed, then the student would have to take Algebra I over again in the 9th grade instead of taking Geometry. This policy kept many students from taking an additional mathematics course that would have better prepared them for college or the workforce. With the shortage of licensed mathematics teachers in this country, students were being penalized for things out of their control. Fortunately, the district did change the policy and the state since then has come up with policies about courses taken in middle school, which was a barrier to "mathematics for all."


## Conclusion

We must raise expectations throughout the educational community for the mathematics achievement of ALL students, especially those from traditionally underserved populations. "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000). Each student is unique and requires that teachers address these different educational needs. We must demonstrate to students that we value them as individuals and want them to succeed in their mathematics courses. From teachers, administrators, parents, and community leaders, everyone's effort is required to ensure the creation of a content rich, supportive, equitable mathematics environment for ALL students.

## References

Allexsaht-Snider, M., \& Hart, L.E. (2001). "Mathematics for all": How do we get there? Theory into Practice, 40(2), 93101.

Apple, M.W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. Journal for Research in Mathematics Education, 23, 412-431.
Bishop, A. \& Forgasz, H. (2007). Issues in access and equity in mathematics education. In F. Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp. 1145-1167). Charlotte: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.
National Research Council. (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.

## Mini-grants

The North Carolina Council of Teachers of Mathematics through its mini-grant program, provides incentive funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research, which will enhance the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. Available to current members of NCCTM, the mini-grants are awarded by each of the three regional organizations to members within their geographic boundaries (If you incorrectly identify with the wrong region, your proposal will be ineligible for funding). A total of $\$ 15,000$ is available each year for the state's mini-grants, with each region awarding approximately $\$ 5000$ in grants to its members. In recent years, approximately $30-35$ proposals have been partially or fully funded, for an average grant of just less than $\$ 800$.

Grant proposals must be postmarked by September 15 (or emailed on that date), and proposals selected for funding will receive their funds as soon as possible after the state conference. You will receive an email confirmation of receipt once your proposal has been received. If you do not receive a confirmation within one week, it is your responsibility to follow-up with the Mini-grant Coordinator.

## Directions

The directions and application are available on the NCCTM website [http://www.ncctm.org](http://www.ncctm.org). Please read all directions carefully and fill out application and cover sheet completely, as directed. Failure to correctly list the NCCTM region and membership number will cause your application to not be considered. Be sure that your NCCTM membership is current and active for the 2007-2008 school year! Be sure that it is a NCCTM membership and not NCTM or some other organization. Each year we have applications that cannot be considered because of the membership requirement.

## Are you wondering what you could ask for from the mini-grant program?

Possible projects for consideration include: Math clubs, field days, contests, math activities and laboratories, research projects/topics, family math, and parent workshops, etc. This list is not meant to limit you to these ideas. Creativity is encouraged!

For a specific example of a funded project, and the impact it had on both middle grades and high school students, see the mini-grant report on the Brunswick County Early College High School River Project on the next page.

## Mini-grant Report

Music in Mathematics<br>Ashley Cullipher<br>Northside High School, Pinetown, North Carolina

As the recipient of a 2007 NCCTM mini-grant, I held a "Music in Mathematics" competition at my school this semester. The purpose of the competition was to get students motivated to learn through creating songs about topics in mathematics. Students must value their education and feel empowered to develop their own understanding of mathematics. This project is my approach to engaging students in learning mathematics.

Students have difficulty learning mathematical concepts and remembering mathematical formulas. However, they easily recall the words and music to all their favorite songs; so, we, as teachers, need to take advantage of the brain's ability to learn information set to music. Through personally creating songs about difficult topics that the student has encountered over the years, the learning process transitions from meaningless memorization to a powerful conceptual understanding of mathematics. Through the engagement of song writing, students will connect by rhyme or song, the mathematics that we, as educators, desire them to know.

It has been proven through brain research that music can boost thinking and intelligence and carry words to the unconscious mind (Jensen,1995). The use of music is an effective strategy to learn new or difficult concepts, to engage students' attention, to increase energy levels, to stimulate creativity, and, in the case of this project with students creating their "own" math song, giving students a sense of ownership (Rogers, Ludington, \& Graham, 1999). A sense of ownership is created when students are held accountable for their education. "When students feel personally involved in the material, they tend to look harder to find the value in the information, and how it might apply to them" (Allen, 2002, p. 93). By developing this sense of ownership, students are motivated to learn what is being taught as opposed to the teacher's trying to force them to learn the material. Everyone appreciates feeling valued for his/her contributions, so why not learn some math in this process?

## Project Description

The project consisted of a talent competition involving the creation of mathematics lyrics. The competition was open to all school members to participate. A week prior to the competition, I used our school news to broadcast the upcoming competition. This announcement was published daily in the school announcements posted in each classroom and read aloud to the students during second period each day for a week. The call was for students who had a knack for writing and those who enjoyed music to come and help the students of our school make mathematics more interesting and meaningful by competing to write math songs for publication. It was announced that the ten best songs would be produced on a CD made available to all of our math students, at no cost to them, to help them learn the difficult material that was recorded in the selected songs.

All interested students were told to research a topic in mathematics that they had experienced difficulty in learning and to write down all the facts that were needed to learn the mathematical concept. They were told to find a familiar tune and to rewrite their facts to the beat of their song of choice. They were given the suggestion to use commonly known songs, like camp songs, nursery rhymes, folk tunes, or even their own rap song for their melody. The students were given three weeks to submit their song lyrics. They were encouraged to work with a partner for assistance with ideas and support.

## Requirements for the Project

In order to complete this project, some materials were required to be purchased. With the help of a NCCTM mini-grant that I was awarded in October 2007, I was able to purchase everything that I needed to make this a successful, annual project. I purchased Roxio Record Now 9 Music Lab (PC) and the system requirements that my computer needed to run the program, an external CD/DVD burner, a microphone, a CD Boom box for class use, and enough CDs, CD cases, and CD labels to fund the project for the first year.

## Implementation of the Project

By the end of the three weeks, I had 25 songs submitted for review. The songs submitted were topics representing Occupational Math, Pre-Algebra, Algebra I, Geometry, and Algebra II. There were many great
songs submitted, and it was a difficult decision narrowing it down to just ten songs. It was so hard in fact that I actually decided to select 13 of the songs for publication, with at least one song from each of the participating subjects.

The chosen song composers came to record their songs during my planning period or after school, which ever they could fit into their schedule. Many requested to have a support group with them to help them sing. I fully supported this request. The more students who got involved in this project increased the number of students who were impacted by learning mathematics. After all 13 songs were recorded, a team of computer savvy students from our school worked after school and during lunch break to publish the songs. Some of the chosen song composers were so excited that they came to practice their song using their guitar or beat producer as shown in the pictures below. All students in all math courses this semester were given a CD before Christmas break as an early Christmas present.

## Student Feedback

In the hand-out that was distributed to competition participants, I had suggested students submit a paragraph of feedback for me to use for future reference about their thoughts on the project. Students had been asked if they enjoyed the project and if they thought it was beneficial. Most of the students complied with my request, and I was able to get great feedback in terms of the impact of the project on the students. Students repeatedly said that it was a challenging but enjoyable experience.

In general, students doubted their ability to write the lyrics to a song but admitted that after completing their song the mathematics process involved had become much easier to do. One student summed it all up when he said, "Upon completion of our song, I felt satisfied and somewhat proud of our little creation. I know that our song will help me do my math because when I sing the song to myself, it answers any questions that I might have about functions" (Thack Cutler "What are Functions?").

Overall, the students loved the competition and everyone has enjoyed and benefited from the publication of our math music CD. The song composers were excited about producing the CD, and all students were excited and appreciative of receiving the finished product. I even had students who were not currently in a math class request a copy of the CD. I have had students come to me already with ideas for our next competition. This was a great experience for our school.

## "What are Functions?" Beat by: Yamaha Authors: Thack Cutler and Grant Fath

You got your $X$, now that's your domain. Don't forget your Y , which is the range. A relation is a set of ordered pairs. Now you know why that is there.

Yeh, Functions have strict rules! But that's alright, cause they are cool. If a relation makes a U or a V ,
Then it's a function, now don't ask me. But if it makes a C, then no way
It ain't gonna work, cause it touches both ways.
Yeh, functions have strict rules
But that's alright, cause they are cool.
To check a function use the vertical line test Cause it works the very best. Now think about what you learned today How to tell functions, the right way.

Yeh, functions have strict rules But that's alright, cause they are cool Word to your mother, peace out!

## "Multiply Binomials" <br> (To the tune of "I'm Walking on Sunshine" by Katrina \& the Waves)

Authors: Catt Hoagland and Emily Stutts
I used to think monomials confused me, but now they're conquered,
And I just can't wit 'til binomials are done too.
I can add and subtract but I can't get multiplying down.
So I'm just gonna "F-O-I-L" like I was taught to do!
Multiply binomials, whoahh-h,
Multiply binomials, whoahh-h,
Multiply binomials, whoahh-h,
And don't feel good!
Hey, alright now
And don't feel good!
Hey, yea!
I would add and not multiply, but now I know how.
You gotta times the First factors in line.
Then you multiply Out in parenthesis,
It's not that confusing.
You only times Inner and Last and you can simplify now.
I can multiply-y, whoahh-h,
Multiply binomials, whoahh-h,
Multiply binomials, whoahh-h,
And it's so easy!
Hey, alright now
It's just so easy,
Hey, yea, oh yea
And don't it feel good!
I multiply; I do the first, then the outer, that's really real. I multiply, next it's inner, and the last, now simplify.

I'm multiplying, baby, oh!
I'm on sunshine, baby, oh!
Multiply binomials, whoahh-h,
Multiply binomials, whoahh-h,
Multiply binomials, whoahh-h,
And it's so easy,
l'll say it again, now!
It's just so easy!
"Trig Ratios"
(To the tune of Winnie the Pooh)
Authors: Byant Fletcher and Kasey Clayton
Sine, Cosine, Tangent
Sine, Cosine, Tangent
Sine is opposite over hypotenuse
It's in right triangles
Sine, Cosine, Tangent
Opposite over hypotenuse
Cosine, Tangent Sine
Cosine, Tangent, Sine
Cosine is adjacent over hypotenuse
It's in right triangles
Cosine, Tangent, Sine
Adjacent over hypotenuse
Tangent, Sine, Cosine
Tangent, Sine, Cosine
Tangent is opposite over adjacent
It's in right triangles
Tangent, Sine, Cosine
Opposite over adjacent


Pictured with permission left to right: Austin Latham and Colby Nixon

## Future Plans

Since the materials required in creating and recording music for publication has been funded by NCCTM, this project can be managed at a reasonable price each year. The only ongoing cost would be that of purchasing blank CDs, and the math club could raise the funds needed there. Therefore, this is a very feasible project to continue each year that will benefit all math students in the present and for years to come. I plan for this to be an annual project held at our school. I have heard positive feedback from both students and fellow faculty members. The best thing is when you hear students and faculty members singing songs about math in the hallways or in class. If you want a new, motivating teaching strategy for your mathematics classroom, give "Music in Mathematics" a try.

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## Math in the Middle

Reminiscing, Problem Posing, and Problem Solving with Magic Cards<br>Greg Harrell<br>Valdosta State University, Valdosta Georgia

How well we do is determined not just by our gifts and grit, but also by social background, networks, family connections
and - powerfully - by how well our parents did.
Life is not merely a footrace, but a relay race.
It matters a lot how much headway the previous runner
has made when he hands you the baton.
-William Raspberry, Washington Post
As we go through life we cannot help but look back now and then at our past. As teachers, we realize that our journey has been one of cooperative learning. We have learned a great deal from others and, hopefully, others have learned from us. I had many teachers and friends who were good influences both within and outside of academics. Many of my most vivid memories are from the time that I was in elementary and middle school. The small school that I attended had fewer than 200 students in grades K-8. We had sports teams that competed against other schools. There was a strong sense of community.

This time also played a key role in increasing my interest in mathematics. Fortunately, I had many good teachers. One of those teachers, my father, taught me mathematics in grades 6-8. At the time, I did not envision myself as a future mathematics teacher. Yet, unknown to us both, my father was passing the baton to me. In class one day, he brought in a set of five cards, each card with 16 numbers between 1 and 31 (Figure 1). Using these cards, he could determine a student's birth month or day as well as any other number between 1 and 31. "Wow, he found my birthday!" students thought. As the class discovered the birthdays of several students, we found meaning and interest in what was being done. Of course, we wanted to know how it was done so we could do it, too!

My father made the magic cards for a group project while working toward his master's degree (M.Ed., 1974). Several years ago, he gave me his set of magic cards. Since then, I have enjoyed exploring the mathematics behind the use of the cards, and have developed mathematics activities using the cards. I have


Figure 1. Magic Cards used them with both middle school students and future middle school teachers, as I strive to pass the baton of an enjoyment of mathematics to many others. It is my hope that you can reminisce while doing some problem solving, then let your students do some problem solving as well. The activity lends itself well to individual work, small-group work, and whole class discussions.

## Initial Investigations

During the summer, I worked with sixth, seventh, and eighth grade students together in a single classroom. After a class session on exponents, I started the magic cards activity. Each card is approximately $1 \mathrm{ft} x 1 \mathrm{ft}$ in size, so students throughout the classroom can see the numbers on the cards. I hold the five magic cards in a stack and display each card one at the time. I ask a student to tell me whether or not his or her birthday is on the card. I then move the card to the back of the stack. I continue this process until I have a "yes" or "no" response from this student for each of the five cards. The sum that results from adding the numbers in the top left corner of the cards for which the student replies "yes" gives his or her birthday.

Numerous volunteering hands shoot up. "Find my birthday." After the third or fourth time I ask, "How am I doing this? How does it work? If you think you know how to use the cards, let me know, and I will give them to you to try." Within minutes, Steve wants to try. "Find out his birthday," someone said to Steve, referring to me.

Steve says, "Ok. Is it on this card?" "No," I reply. Steve looks carefully at each card for which I reply "yes." "Your birthday is on the 5th," he says. "That's right, how did you do it?" I ask. Steve says that he just looked at each card carefully to try to find the number that was on all of the "yes" cards. Another student points out that there are other numbers on the two cards that I replied "yes" to besides just the number five.

As we continue the investigation, I place the magic cards with "yes" responses together on the left side of the whiteboard tray and the cards with "no" responses on the right side of the whiteboard tray so that the students can study all of the cards simultaneously. I also give the hint "Remember the sequence of numbers 1,2, $4,8,16, \ldots$ that we saw the other day? Does anyone see that sequence of numbers on the cards?" After looking at the cards, a student sees the powers of two in the top left corner. I ask them to focus on that sequence of numbers and the cards that have "yes" replies. It's not long before Steve again wants to try the cards. This time, he doesn't try to memorize the cards. He quickly gets the right birthday. "How did you do that?" other students ask. Continuing the investigation, other students figure it out, then they let the remaining students know how to use the cards. The students have seen magic transition into mathematics.

## Further Explorations

After the students see that the number is obtained by adding the top left numbers on each "yes" card, I pose some problems for them and give them time to explore and look for patterns so they can pose their own problems. A good problem is said to generate more problems (Curcio, 1987, p. 11). With the magic cards, many questions and problems naturally arise. By using the "what-if-not" strategy of problem posing (Brown \& Walter, 1990), the following questions may arise:
(1) What if all of the numbers were not on the magic cards? Can the missing numbers be determined?
(2) What if there were not five magic cards, but four or six instead?
(3) What if the person choosing the number was not choosing as high as 31 ? Do we still need five magic cards?
(4) What if we were limited to the numbers 1 to 24 ?
(5) Can we construct magic cards that only include the numbers 1 to 15 ?
(6) Can we construct magic cards that include the numbers 1 to 50 ?
(7) What is the lowest number above 31 that will give magic cards that each have the same number of numbers on them?

Our class first explores the set of magic cards with numbers missing from each card-(2) on the Activity Sheet. The students look for patterns to complete the missing numbers on all of the cards. When asked which card is hardest to complete, most of the students reply that the card with 2 in the top left corner was hardest. Almost all of the students who complete this card tend to see an "add 1 , add 3 , add 1 , add $3, \ldots$ " pattern in the numbers by looking at them in text-reading order. Few notice the add-eight pattern looking down the columns.

By removing more numbers from the magic cards or reordering the numbers so there is no apparent pattern, the focus then moves to the numbers in the top left corner. Students must then determine what cards should include each number. For example, should the card with 16 in the top left corner include the number 22 on it? What other cards should include the number 22? Are there any numbers between 1 and 31 that should not be on the cards?

Given the powers of two ( $1,2,4,8,16$ ) from the "yes" responses to the cards, students have no trouble adding the numbers to get the resulting birthday. They do, however, have trouble when I give the birthday (such as the 21 st), then ask them to find the powers of two that must be added to give that birthday $(21=16+4+1)$. Once they get better at doing this, I ask them if any counting number between 1 and 31 can be written as the sum of powers of two. I then ask the students to make their own set of four magic cards that includes only the numbers from 1 to 15 on them.

## A Deeper Understanding

When making the cards with powers of two in the top left corner, a final question seems to naturally arise. I ask the students, "Can we write all counting numbers as the sum of selected powers of two without using any numbers in the sum that are not powers of two?" To try to reason out this question, the students write a mixture of numbers beyond 31 as the sum of powers of two. Some students want to use the same numbers twice, so I show them the cards again so that they realize that each power of two appears on only one card.

According to the Principle of Expanded Form, we can write all base two numbers as the sum of powers of two (Butts, 1973, p. 44). Since we can write all base ten counting numbers in base two, then we can write all base ten counting numbers as the sum of powers of two as well.

## Student Comments and Evaluations

The middle school students did five activity-based projects with me during a six-week summer session. After the students completed all five activities, I gave them a survey with the question, "Which of the projects did you enjoy the most? Why did you enjoy it the most?" More students selected the magic cards activity as "most enjoyed" that any other project. They explained that they liked the magic cards the most because "it was a challenge to figure out;" "they made us think and at the same time have fun;" "I was getting to learn while having fun."

## Conclusion

Little did I realize as a middle school student that I would one day teach with the same magic cards that I learned with as a student. Nor did my father anticipate the explorations that giving the cards to me would generate. After receiving the cards, I once again explored the mathematics behind the cards. This time, however, I explored the cards as a teacher, not as a middle school student. As I explored, I learned more mathematics and developed an inductive proof that proves that any counting number can be written as the sum of the powers of two. I would enjoy passing the baton and sharing it with you! In addition, I developed activities for middle school students that are based on the magic cards. Over time, I organized these activities in the Magic Cards Activity Sheet on the next page of this issue.

The magic cards have also generated reflection on my journey as a teacher. The personal connection to the cards has prompted me to read books such as The Courage to Teach by Parker J. Palmer. He states that "knowing myself is as crucial to good teaching as knowing my students and my subject" (Palmer, 1998, p. 2). As I reflected on my teaching journey, I also reflected on the first NCTM curriculum standards, which were written some 15 years after I was introduced to the magic cards as a middle grades student. The Curriculum and Evaluation Standards for School Mathematics for grades 5-8 (NCTM, 1989, p. 66-67) focused on investigations that motivate students, are relevant to the students, and actively engage the students as learners. The magic cards do an excellent job of meeting these standards. This has shown to me that the Standards existed in the minds of many teachers well before they were written. My father always stressed the importance of motivation and relevance, which laid the groundwork for me as a beginning teacher. I give thanks to him and to all of the others who have helped shape me as a teacher and a person.

It is my hope that you can enjoy the magic cards with your students. I also hope that you can reflect upon your mathematical experiences as a student. Undoubtedly, there are some mathematical batons that you can pass on to your students. As you reflect upon your mathematical experiences as a student, you may learn some more mathematics and learn more about yourself as a teacher as well.

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## Middle Grades Activity: Magic Cards

1. Explain how to use the set of Magic Cards.
2. Fill in the blanks to complete the set of Magic Cards.

| 1 | 3 |  | 7 |
| :---: | :---: | :---: | :---: |
| 9 | 11 |  | 15 |
| 17 |  | 21 |  |
|  | 27 | 29 |  |


| 8 | 9 |  | 11 |
| :---: | :---: | :---: | :---: |
| 12 | 13 | 14 |  |
| 24 |  | 26 | 27 |
|  | 29 | 30 | 31 |


| 4 | 5 | 6 |  |
| :---: | :---: | :---: | :---: |
| 12 | 13 |  | 15 |
| 20 |  | 22 | 23 |
|  | 29 | 30 | 31 |


| 2 | 3 | 6 | 7 |
| :---: | :---: | :---: | :---: |
| 10 | 11 | 14 | 15 |
| 18 | 19 | 22 | 23 |
|  |  |  | 31 |


|  | 17 | 18 |  |
| :--- | :--- | :--- | :--- |
| 20 | 21 |  | 23 |
| 23 |  | 26 | 27 |
|  | 29 | 30 | 31 |

3. If someone's birthday is on the following dates, give the numbers that you will add to get their birthdays.
(a) 14
(b) 21
(c) 30
4. Suppose you had a set of Magic Cards that will give larger numbers than the set of five Magic Cards. How will you get the number based on someone's responses to you as you display the cards? Give the numbers that you will add to get someone's number if his or her number is:
(a) 86
(b) 127
5. Do you think that all counting numbers can be written as the sum of powers of two? Explain.
6. Write the following numbers as the sum of powers of two. Complete the table based on the powers of two used in the sum. If a power of two is used in the sum, write "Yes," otherwise write "No." Notice that this is the same "Yes" or "No" response that you will get to the question "Is your birthday on this card?" when using the Magic Cards. If you entered "Yes," then write "1" below your "Yes." If you entered "No," then write "0" below your "No."
(a) 14
(b) 22
(c) 31

|  | $2^{4}=\mathbf{1 6}$ | $\mathbf{2}^{\mathbf{3}=8}$ | $\mathbf{2}^{2}=\mathbf{4}$ | $\mathbf{2}^{\mathbf{1}}=\mathbf{2}$ | $\mathbf{2}^{0}=\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (a) $\mathbf{1 4}$ | No | Yes | Yes | Yes | No |
|  | 0 | 1 | 1 | 1 | 0 |
| (b) 22 | Yes | No | Yes | Yes | No |
|  |  |  |  |  |  |
| (c) 31 |  |  |  |  |  |
|  |  |  |  |  |  |

7. Binary Number System. The binary number system is a base-two number system. Similar to our baseten number system, the base-two number system uses place value. The base-two number system only has two digits, 0 and 1. These binary digits are usually called by their abbreviated name, "bits." The binary system is the language of computers. When you completed the table above, you were actually constructing a binary number with five place values. If you write down the digits in order left to right, then you have written a binary number. As an example, we write 14 as the binary number 01110 , or 1110 (since we typically don't record leading zeros). Write the following base-ten numbers as binary numbers:
(a) 22
(b) 31
(c) 17
8. Use the $3 \times 5$ index cards provided to make your own set of four Magic Cards. What limitation will you place on the numbers when you ask someone to "pick a number?" Explain.

# Women and Minorities in Mathematics 

Incorporating Their Mathematical Achievements Into School Classrooms

Evelyn Boyd Granville: Complex Solutions to Real-Life Problems<br>Sarah J. Greenwald<br>Appalachian State University, Boone North Carolina



Evelyn Boyd Granville in 2001

In 1949 Evelyn Boyd Granville became the second black woman we know of to earn a Ph.D. in mathematics. Granville is well known for examining diverse and complex solutions to real-life problems - quite literally, as her doctoral work was in the field of complex analysis, but this theme can also be found in her approach to life and in her teaching philosophy. Over the course of her career, she worked for NASA and others in support of space missions, and then she transitioned to college teaching, where she focused on the mathematics education of future teachers. Today she continues to speak as an advocate for mathematics. Granville advises that: "Life is best lived when you try to leave the world in better shape." (Smith, 2007)

By applying Granville's own definition to her efforts to make the world better, it is easy to see that Granville has led a rich and wonderful life.

## First Black Women Mathematicians

The earliest black women who studied mathematics faced many barriers:
Over the years, black women who might be disposed to pursue a career in mathematics faced the "double whammy" of racism and sexism. Like blacks, women were not considered to have the mental skills necessary for advanced mathematical inquiry. For all women, and especially for black women, the field of mathematics was essentially shut tight. (The Journal of Blacks in Higher Education, 2001)

## Martha Euphemia Lofton Haynes

In 1943 Martha Euphemia Lofton Haynes (1890-1980) became the first black woman we know of to conquer the race and sex barriers at the Ph.D. level in mathematics. She earned her Ph.D. from Catholic University and then she had a distinguished career in Washington, DC. She taught in the public schools for 47 years, and she occasionally taught part time at Howard University. Her husband was a deputy superintendent of public schools. After her retirement she chaired the DC Board of Education, where she had a central role in the integration of DC public schools. In 1976, she reflected, "I have been a mathematics scholar all my life, through high school, through college, and then to get my doctor's degree in mathematics. . . I I didn't expect to get my doctor's degree, never, in mathematics, but I wasn't surprised in other areas because I enjoyed it so much" (Kenschaft, 2005).


Martha Euphemia Lofton Haynes ca. 1900-1910

## Evelyn Boyd Granville

Evelyn Boyd Granville also grew up in Washington, DC. She was born on May 1, 1924. Haynes' successful efforts to desegregate the DC public schools came much later, so it is not surprising that Granville attended a segregated high school. The school was excellent and Granville feels that this was due to teacher training and dedication:
"Although the systems were separate, the colored system was in no way inferior to its counterpart. The system achieved a national reputation for excellence because teachers and administrators were well
trained in their subject areas and were dedicated to providing the kind of education that students needed to be able to compete in a larger community." (Case \& Leggett, 2005)

Inspired by her high school teachers and with the encouragement of her mother and aunt, she graduated as valedictorian of her class, and then received a partial scholarship to attend Smith College. Her mother and aunt made many sacrifices in order to help her make up the difference between her scholarship and her bills (Kenschaft, 1981). Granville earned degrees in mathematics and physics from Smith. She earned her Ph.D. from Yale University in 1949, becoming the second black woman we know of to obtain a doctorate in mathematics.

Granville spent a year at a research postdoctoral position. She then interviewed for jobs but she encountered discrimination. At one school, she later found out, when the hiring committee discovered that she was black, "they just laughed," and the dean said they "would have to change the plumbing" (Murray, 2000). At the national level, she advocated for the integration of mathematics conferences and she was an outspoken critic of discrimination (Inniss and Bozeman, 2006). She accepted a position at Fisk University, a historically black institution, but after two years at Fisk, she moved to jobs in government and industry, which offered more opportunities. She used numerical analysis to aid in the design of missile fuses, and she later worked on trajectory and orbit analyses for the Vanguard, Mercury, and Apollo space projects:
"I found employment in government and private industry, where I had to study on my own areas of mathematics (mainly numerical analysis) needed to do the projects assigned to me. Whenever I speak to groups of young people I always advise them that learning never ends. The projects I worked on were in no way related to my thesis topic." (Granville, 2007)

In 1960 Granville married Reverend Gamaliel Mansfield Collins. After her first marriage ended in divorce, Granville returned to the academic world in 1967 to concentrate on teaching:
"How do you teach the beauty of mathematics, how do we teach them to. . . solve problems, to acquaint them with various strategies of problem solving so they can take these skills into any level of mathematics? That's the dilemma we face." (O'Connor \& Robertson, 2001)

She married again in 1970, to Edward V. Granville, a real estate broker, and has remained happily married. She has received numerous honors and awards, including an honor from the National Academy of Sciences in 1999. Granville has retired a number of different times, but she still visits schools and universities, and seems to be drawn to continue her efforts to show students the beauty of mathematics. In the summer of 2007, she taught a "Math Warm-Up" class for students entering grades 6-7 (Granville, 2007). Just as Granville was inspired by her teachers, she continues to inspire students.

## Changing Conditions for Black Women

Conditions began to change as black women obtained doctorates in mathematics, making it easier for others who followed, but there is still much work to be done. As of 1999 , it was estimated that there were only 40 total African American women who had earned doctorates in mathematics, and only 20 more black women mathematicians elsewhere in the world (Kenschaft, 1999). More recent data is not available due to an increased emphasis on privacy issues.

## Activities and NCTM Standards

The following activities relate to Evelyn Boyd Granville and address numerous points in the NCTM Principles and Standards for School Mathematics.

Activities Related to her Thesis Work: Granville says that her thesis was not at all impractical, since it prepared her well for everything in life and work (Clayton, 2000). Curious students can examine Granville's thesis abstract (Boyd, 1949), but the content on Laguerre series in the complex domain is too advanced for school classrooms. The NCTM number and operations standard specifies that students should understand complex numbers as solutions to quadratic equations with no real solutions, and Granville could be mentioned in that context.

Activities Related to her NASA Work: Evelyn Boyd Granville loved working for NASA's space programs: "I can say without a doubt that this was the most interesting job of my lifetime - to be a member of a group
responsible for writing computer programs to track the paths of vehicles in space" (Granville, 1989). A slide containing pictures of Granville and this quotation is available to print and project (Greenwald, 2008).

Many related student worksheets and activities are available. In fact, NCTM and NASA partnered to produce aerospace activity books that align with the standards for Pre K-2 (Hynes \& Blair, 2005), grades 3-5 (Hynes \& Hicks, 2005), grades 6-8 (Hynes \& Dixon, 2005), and grades 9-12 (House \& Day, 2005).

Many on-line lesson plans can also be found. For example, students in grades 6-8 can learn about the time and distance required for travel in the solar system (National Council of Teachers of Mathematics, 2008) or they can become scientists and engineers as they launch spacecraft (Space Explorers, 2008). Teachers of grades 5-6 can even access lesson plans on trajectory and projectile motion designed for them by grade 9 students (Leaf, 2008).

Students interested in Granville's orbital computations can access an article detailing the contributions, procedures, and equipment of mathematicians, engineers, and programmers who worked on Project Mercury (Gass, 1999). Advanced high school students in calculus or physics can also explore a technical article related to Project Mercury (National Aeronautics and Space Administration, 1962) that is dated from the time that Granville was working on the Mercury program.

## Activity Sheet: Granville's Challenge

Granville says, "My advice to teachers of mathematics is to stress problem solving and stress techniques available for problem solving" (personal communication, 2007). She then illustrates her teaching philosophy by sharing her favorite challenge. The activity sheet on the page after the end of this article explores this challenge.

The first portion of the activity sheet is designed for young children and relates to the algebra standard for Pre-K-2, which specifies that students should sort, classify, and order objects by size, number, and other properties. Wang explored the challenge with his five-year old child and the questions listed under Method 1 are adapted from his reflections (Wang, 2003).

The remainder of the activity sheet is designed for students in grades 7-10. Students use a variety of methods to solve the challenge, including algebraic and geometric techniques. The problem solving standard specifies that students should apply and adapt a variety of appropriate strategies to solve problems. The extension questions in the worksheet are also designed for these types of students.

Advanced students, such as those in linear algebra, can use matrix methods to solve the challenge. The number and operations standard for grades $9-12$ specifies that students should understand vectors and matrices as systems and should develop their understanding of properties and representations of multiplication of matrices, and this activity sheet could be used as a way to introduce or review matrix methods.

Activity sheet solutions can be found at http://www.mathsci.appstate.edu/centroid/.

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Picture Credits: The 2001 picture of Granville was taken at Yale University when Granville received an honorary degree (Spangenburg \& Moser, 2003). The picture of Haynes is from the Catholic University of America archives. The 1997 picture of Granville was taken by Margaret Murray (Murray, 2000).

## Crossword

Want to try to make one for your students? Check out [http://crosswordpuzzlegames.com](http://crosswordpuzzlegames.com)!

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ACROSS

1. measured in degrees
2. compare those ratios
3. on the bottom
4. no straight sides
5. circle part
6. square
7. not a square
8. connects two points
9. power
10. middle number
11. both sides the same
12. no fractions involved

DOWN
2. on the top
3. circumference to diameter
5. out of 100
7. chances are
8. nothing at all
13. power of 10
14. average
18. break into parts

## Activity Sheet: Evelyn Boyd Granville's Favorite Challenge



Evelyn Boyd Granville in 1997

Evelyn Boyd Granville was the second black woman we know of to receive her PhD in mathematics. Dr. Granville's original research related to complex numbers but she also worked on numerous space missions, including Project Mercury, the first manned space flight program: "I can say without a doubt that this was the most interesting job of my lifetime - to be a member of a group responsible for writing computer programs to track the paths of vehicles in space" (Granville, 1989). In this worksheet we will explore topics related to her favorite challenge.

My favorite challenge to teachers and children is to solve the following problem using three different methods: Rabbits and chickens have been placed in a cage. You count 48 feet and seventeen heads. How many rabbits and how many chickens are in the cage? (personal communication, 2007)

## Method 1

1. Sketch seventeen circles to represent the seventeen heads.
2. How many feet do chickens have?
3. How many feet do rabbits have?
4. Notice that rabbits and chickens each have at least two feet. Draw two feet attached to each head.
5. How many feet did you draw?
6. How many feet remain from the 48 total feet?
7. Do the remaining feet belong to chickens or rabbits?
8. Distribute the remaining feet on some of the heads to complete the pictures.
9. How many heads have two feet?
10. How many heads belong to the chickens?
11. How many heads belong to the rabbits?

Additional Methods Let $x=$ the number of rabbits and $y=$ the number of chickens
12. In terms of $x$ and $y$, how many heads are there?
13. In terms of $x$ and $y$, how many feet are there?
14. Solve these equations for $x$ and $y$ using at least two different methods.

## Extensions

15. In real-life we know there are more feet per chicken and rabbit than heads, but can the number of heads and feet ever equal each other mathematically? If so, find a general criterion.
16. Can you find general criteria for the numbers of heads and the feet that result in an equal number of rabbits and chickens? Do the solutions always make sense in real-life?
17. Given a certain number of heads and feet, must a mathematical solution for the numbers of rabbits and chickens always exist? Explain why or find a counterexample.

## Problems to Ponder

Fall 2008 Problems<br>Gregory S. Rhoads<br>Appalachian State University, Boone, North Carolina

Grades K-2 Gary wanted to help Mrs. Smith by mowing her lawn and trimming her trees. He started working when the clock said

(in)and finished when the clock said
 How many hours and minutes did he work for Mrs. Smith?

Grades 3-5 Students are given a test that has 24 multiple-choice questions and 4 short-answer questions. For the multiple-choice questions, you earn 4 points for each correct answer, 0 points for each incorrect answer and 1 point for a question you leave blank. Julie got half of the questions correct, one-third of the questions incorrect, and left the remaining questions blank. How many points did Julie earn on the test?

Grades 6-8 List all of the numbers between 2 and 36 that have no common positive factor with 36 other than 1 (i.e. whose greatest common factor with 36 is 1). Example: 27 is not an answer since 27 and 36 both have the common factor of 9 ; however, 23 is an answer since 23 and 36 have no common factors except 1 .

Grades 9-12 My old car got 38.5 miles per gallon and my new car gets 27 miles per gallon. If the trip from my house to my work is 14.5 miles one way and the price of a gallon of gas is currently $\$ 3.879$ per gallon, how much money will I save by driving my old car to work and back this week ( 5 days) and not driving in my new car?

## Directions for submitting solutions

1. Neatly print the following at the top of each solution page:

- Your full name (first and last)
- Your teacher's name
- Your grade
- Your school

2. Submit one problem per page.

Students who submit correct solutions will be recognized in the next issue of The Centroid. We wish to publish creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission. Keep in mind that proper acknowledgement is contingent on legible information and solutions.

## Send solutions by 20 November 2008 to:

Problems to Ponder, c/o Dr. Greg Rhoads
Dept. of Mathematical Sciences
Appalachian State University
Boone, NC 28608
As these problems are intended to stimulate independent thinking, it is expected that a submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

## Grades K-2 Spring 2008 issue

John and Mary are playing a game of bowling. They have 3 chances to knock down as many of the 18 pins as they can. John knocks down 8 with his first ball, 4 with his second ball and 1 with his third. Mary knocks down 5 with her first ball, 5 with her second and 4 with her third. How many pins do each person leave standing? Who knocked down more pins?

Solution: By Nina Ward, $1^{\text {st }}$ grade, Marvin Elementary (Teacher: Mrs. Janie Kendrick).


Editor's Note: I liked the fact that Nina wrote her answer in a sentence. You can't start early enough having students explain their answers in the context of the problem.

Correct solutions were received by Brandon Bartolome, Silas Kelly, Cody Marlowe, Cassidy Mateo, Lauren Menendez, and Tyler Smith of Antioch Elementary, Kevin Angeles, Caitlyn Brown, Quinten Burrus, Henry Helms, Jayden Helms, Sheldon Hooks, Gisselle Jarquin, Erik Ramos, Kylia Rushing, Ruri Silva, Josh Thomas, Alec Westmoreland, and Jamison of Benton Heights Elementary, Rosy Acosta, Hugo Arteaga, Monica Barrios, Tedarren Blount, Bobby Brown, Gabriela Castaneola, Uriel Cortez, Vanessa Coyotl, Brandon Duran, Pablo Guerrero, Isaiah Hampton, Daleth Herrera, Zeeondrea Lassiter, Marco Nava, Destiney Sturdivant, April , Gustavo , Itzel , and N'Kya of East Elementary, Lauren Doane, Ruben Gomez, Johnson Krajewski, Luke Motsinger, and Nia Studstill of Indian Trail Elementary, Owen Phillips of M.B. Hubbard Elementary, Evan Humphrey and Nicholas Rademaker of Marshville Elementary, Michael Jewell and Nina Ward of Marvin Elementary, Yovani Chinas-Flores, Alyssa Drake, Lisabeth Guillen, Steven Lopez, Joella Sabatino, Maddy Sproles, and Alex-Valdiria Torres of Porter Ridge Elementary, Brooke Austen, Kendall Collins, Grace Cooper, Lochainn DiMarco, Julia Gundlach, Nathan Peng, Kyle Romenick, Anna, and Ludovico of Sandy Ridge Elementary, Emily Alvin, Dominick Battaglia, Kevin Callahan, Sydney Clark, Jonah Coombs, Chloe Doane, Jake Myers, and Aunika Pearson of Sardis Elementary, Shannon Birlet, Tyler Boone, Mary Burnette, Elizabeth Cram, Tohala Gianluca, Marissa Haskell, Alanis Herbst, Miranda Hunt, Philip Johnson, Dmitry Litoshik, Caroline Lopp, Alex Macari, Mykelanne Madonna, Jordan Melton, Kendall Panos, Jason Reynolds, Abbie Thompson, Jake, Kyle, Marco, Steven, and Taylor of Shiloh Elementary, Sam Gunn of Unionville Elementary, Caleb Galek, Weston Carpenter, Jaime Castlejam, Kendall Rose Heinen, Madison Paige Hunter, Christian Martinez, Jacob Mendlin, Benjamin Polk, Julian Ramos, Ricardo Sanchez, Shawn Sizemore, Josh Sturdivant, Emily Taboada, Bricara Wilson, and Hunter Wright of Wingate Elementary.

Editor's Note: I was happy to see submitted solutions from kindergarten students (we've gotten very few in the past) and have presented Jason Reynolds' solution below. He attends Shiloh Elementary and Mrs. Karyn Cantville is his teacher. EXCELLENT JOB!!


## Grades 3-5 Spring 2008 Issue

The student council is making and selling pizzas as a fundraiser. Each pizza can be cut into 8 slices and they will sell each slice for $\$ 1.50$. If the cost of the ingredients for 50 pizzas is $\$ 100.00$ and if the student council sells every slice in the 50 pizzas, how much money will they make?

Solution: By Sydney Schwai, $3^{\text {rd }}$ grade, Sandy Ridge Elementary (Teacher: Mrs. Kristin Ross).

## Correct solutions were received by

 Maria Correa, Emily Lynch, and Erin November of Antiock Elementary, Zoe Branson, and Kyrsten Rudock of Benton Heights Elementary, Victoria Coss, Hugo Cuenea, Jarod Dahr, Eri Dominguez, Emily Guerrero, Marisa Sanchez, Mariah Springs, and Juan Valenciano of East Elementary, Tara Lavrik of Kensington Elementary, Layla Ballew, Robert Britt, Austin Chaney, Julian Corey, Dalton Davis, Meghanne Edwards, Kayla Funderburk, Charity Hamilton, Lola Helms, Taylor Helms, Garrett Henry, Allyson High, Matthew Lang, Kyle Lily, Christian McKeown, Sarah Mills, Evonne Nguyen, Hallie Sholar, Melissa Silsby, and Quentin Smith of Marshville Elementary, Taylor Earnheart, Jonathan Gras, Mary Blair Salati, Julia Spooner, and Colin Stewart of Marvin Elementary, Anna Poole, Sydney Schwai, and Arpad Attila Voros of

Sandy Ridge Elementary, Troy Blacke, Michele Helms, Phillip Price, and Wyatt Rowell of Unionville Elementary, Estefany Avila G., Cassie Brown, Joshua Dieguez, Tariah Harrell, Bryce Helms, Storm Hicks, Luis Jimenez, Argenis Marino, Susana Martinez, Austin Miller, Diego Morgado, Bryan Munoz, Brianna Plessner, Logan Brooke Polk, Brajan Santiago, Veronica Solis-Ortiz, Javon Swanston, Amber Theroux, Seth Whitley, and Dustin Wilson of Wingate Elenentary.

## Grades 6-8 Spring 2008 issue

A total of $11,540,000$ people visited Myrtle Beach during 2005 and in 2006, there were $12,182,000$ visitors to Myrtle Beach. The number of visitors in 2007 increased by the same percentage over the previous year as did the number of visitors in 2006 over its previous year. How many people visited Myrtle Beach in 2007?

Editor's Note: This was a difficult problem. Many students submitted solutions where the actual increase was the same, not the percentage increase. They said the increase from 2005 to 2006 was 642,000 so they added 642,000 to the total from 2006 to get the answer. This would imply the number of visitors would increase linearly. However, if you assume the percentage increase is the same from year to year, then the number of visitors would increase exponentially (students are not expected to know exponential growth and you don't need to know that to do the problem). It may be interesting to have students compute the visitors under both types of growth and graph them to see the differences. It was also interesting to note the differences in how students represented the percentage. Some rounded to one or two decimal places and some rounded to the nearest whole percentage, all causing large differences in the reported answer. I've heard criticism that we use too many whole numbers in the problems we give and students don't know how to deal with numbers that are more complex.


Solution: By Juan Paulo Villamor, $8^{\text {th }}$ grade, Bertie Middle School (Teacher: Mr. Ramil Orbita).

## Correct solutions were received

 by Tykevis Bazemore, Mattison Bond, Brook Bowes, Ryann Bryant, Jusselynn Carter, Jaliah Harmon, Chelsea Hoggard, Michael Johnson, Laarni Lapat, Lyndsae Peele, Joella Quiroz, Anthony Spruill, Carmen Villamor, Juan Paulo Villamor, Caroline White, Isaac William, and Raekwon Williams of Bertie Middle School.
## Grades 9-12 Spring 2008 Issue

Suppose the lengths of the sides of a trapezoid are $5,5,5$, and 11 . What is the area of the trapezoid?
Solution: By Jordan Cox and Summer Goerler, $10^{\text {th }}$ grade, Jones Senior High School (Teacher: Mrs. Sudha Srivatsa).

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\begin{aligned}
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& \text { To find the arca of the trapezoid, we have } \\
& \text { to Construct the heights } A E \text { and BF. } \\
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& \text { Sina it is an isosceles tropezoid, } l(D E)=l(C F)=3 \\
& \begin{array}{l}
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\text { therrem, we get } A E^{2}+D E^{2}=A D^{2} \text { or } A E^{2}+3^{2}=5^{2}
\end{array} \\
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A E^{2}=25 \\
U(A E)=4
\end{array} \\
& l(A E)=t(B F)=4 \\
& \text { Now area of } \triangle A E D=\text { area of } \triangle B F C=\frac{1}{2} \text {. base. height. } \\
& \text { Area of } \triangle A B F E=b \cdot h=5 \cdot 4=20 \text { Sqe }=\frac{1}{2} \cdot 3 \cdot 4=6 \text { squis } \\
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& =6+20+6=32^{A(\triangle B F C)} \text { Square umits. }
\end{aligned}
$$

Correct solutions were received by Jordan Cox and Summer Goerler of Jones Senior High School.

## Professional Development Opportunities for NC Math Teachers

Know of any summer 2009 workshops or other professional development activities for math teachers? Let us know so we can put them in the Spring Issue of the Centroid, made available to NCCTM members in early February. Submit a title, dates, a short description ( 50 word or less), and contact information via email to Holly Hirst (HirstHP@appstate.edu).

## NCTM National Organization News

## NCTM Seeks Feedback from Educators

In 2006, a task force appointed by NCTM recommended that a framework be developed to guide future work in high school mathematics. A writing group was appointed and a draft is now available.

NCTM has requested feedback on the document "Focus in High School Mathematics: Reasoning and Sense Making." The draft is being made available on the NCTM website for comment until September 19, 2008 [http://www.nctm.org/highschooldraft.aspx](http://www.nctm.org/highschooldraft.aspx). Feedback is welcomed from NCTM members as well as nonmembers.

From the Preface of the document:
This document proposes that all high school mathematics programs should be focused on reasoning and sense making. The audience for this document is intended to be everyone involved in decisions regarding high school mathematics programs, including formal decision makers within the system; people charged with implementing those decisions; and other stakeholders affected by, and involved in, those decisions.

Chapter 1 describes what constitutes reasoning and sense making in the mathematics classroom, why they should be considered as the foundation for high school mathematics, and how they link with other Process Standards. Chapter 2 describes in more detail the reasoning habits that students should continue to acquire throughout their high school mathematics experiences. Chapter 3 demonstrates with examples how reasoning and sense making can be incorporated into five overarching areas of high school mathematics-number and measurement, algebraic symbols, functions, geometry, and statistics and probability. Chapter 4 focuses on what it means to provide equitable opportunities for all students to engage in reasoning and sense making. Chapter 5 addresses the importance of coherent expectations regarding curriculum, instruction, and assessment in promoting reasoning and sense making. Finally, chapter 6 presents questions to consider as stakeholders work together to improve high school mathematics education. (p. viii)

## NCTM Annual Meeting and Exposition - Save the Date!

April 22-25, 2009
Washington, D.C.

- Over 1,000 sessions and workshops
- Preview the latest teaching tools
- Network with friends and meet new ones

This is a professional development opportunity you can't afford to miss! Additional information on the conference is coming soon. Registration and Housing will open in November. Interested in volunteering at the conference? Learn more at [http://www.nctm.org/conferences/default.aspx?id=52](http://www.nctm.org/conferences/default.aspx?id=52)

## 2008-09 Professional Development Focus of the Year: Equity means ALL

The National Council of Teachers of Mathematics Principles and Standards for School Mathematics asserts that "All students regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study-and support to learn-mathematics."

The intent of this year long emphasis on equity is to help teachers, school leaders, and teacher educators examine equity and its salience in the mathematics teaching and learning process. Find resources and learn more at [http://www.nctm.org/profdev/content.aspx?id=15589](http://www.nctm.org/profdev/content.aspx?id=15589).

## Awards

## 2008 NCCTM Mathematics Logo Contest Winners Reported by Lisa Carnell, High Point University, High Point North Carolina

Over 2900 entries were submitted for the 2008 NCCTM Mathematics Logo contest this year! The following winner and regional finalists were chosen. Well done to all who submitted; choosing the winner was very difficult!

## Winner: Life is Built on Math

Erica Perine
6th grade
The Academy at Lincoln, Greensboro
Teacher: Mrs. Schram

## Regional Finalists



Hannah Templeton, 5th grade, Canterbury School, Greensboro
Teacher: Kelly Wesney
Sunny Su, 8th grade, Martin Middle School, Raleigh
Teacher: Lucy Kay
Steven Seward, 2nd grade, Coleridge Elementary School, Ramseur
Teacher: Mrs. Linda Beddingfield
Hayden Wood, 5th grade, Pasquotank Elementary School, Elizabeth City
Teacher: S. Lindsay
Trent Dameron, 2nd grade, Lockhart Elementary School, Knightdale
Teacher: Traci Behrendt
John Short, 12th grade, Sanderson High School, Raleigh
Teacher: Ms. J. Blackwell
Cassronda Sholtzhauer, 2nd grade, Rocky River Elementary School, Monroe
Teacher: Brenda Todd
Madison Sain, 4th grade, Lincoln Elementary School, Vale
Teacher: Denise Smith
Dana King, 6th grade, Marvin Ridge Middle School, Waxhaw
Teacher: Carmen Harpham
Michael Yarbro, 12th grade, Kings Mountain High School, Kings Mountain
Teacher: Jamey-Anne Croft
Mona Xiao, 12th grade, Career Center High School, Winston-Salem
Teacher: Ms. Kay Endriss

## Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in the State. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

If you have nominated someone in the past who has not received the award to date, or if you would like to nominate someone new, please submit as much of the following information as possible!

Nominations are accepted at any time.

Please submit the following information. Use as many typewritten pages as needed. If possible, attach a vita of the nominee.

- Name of the nominee
- Current position
- Your relationship to the nominee (e.g. principal, co-worker, etc.)
- The nominee's contributions to mathematics education, NCTM, NCCTM, etc. (Please include information on specific offices held and honors received by the nominee.)
- Any information about contributions to the community, teaching, and education that would be of value to the Rankin Award Committee in its deliberations
- Other relevant information
- Letters of endorsement from other colleagues may be included.
- Date of nomination

| Nominator* | Name |
| :--- | :--- |
|  | Current position; Business or educational institution |
|  | Preferred mailing address; Preferred telephone number |

*The Rankin Award Committee reserves the right to use portions of nomination information in the presentation of the award if the candidate is selected.

Send to: Ms. Jan Wessell
23 Shore Drive
Wrightsville Beach, NC 28480

## National Math Panel Report

Foundations for Success: The Final Report of the National Mathematics Advisory Panel
On March 13, 2008, the National Mathematics Advisory Panel presented its final report to President Bush and Secretary of Education Spelling. The report contains 45 findings and recommendations. In response to one of the recommendations, the Department of Education and the Conference Board of Mathematical Sciences are hosting a National Math Panel Forum on October 6-7 to bring together interested parties to discuss ways to engage their members or constituents in discussions about the National Math Panel's findings and recommendations.

Read the math panel report and supporting materials online at
[http://www.ed.gov/about/bdscomm/list/mathpanel/index.html](http://www.ed.gov/about/bdscomm/list/mathpanel/index.html).

## Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The purpose of this award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state. Further, this program must have been sustained for a period of at least three years. A number of organizations have made significant contributions to mathematics education in North Carolina; the Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations by sending in the form below. Nominations will be retained in the active file for at least three years.

## NOMINATION FORM

Name of Nominee: $\qquad$
Present Position: $\qquad$
Outstanding contributions to mathematics education in North Carolina which serves as the basis for this nomination:

Additional information that would be of value to the selection committee:

Signature: $\qquad$ Date: $\qquad$
Name (print/type): $\qquad$
Position: $\qquad$
Business or Institution: $\qquad$
Address: $\qquad$
Phone: Business $\qquad$ Home: $\qquad$
Email: $\qquad$
Send to: John Parker
316 West Soundside Road
Nags Head, NC 27959

## NCCTM Trust Fund Scholarship

North Carolina Council of Teachers of Mathematics
$\$ 600$ scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction.

Applicants must be:

- Currently employed as a pre-K - 12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting this application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are:

- March 1
- October 1

Direct inquiries to:
NCCTM Trust Fund Chairperson
John R. Kolb, Chairperson
phone: (919) 787-8116
e-mail: JKolb1@nc.rr.com
(Please print all information.)

## PERSONAL INFORMATION:

Name $\qquad$
Home address: $\qquad$
Street
$\qquad$ , $\quad \mathrm{NC}$ $\qquad$
Home phone: $\qquad$ Home e-mail: $\qquad$
NCCTM membership number: $\qquad$
EMPLOYMENT INFORMATION:
How many years of teaching experience? $\qquad$
Currently employed in what school system? $\qquad$
School name: $\qquad$
School address: $\qquad$
School phone: $\qquad$ School e-mail: $\qquad$
Current teaching assignment: $\qquad$
Principal's name: $\qquad$

COURSE INFORMATION: (One course only)
Institution of higher education:
Graduate degree program in which you are currently enrolled: $\qquad$
Course name $\qquad$ Course number: $\qquad$
Dates of enrollment: (circle one) Fall semester Spring semester Summer session Year: $\qquad$
Name of course instructor: $\qquad$

PROFESSIONAL ACTIVITIES WITHIN PAST 5 YEARS WITH EMPHASIS ON ACTIVITIES RELATED TO MATHEMATICS EDUCATION:

## BRIEF STATEMENT OF FUTURE PROFESSIONAL GOALS:

## REQUIRED SIGNATURES:

Applicant signature: $\qquad$
Principal's signature: $\qquad$
Instructor signature (if currently enrolled): $\qquad$

Date: $\qquad$
Date: $\qquad$
Date: $\qquad$

## REQUIRED ATTACHMENTS:

Please attach a copy of

1. A letter of acceptance to an accredited graduate program in North Carolina;
2. Official verification of enrollment in the graduate course described in the COURSE INFORMATION above if the course is currently being taken, OR official transcript containing the grade awarded to the applicant if the course described in the COURSE INFORMATION above has been completed.

NOTE: Applications must be complete to be considered. If your application is approved, an official course grade report must be submitted to verify successful completion of the course before scholarship funds will be issued.

NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS
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## MEMBERSHIP - NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

Name: $\qquad$ Home Telephone: $\qquad$ ) - $\qquad$
Address: $\qquad$ School Telephone: $\qquad$ ) - $\qquad$
City: $\qquad$ State: $\qquad$ Zip: $\qquad$ E-mail:

School System: $\qquad$

## POSITION

## LEVEL

Teacher

- K-3
-4-6
Junior High/Middle School
$\square$ Senior High
2-Year College/Technical
- 4-Year College/University


## MEMBERSHIP STATUS

$\square$ New Former/Renewing Member \# $\qquad$

## MEMBERSHIP DUES

$\square 1$ year:
$\square 3$ years:
Full-time Student: $\quad \$ 10.00$ $\qquad$
$\square$ Contribution to Trust Fund:
Total Payment Enclosed:
Payment by $\square$ Check $\square$ Visa MasterCard
Card \#
Exp. Date
Signature
\$20.00 $\qquad$
$\$ 50.00$ $\qquad$
$\qquad$
$\qquad$

Please make your check or money order payable to NCCTM. Send this form and your payment to NCCTM
P.O.Box 4604

CARY, NC 27519
Payments by credit card may be mailed or faxed to
919-859-3342


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