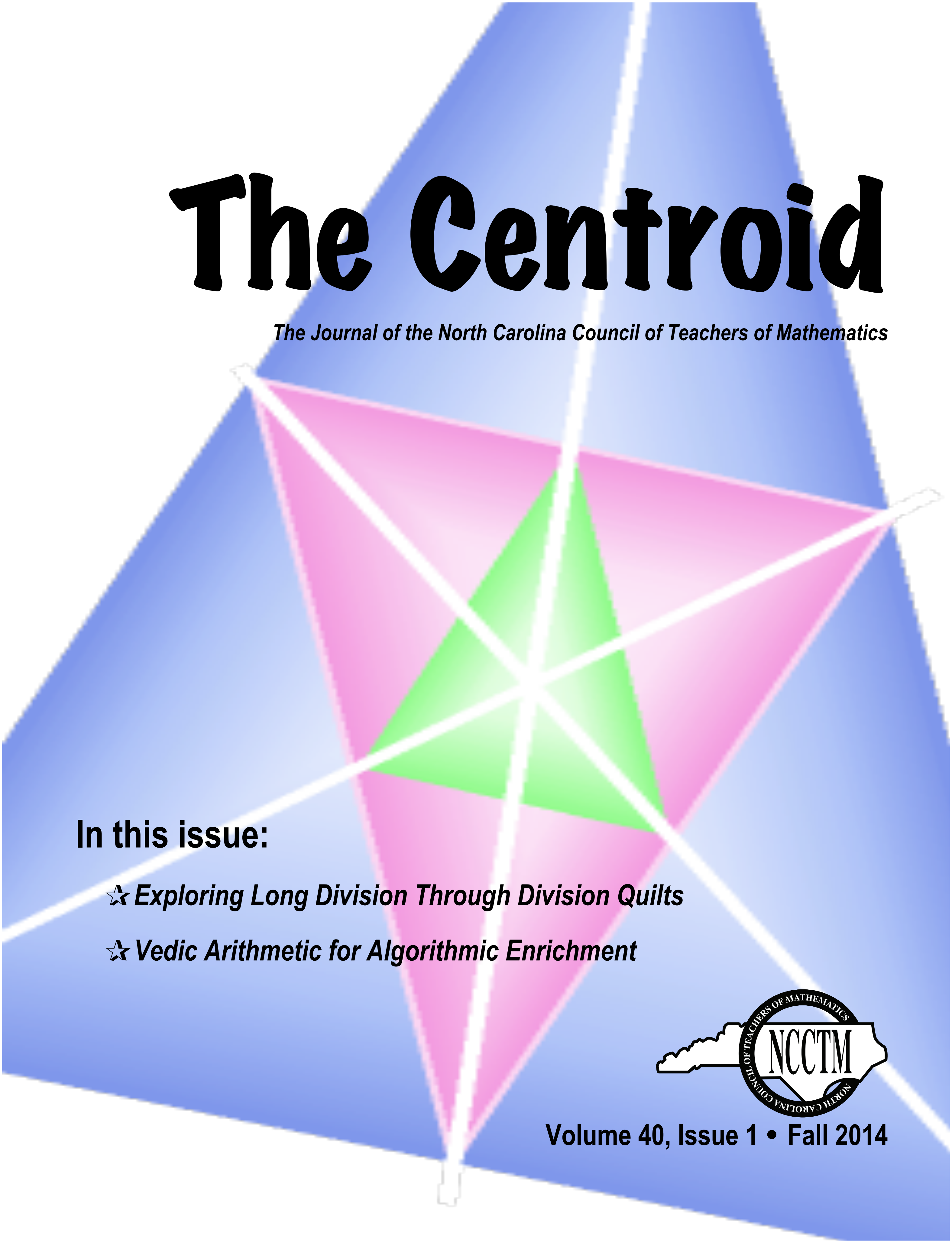

The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

- ✰ *Exploring Long Division Through Division Quilts*
- ✰ *Vedic Arithmetic for Algorithmic Enrichment*

NCCTM
NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

Volume 40, Issue 1 • Fall 2014

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

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Submission of News and Announcements

We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts

We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
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Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:

  Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:

  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, 7, 196–199.

  Chapters or sections of books:

  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:

  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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October 30th and 31st
Koury Convention Center in Greensboro, NC.
Big Ideas for Teaching and Learning Mathematics.

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Juli Dixon: *Five Essential Instructional Shifts for Transforming Middle School Common Core Classrooms*
Wade Ellis: *The Learning Trajectory of Proportional Reasoning in High School Mathematics*
Cindy Neuschwander: *What's the Story behind Math Literature?*

See http://ncctm.org for more information and to register!

President's Message

State President Debbie Crocker
Appalachian State University, Boone, NC
crockerda@appstate.edu

This is my second year as President of NCCTM. It is hard to believe that a year has already passed! I have enjoyed communicating with many of you by e-mail, phone, and in person. Let's continue that communication. I want to thank the Board of Directors and members of NCCTM for all of your help. I enjoy serving as your President and representing our organization. It takes all of us to improve the teaching and learning of mathematics in North Carolina.

I hope you are planning to attend the NCCTM Leadership Seminar and the 44th Annual State Mathematics Conference! The theme for the Leadership Seminar is: "Teaching and Learning for All Students" and the theme for the NCCTM 44th Annual State Conference is: "Big Ideas for Teaching and Learning Mathematics". Go to http://ncctm.org/ to find links to registration for each event, a preliminary program for the Conference, and to hotel reservations. When making a hotel reservation at the Sheraton Fours Seasons, use the code MATH14 to receive the conference rate for rooms. Reservations must be made by October 7 to guarantee availability of rooms.

Tracey Howell, Tony Thompson, and Ryan Dougherty, Conference Co-chairs; Lisa Carnell and Amy Travis, Program Co-chairs; and Karen McPherson, Program Booklet Editing Chair, along with all of the conference committees and volunteers, have worked hard to provide you with this wonderful professional development experience! We are sure this professional development opportunity will provide you with a wealth of knowledge and information to take back to your district, school, and classroom as well as time to network with friends and colleagues to share ideas. Be sure to visit the exhibit hall and examine the materials and print resources available to enhance your classroom.

Once again this year, we are using the conference app, Grupio and will have workshops with no tickets. The seating in workshops will still be limited to 40, but the seats will go to the first 40 participants at the workshop. We are also continuing the on line evaluation form for the conference. You will find a link to the evaluation form in the program booklet for the conference. Be sure to fill out the evaluation at the end of the conference. We value your comments.

Make plans to join us for the NCCTM Awards Celebration on Friday, October 31 from 7:00 - 8:45 a.m. in the Guilford C room. NCCTM will provide a continental breakfast just before the Awards Celebration begins. We will recognize Outstanding Secondary Teachers from across North Carolina, present the Outstanding Pre-service Teacher awards, the Innovator Award, and the Rankin Award, among others. Join us to congratulate all of the recipients.

Check the web site, http://ncctm.org/ , for information on NCCTM Spring activities. We will have regional conferences, regional and state math fairs, regional and state math contests, the spring Leadership Seminar, and more! Spring is a great time to involve yourself and your students in these opportunities.

Look for a link soon at http://ncctm.org to submit proposals for next year's conference! Watch the web site for information on the Spring 2014 Leadership Seminar. It will be on Friday, March 27th! Don't miss it!

Exploring Long Division through Division Quilts

Tina Lupton, Davidson County Schools, Lexington, NC
Sarah Pratt, University of North Texas, Denton, TX
Kerri Richardson, UNC at Greensboro, Greensboro, NC

New teachers in elementary schools often follow the same strategy for teaching division as they experienced in school, memorizing the algorithm. In this article Tina Lupton shares her experiences in the transition from helping students "understand" division – by reciting "divide, multiply, subtract, bring down" again and again until they could perform proficiently – to affording opportunities for students to "imagine" division.

As a collective, all of the authors agree that at some point in our teaching careers we recognized that there were minimal ways to demonstrate division when teaching the algorithm in isolation; furthermore, there are rare opportunities to adhere to the expectations in mathematics education (Common Core Standards Initiative, 2011; NCTM, 2000) that students should be able to identify and use relationships between operations. Imaging is an important activity (Richardson, Pratt & Kurtts, 2010; Wheatley, 1998) that allows for such opportunities. In this article, we outline a series of activities that provide students different representations of division to help them achieve understanding and proficiency of the algorithm due to their understandings of the visual aide. It is important to state here that we believe students should be able to use the division algorithm, but only as it is attached to meaning making and images. We argue that by using Division Quilts, students are better able to demonstrate their comprehension of division, through student work as well as standardized assessments.

Multiplication and division are inverse operations, and both can be understood through models of grouping/fair sharing, arrays and area model (Ma, 1999; Reynolds & Wheatley, 2010). Progressing through understandings of these three models strengthens connections between multiplication and division, as well as expanding understandings of algebraic structures and geometric structures. Starting students with Base-10 blocks to represent division as fair sharing allows connections between personal experiences and mathematical concepts. Moving to division as an array and an area model provides more images with which students can work to achieve deeper understandings and connections among mathematical concepts. We provide Division Quilts as one way to foster opportunities for conceptual development, promoting exploration of patterns, shapes, measurements, spatial relationships, and number sense (Dark, 2007). Specifically, there is a cognitive leap that occurs between grouping/fair sharing and area models to represent division. Division Quilts provide an opportunity for individual students to transition to the more sophisticated model in their own time.

The data collected in this study comes from multiple 4th grade classrooms in a Title 1 School (2008-2012) where over 80% of the students are considered living at or below the poverty level[1]. The demographics of the classrooms over the years have consisted of approximately of 13-15 boys, 10-12 girls, 5-8 English as a Second Language Learners, 2-4 EC students (1 B.E.D., 2 ADHD, 1 OHI), and 2-4 Academically Gifted students. Division Quilts are used as a strategy that meets the needs of these diverse learners.

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[1] In 2010 NC adopted the Common Core State Standards creating a lapse in comparable data.

Based on the data collected from the 2008 and 2009 Benchmark Assessments, the content standard for three digit by two digit division increased 12% after the first year of implementing Division Quilts. The 12% increase meant that students were now performing at 61% proficiency with division. Division Quilts made a significant impact on the comprehension of number sense. The greatest impact that Division Quilts had was on standard for strategies for multiplying and dividing numbers, which increased 22% from 2008 to 2009.

Division Quilts

Division Quilts are created on grid paper by outlining the dividend then shading the divisor with different colors to reflect the quotient. The outlined polygon is referred to as the dividend, and once shaded by the pattern of the divisors, it is now a "quilt." To engage students in thinking about division, the teacher begins the lesson by asking students to represent 25 ÷ 7 using Base-10 Blocks. The representations created by the students exhibit a fair sharing model, which is an easy model for students to understand. From there, the teacher will transition to the Division Quilts, using the same division problem of 25 ÷ 7. The problem is selected because it includes remainders, skip-counting by 5, and it is a small enough dividend that it is not numerically challenging. The first quilt should be done the same by all students. The teacher can suggest to students to find the dividends by counting by a factor of 25, which are 1, 5, and 25. In this problem counting by 5 is the most efficient choice. Starting with the first unit/block/square at the top left of the grid paper, everyone counts to the right 5, then from that unit count down rows by multiples of 5 until the dividend of 25 is outlined (Figure 1a). Once the dividend is verified (Figure 1b), the outline is then colored with a heavy marker to enclose the dividend for the quilt (Figure 1c). For future Division Quilts, students choose the polygon to be outlined, based on the level of the student. For some students, it may be best to have a pre-drawn outline and have them record the dividend. For other students it may be enriching to tell them the dividend and let them decide how to draw the outline. As long as the outline encloses the dividend as a polygon (closed, straight, non-overlapping figure) then the quotient can still be found. It is not necessary that the outline is an array, square or rectangle; some quilts may need to have "loose strings" where a few units are needed that exceeds a quadrilateral (Figure 2). The less complex the outline, the easier the next steps will be, and as students become more proficient, they will move toward creating rectangles. To differentiate, the teacher can guide students to group the units by a factor of the number and skip count until they get to the dividend. If students find it difficult to skip count by the divisor, then it may be easiest to count by 5's or 10's until they are close to or on the number.

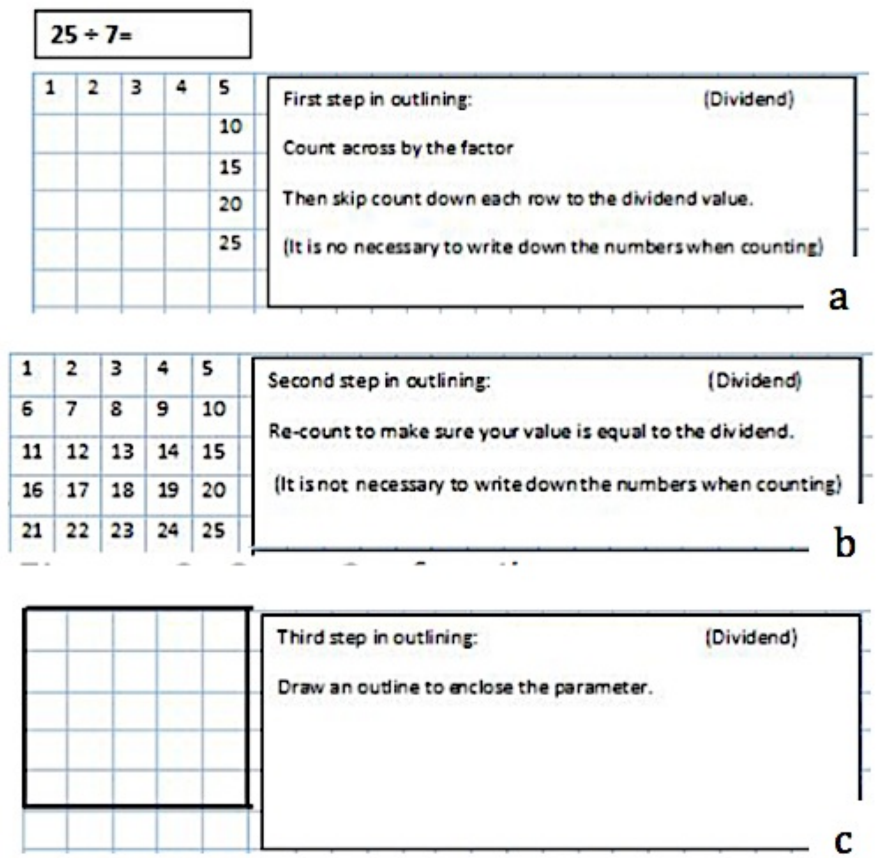

Figure 1: Building the Quilt in Steps

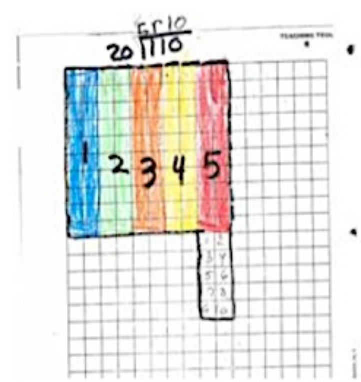

Figure 2: A non-rectangular quilt

Now that the outlined dividend creates the parameter and is verified, the next step is to color in the divisor. In this case the divisor is 7, so each student picks a color and shades in 7 blocks (Figure 3). Count each unit one at a time 1-7. Show the students that they can count vertically, horizontally, or in a pattern that makes sense, as long as the 7 units are adjacent. Students should be discouraged from doing "checkerboard" style coloring. Remind them that the divisor is a collection, so the group is colored together.

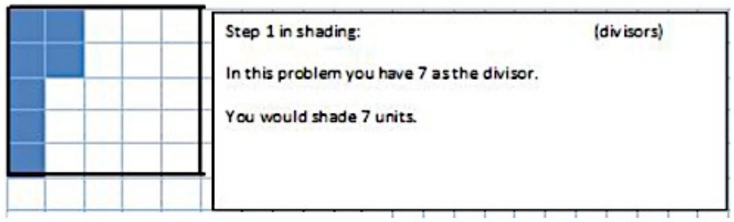

Figure 3: Step 4 of the Quilt

Next have the students count 7 new units and color them a different color (Figure 4). Students will continue a pattern of changing colors and coloring groups, until they cannot make any more groups of the divisor. In this example, there are 4 white spaces left, representing the remainder. Number each white unit that is remaining. Now that the quilt is colorful and remainders are labeled, students number each of the groups of 7 with a heavy marker. The labels allow students see the image of the quotient.

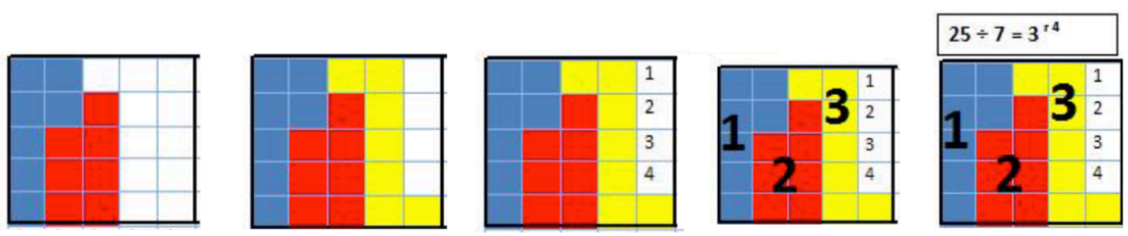

Figure 4: Steps 5 through 9

Once the quilt has been made, the teacher should review the process. Reviewing the steps allows the teacher to answer questions, clear up misconceptions, review the vocabulary, label the vocabulary terms on the quilt, or go over alternative options in making the quilt. The next step is to connect the process of creating the Division Quilt to the division algorithm. The process of creating the quilt can relate to the "divide, multiply, subtract, bring down" strategy. Each time a group is colored, a piece of the dividend is being removed, which is subtraction. With the first colored group (25 - 7), 18 blocks of the dividend remain. As long as the number of blocks remaining is greater than the divisor, then the process is to continue dividing the quilt into groups, explaining this can help clarify to students why we "Bring down" in the algorithm. The repeated grouping of 7 shows the meaning of division. Each additional group increases the number of groups of size 7 that are colored. Once all sets of the divisor (7) are colored (3 groups of 7), the number of groups times the divisor gives the dividend with the remainder of the uncolored blocks (3 times 7 is 21 units with 4 units left). Through the process the teacher can ask students to pause and count how many groups have been made, if they are to multiply the divisor by the number of groups, what part of the quilt would that answer represent? (The answer is the dividend.) It is important to not only have students go through the process, but to also have conversations and engage mathematical thinking about the process.

Advancing to the next step of interpreting the remainder is an easy progression. Ask students what would happen with the 4 units that are left over. Prompt two different scenarios to suggest when the remainder affects the quotient from being simply interpreted as a number versus a logical answer. For example, if there are 25 students and 7 can fit in a canoe, how many canoes are needed? Students are required to think logically and take into consideration what a remainder represents. In this scenario it means that 3 canoes are not sufficient because 4 students would not be able to partake in the trip, so a fourth canoe would be required in order to include those that did not make a complete group. Another scenario is a wedding photographer creating a 30-page album with 236 photos, and the bride and groom want the same number of pictures on each page. What is the maximum number of photos that can be placed on each page? With this scenario the interpretation of the remainder provides the number of pictures that will not be included in the album, but it will not affect how many pictures will be placed on each page. The photographer could place as many as 7 pictures on a page, using 210 pictures, and so 26 pictures would be left out of the album. Students are required by state standards to analyze remainders in real life word problems. They are expected to consider situations where the remainder indicates that the value of the quotient is not enough to accommodate, to consider that the remainder is the desired answer, or to consider that the value of the quotient is sufficient as is.

Once students have completed their first Division Quilt along with the teacher's modeling and have had time to process the strategy, they can proceed with guided modeling. With guided modeling the teacher will present another problem, such as 56 ÷ 8, and pose questions to guide the students through the steps. The teacher would start by asking, "If I am dividing 56 by 8 then what is my dividend?" 56. "If my dividend is 56 then what is the first thing I need to do to set up the problem?" Find a factor of 56. "What are the factors of 56?" 1, 2, 4, 7, 8, 14, 28, 56. "So which factor should we use?" Appropriate factors would be 4, 7, or 8. This is where the students decide which factor they want to count by or if they want to count by 5's or 10's, depending on their understanding of factors and multiples. If they count by 5, then they would be able to get to 55 and then would need to include one extra unit underneath to have the whole dividend.

Once students have decided which factor works best for them, prompt students to consider the different shapes and ask if it will change the quotient. The answer is that it will not because all of the shapes have the same dividend, even if they look differently. While students are verifying their outlined dividend, check students' understandings. The next step is for students to color a group of size 8 each time to show the divisor. The teacher transitions between modeling for all to see and monitoring students. When students have finished, ask "How was this quilt different from the first quilt?" One answer is that the second quilt has no remainder so all units are colored in with nothing left over. More questions can be used as needed, depending on the struggles and success of the students.

Gauge each student's ability to determine which students need pre-outlined dividends and which are able to outline the dividends on their own. After the first two quilts have been modeled, it would be acceptable to have students create their own problems or give them several to try on their own alternating between problems that have remainders and those that do not. Some students may be able to do triple digit dividends with single and double-digit divisors. (Note: For these advanced problems, use 1 cm grid paper so there are enough units. Figure 6 shows an example of this.)

Mathematical Purposes of Division Quilts

Division Quilts address the mathematical ideas of quotient, divisor, dividend, remainder, the meaning of division, and how multiplication and division are inversely related. Modeling can help promote problem solving and provide rich learning experiences (English, Fox, & Walters, 2005). Initially, Division Quilts should be introduced by pre-drawing a quilt outline for students for the purpose of primarily teaching them about the groups, or parts (divisor). While doing this activity, it is important to prompt students' thinking by asking questions such as: "Why does each group have x number of blocks?" "Why would I want to use a new color when I build a new group?" "How can I draw a group?" When focusing on the dividend they will see the "whole" that is being divided in the problem and help them understand what they are dividing. During this phase, questions can be asked, such as: "How many total blocks did you need?" "How did you find your outline?" "Could there have been an easier way to find your quilt?" "Was there a pattern?" The result then becomes a natural focal point, leading to

Figure 5: Modeling the Division Quilt

discussions about the quotient. Finally, problems can be introduced in which remainders becomes a part of the result, from the image of an excess of white blocks/squares/units.

Once students have all possible groups colored, prompt student thinking with questions such as: “How many blocks are left?” “Why should we not color them?” “How many more blocks do you need to make a group?” “Can you have more white blocks left then the value of your divisor”?” What if you did not have any white blocks left?”
It is best for students to use an array of colors so they can see each group individually. Students will determine what they want their groups to look like based on the amount of the divisor and the length of the Division Quilt. Some students will count their groups horizontally across the quilt; others will make a pattern/tessellation. After one group is formed, it is important for students to keep their patterns similar and interchange colors so they can determine where one group ends and another begins (Figure 5).

Concepts in Division Quilts
With the use of Division Quilts, students are exposed to the terminology of the long division algorithm; understanding the terms in division helps students in problem solving with division. Division Quilts create an image for the dividend, the inside of the quilt. A common misconception for students is being able to understand what they are dividing, so if students can start division by physically seeing what they are dividing, there is an increase in their success to interpret division problems because what they understand is more concrete.

Another common misconception for students is the importance of remainders. Division Quilts are a great way for students to visually see what a remainder is, as well as how the value of the remainder may or may not change the interpretation of the quotient depending on the situation and whether to exclude or include the remaining pieces. Some students believe that the remainder is how many groups are left, instead of thinking how many individual items are left. Some students’ misconceptions of remainders stem from inexperience of manipulating a problem. Many times students will go through the mathematical process of division and know that there is a remainder but do not know what it means. By seeing the white space in the quilt, students can recognize that it is part of the dividend, but not part of the divisor or the quotient. In addition, Division Quilts can help students make connections to multiplication. Prompting students by asking questions that relate the image to multiplication as well as division will generate opportunities for them to recognize this relationship and will help them build their understanding between operations.

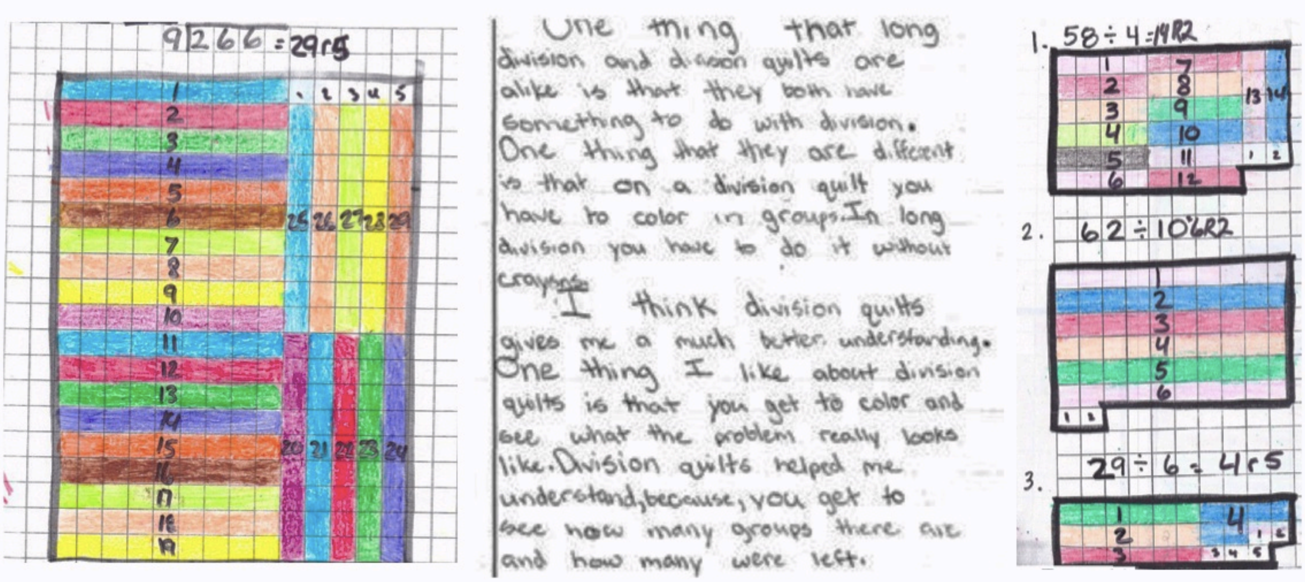

Figure 6: One of Daniel’s Division Quilts and his description of how it helped him

Examples of quilts made by Mrs. Lupton’s students and their comments on Division Quilts are shown in Figure 6. Richardson and Stein (2008) state that when students make connections with prior knowledge, they are able to speak more “mathematically.” The use of Division Quilts supports this statement, as exhibited in the students’ comments and models in Figure 6.

Final Thoughts

Division Quilts have a significant impact on a student’s understanding of division and multiplication. Buschman (2003) states that students enjoy problem solving more when the way they solve the problem makes sense to them. Based on students’ perspectives, Division Quilts provide a clear explanation of how to solve division problems, whereas the long division algorithm allows for many possibilities to make mistakes, especially if the students are not confident and knowledgeable of the algorithm. Modeling of division affords opportunities for meeting the needs of students on any learning level. Division Quilts build students’ interest in division and provides a way for them to engage meaningfully when exploring division problems.

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Vedic Arithmetic for Algorithmic Enrichment

Donald Hooley, Bluffton University, Bluffton, OH

Three Vedic arithmetic algorithms are presented which are of interest for student enrichment, possible alternative computational methods, and as wonderful exercises for developing number sense. As background, Bornemann and her co-authors discussed the role of algorithms in elementary instruction (2008). In particular, an introduction to alternative algorithms for whole-number operations was presented by Carroll (1998).

This article explores a few of the 16 "Vedic Mathematics" sutras advocated in the 20th Century by Hindu cleric Bharati Krishna Tirthaji, each of which describes a mental mathematics technique using an algorithm different from the traditional approach. For more information on the Vedic Mathematics sutras, see: www.vedicmathsindia.org

The algorithms illustrated in this article are referred to as Vedic mathematics because they are based on cryptic phrases called sutras in the writing style of ancient Indian Vedas. The Vedic mathematics sutras were developed fairly recently by the Indian Jagadguru Swami Sri Bharati Krsna Tirthaji during the years 1911-1918 (Bathia, 2005). Students preparing for the very competitive Indian university entrance exams where calculators are not allowed commonly learn these one-line arithmetic shortcuts. My pre-service elementary teachers find these algorithms interesting, and you may also find them useful additions to your understanding of arithmetic.

Subtraction

Which is easier to remember, 13 - 8 or 3 + 2? We can avoid this subtraction from teens by using the Vedic mathematics application of the sutra "the last from ten." The resulting algorithm is slightly different from both our usual method and the Austrian-German algorithm.

Example 1. Consider 55 - 17. In the first step instead of 5 – 7, "taking the last from ten" we think

$$10 - 7 = 3$$
$$-7 = 3 - 10.$$

Thus, subtracting 7 is the same as adding 3 and then subtracting 10. Starting with 55 - 17, since 5 – 7 = 5 + 3 – 10 we can use a superscript -1 to indicate the carried -10 giving

$$\begin{array}{r} 5^{-1}5 \\ \underline{-1\ \ 7} \end{array}$$

5 - 1 - 1 = 3 so complete the subtraction as

$$\begin{array}{r} 5^{-1}5 \\ \underline{-1\ \ 7} \\ 3\ \ 8 \end{array}$$

Example 2. $\begin{array}{r} 547 \\ \underline{-284} \\ \end{array} \rightarrow \begin{array}{r} 547 \\ \underline{-284} \\ 3 \end{array} \rightarrow \begin{array}{r} 5^{-1}4\ 7 \\ \underline{-2\ \ 8\ 4} \\ 6\ 3 \end{array}$ since 4 – 8 = 4 + 2 – 10, $\begin{array}{r} 5^{-1}4\ 7 \\ \underline{-2\ \ 8\ 4} \\ 2\ \ 6\ 3. \end{array}$

Multiplication

The Vedic mathematics one-line shortcut for multiplication is a modification of our traditional algorithm using the memory aid sutra “vertically and crosswise” (Tirthaji, 1992, p. 34).

Example 3. 23 x 47 is solved by writing the solution from left to right starting with the vertical multiplication of tens by tens, giving:

$$\begin{array}{r} 2\ 3 \\ \times\ \underline{4\ 7} \\ 8\ \ \ \end{array}$$

Then the cross-multiplications sum to 2 (tens) x 7 + 4 (tens) x 3 = 14 + 21 = 26 (tens). Thus:

$$\begin{array}{r} 2\ 3 \\ \times\ \underline{4\ 7} \\ 8^{2}6\ \end{array}$$

Finally, the ones times ones vertical multiplication leads to 3 x 7 = 21, so:

$$\begin{array}{r} 2\ 3 \\ \times\ \underline{4\ 7} \\ 8^{2}6^{2}1 \end{array}$$

Completing the carries gives the final result:

$$\begin{array}{r} 2\ 3 \\ \times\ \underline{4\ 7} \\ 1081 \end{array}$$

If students work from right to left, completing the carries in their heads, the solution can be written directly. This method can be extended to problems with more digits as in the next example.

Example 4. $\begin{array}{r} 8\ 6\ 3 \\ \times\ \underline{2\ 7\ 4} \end{array}$ can be done from right to left with carries handled mentally or recorded and then solved:

3 x 4 = 12 (1s)	vertical
6 x 4 + 3 x 7 = 45 (10s)	cross
8 x 4 + 6 x 7 + 3 x 2 = 80 (100s)	triple cross
8 x 7 + 6 x 2 = 68 (1000s)	cross
8 x 2 = 16 (10000s)	vertical

$$\begin{array}{r} 8\ 6\ 3 \\ \times\ \underline{2\ 7\ 4} \\ 16^{6}8^{8}0^{4}5^{1}2 \end{array} \quad \rightarrow \quad \begin{array}{r} 8\ 6\ 3 \\ \times\ \underline{2\ 7\ 4} \\ 236462. \end{array}$$

Division

A non-standard division algorithm suggested by the sutra “last from ten” works well for divisors slightly less than a power of 10. This Vedic non-standard algorithm is a naïve repeated subtraction algorithm using the easiest possible choice of the number of subtractions, the leading digit of the remaining dividend. Division having divisors beginning with smaller leading digits, such as six or less, is usually done using the standard long division method.

Example 5. $9\overline{)123}$ can be shown using the format:

$$\begin{array}{rl} \text{Divisor} & \overline{)\text{Dividend}} \\ \text{(difference)} & \text{remaining portions} \\ & \ \text{Quotient} \mid \text{Remainder.} \end{array}$$

First, separate one place for the possible remainder since the divisor has one digit, list the difference from a power of ten and bring down the existing first digit as the first partial quotient to give:

9) 12|3
(1)___
1

This first partial quotient 1 represents subtraction of 10 nines, so the difference times the partial quotient 1 x 1 = 1 ten is the remaining portion not removed from the dividend out of the initial 100. It is denoted with a 1 below the 2 in the tens column as:

9) 12|3
(1) 1
1

Adding down the tens column shows 2 + 1 = 3 tens remain, plus 3 ones. Again the algorithm selects the easy choice of subtracting 3 nines, denoted by placing a 3 in the ones place of the quotient:

9) 12|3
(1) 1
13

Since we subtracted 3 nines, the difference times the partial quotient 1 x 3 = 3 is the portion not removed and so is placed below the 3 in the ones column:

9) 12|3
(1) 1|3
13

We can now add the remaining ones not subtracted to find the remainder:

9) 12|3
(1) 1|3
13|6 so the solution is 13 remainder 6.

We have accomplished this division using the easier elementary multiplication by the difference value and addition, rather than division and subtraction. Larger remainders require a final apportionment as shown in the next example.

Example 6. 8) 234. Note the difference from the power of ten to eight, and separate one possible remainder place:

8) 23|4
(2)

Bring down the first quotient digit, multiply by the difference and enter in next place:

8) 23|4
(2) 4
2

Add un-subtracted 3 + 4, multiply by the difference and enter in last place:

8) 23| 4
(2) 4|^{1}4
27

Add remainder units 4 + 14 = 18, giving:

8) 23| 4
(2) 4|^{1}4
27|^{1}8 18 / 8 = 2 r 2, so 27 r 18 = 29 r 2 is our final solution.

Example 7. Reserve the last two columns for a remainder if the divisor has two digits, as in 87) 1212. Note the difference from 87 to 100, separate last two columns, bring down first digit and multiply giving

```
  87 ) 12|1 2
(13)    1|3
        1
```

Add in the second column, multiply this 3 times difference 13 resulting in

```
  87 ) 12|1 2
(13)    1|3³9
       13| 8 1
```

so the answer is 13 r 81 since 12 + 30 + 39 = 81.

This method doesn't work well with smaller divisors as the difference values, and therefore the remainders, become too large. Other fascinating algorithms have been developed for these cases and can be found on the internet by searching for Vedic mathematics. As a direct extension of this work, it is interesting for students to find and learn a similar technique that can be used to divide by numbers slightly larger than a given power of ten.

References

Bathia, D. (2005). *Vedic mathematics made easy*. Mumbai, India: Jaico Publishing House.

Bournemann, G., Bright, G., Drury, B., & Lewis, K.. (2008). Algorithms. *Washington Mathematics, Fall*, 1-4.

Carroll, M. C. & Porter, D. (1998). Alternative algorithms for whole-number operations. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 1206-114). Reston, VA: National Council of Teachers of Mathematics.

Tirthaji, B. K. (1992). *Vedic mathematics*. Delhi, India: Motilal Banarsidass Publishers.

Trust Fund Scholarships

$600 scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the ncctm website under the "grants an scholarships" link.

The nomination form can be obtained from the "awards" area of the NCCTM Website (http://ncctm.org). more information can be obtained from: Janice Richardson, richards@elon.edu.

Donating to the Trust Fund

If you wish to memorialize or honor someone important to you through a donation to the NCCTM Trust Committee, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

2014 State Math Fair Winners

Betty Long, Appalachian State University, Boone, NC

Primary Division, Grades K-2

1st Place:
Tripp Wilson and Christopher Mustico
"Rockets to Infinity"
Balfour Elementary School, Asheboro, NC

3rd Place:
William Almeida
"Illuminating Lasers"
A.B. Combs Elementary School, Raleigh, NC

Honorable Mentions:
Sophie Mitchell
"Tall or Petite: What Height Could I Reach?"
Ridgewood Elementary School, Winterville, NC

Andrew Stoker
"Chipping Away at your Money"
Cool Spring Elementary School, Cleveland, NC

2nd Place:
Jackson Lowe
"Musical Fractions"
Butler Avenue Elementary School, Clinton, NC

Honorable Mentions:
Ashwin Varadarajan
"Why Must I Brush My Teeth More Often?"
Heyward C. Bellemy Elementary School, Wilmington, NC

Ethan Blackmon
"Super Math"
Spring Valley Elementary School, Durham, NC

Elementary Division, Grades 3-5

1st Place:
Matthew Congiusta
"Which N. American City is it Best to be a Sports Fan In?"
Forest View Elementary School, Durham, NC

3rd Place:
Emily Grubbs and Kailee Grubbs
"The Art of Slope"
Ridgewood Elementary School, Winterville, NC

Honorable Mentions:
Brooke Elliott and Kevin Donahue
"Let the Game Begin"
Candler Elementary School, Candler, NC

2nd Place:
Varun Varadarajan
"Home Field Advantage in the Winter Olympics"
Heyward C. Bellemy Elementary School, Wilmingon, NC

Honorable Mentions:
Madeline Sigmon
"How Far Does my Food Travel?"
Cool Spring Elementary School, Cleveland, NC

Reif Snyder
"Proportional Skyline"
Woodland Heights Elementary School, Mooresville, NC

Middle School Division, Grades 6-8

1st Place:
Sumani Nunna
"Order in Chaos: A Study of Interating Algorithms"
J. N. Fries Magnet School, Concord, NC

3rd Place:
Brandon Hall
"The Effect of Truss Bridge Designs on Weight Bearing Capacity"
Harris Road Middle School, Concord, NC

Honorable Mentions:
Delaney O'Connor and Lexi Lenker
"Extra Credit: Does it Always Make Cents?"
Hope Middle School, Greenville, NC

2nd Place:
Nathan Suri
"The Science of Secrets Demystified"
J. N. Fries Magnet School, Concord, NC

Honorable Mentions:
Matthew Lower
"Golden Marketing"
Hope Middle School, Greenville, NC

Neha Jakkinpali
"Monty Hall Problem"
J. N. Fries Magnet School, Concord, NC

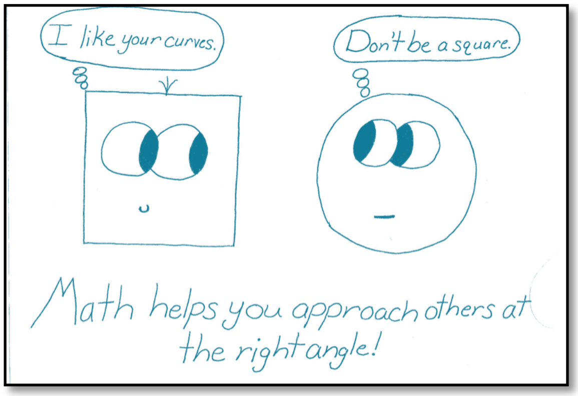
High School Division, Grades 9-12

2nd Place: Veronica Torres and Isabel Hernandez
"Pinata Time: Math and Pinatas"
Mary Phillips High School, Raleigh, NC

3rd Place: Thai Tran and Howard Gaines
"Rocket Power"
Victory Christian Center School, Charlotte, NC

NCCTM Logo Contest Winners

Emily Elrod and Tracie Salinas, Appalachian State University, Boone, NC



The Mathematics Logo Contest is held each spring and submissions are accepted in K-2, 3-5, 6-8, and 9-12 categories. The NCCTM Board selects the winning logo at its Spring meeting.

The 2013 winning logo, pictured, will be available on shirts at the NCCTM State meeting this month.

State Winner: Brooke Jones, 8th Grade student at Cranberry Middle School; Teacher: Laura King

Other finalists:

Student	Grade	School	Teacher
Jovani Santiago	2	Hardin Park Elementary	Lisa Shaw
Brayden Clapp	2	Seagrove Elementary	Lindsey Reynolds
Madison Grizzard	4	Manning Elementary	Lori Pleasant
Claire Smith	5	Charles England Elementary	Ella Frazier
Rylee Smith	5	Norris S. Childers Elementary	Denise Smith
Haley Evans	6	The Magellan Charter School	Wanda Sutton
Kamri Hayward	8	MacWilliams Middle School	Yvonne Wheeler
Anna Bringle	11	RJ Reynolds High School	Cameron Bell
On Kin Cheang	12	Asheville High School	Anuradha Kanakamedala

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, approximately 30-35 proposals have been funded, for an average grant of just less than $800.

The application is available in the "grants and scholarships" area of the NCCTM website, http://ncctm.org. Grant proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement.

Email Sandra Childrey, schildrey@wcpss.net, with questions.

Problems to Ponder

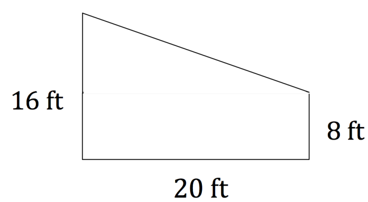
Holly Hirst, Appalachian State University, Boone, NC

Fall 2014 Problems

Grades K–2: Sonia, Megan, and Mike have collected sea shells on the beach. They put 5 shells in each pail. Sonia has 2 pails and 3 shells left over. Megan has 1 more pail than Sonia, but no shells left over. Mike has 2 more shells than Megan. How many shells does each person have?

Grades 3–5: Four classes of students competed in field-day events in which it was possible to score 60 points total. Mr. Smith's class scored 30% of the points. Ms. Jones's class scored half as many points as Mr. Smith's class. Mrs. Green's class scored 150% of the points won by Mr. Smith's class. Mr. White's class scored the remaining points. How many did points did each class score?

Grades 6–8: Tamara wants to figure out how many 18-inch-square floor tiles her parents need to tile their new trapezoidal sun room the dimensions for which are given in the diagram. If purchasing one dozen extra tiles is recommended for replacements, how many tiles total should be ordered?



Directions for submitting solutions:

1. Neatly print the following at the top of each solution page:
   - Your full name (first and last)
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page. Students who submit correct solutions will be recognized in the next issue of The Centroid. We wish to publish creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission.

Proper acknowledgement is contingent on legible information and solutions. Send solutions by 1 December 2014 to:

Problems to Ponder, c/o Dr. Holly Hirst
Mathematical Sciences
BOX 32069 Appalachian State University
Boone, NC 28608

As these problems are intended to stimulate independent thinking, it is expected that a submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

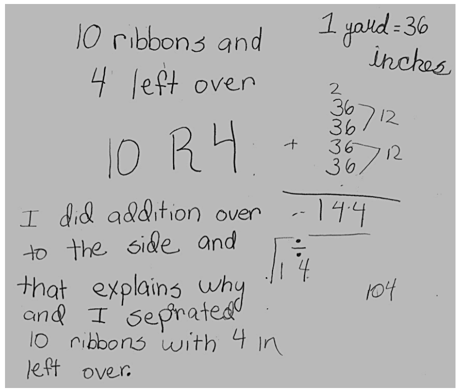
Spring 2014 Problem Solutions

Grades K–2: Jessica wants to take some of her toys to her grandmother's house. Jessica has a puzzle, a stuffed bear, and a doll. How many choices does she have if she can take 2 items to her grandmother's house?

No correct solutions were submitted.

Grades 3–5: A piece of ribbon 4 yards long is used to make bows requiring 14 inches of ribbon for each. What is the maximum number of bows that can be made?

Correct solutions were submitted by Mrs. Schulze's and Ms. Baccus's 3rd grade classes at Ravenscroft School in Raleigh. Here is one example:



Grades 6–8: 12 litres of water are poured into an aquarium of dimensions 50cm length, 30cm breadth, and 40cm height. How high (in cm) will the water rise?

No correct solutions were submitted.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Lee V. Stiff, lee_stiff@ncsu.edu.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The purpose of this award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state. Further, this program must have been sustained for a period of at least three years. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. Nominations will be retained in the active file for at least three years.

The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Bampia Bangura, babangur@ncat.edu.

NCCTM Board

contact information can be found at ncctm.org

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Follow the “Membership Information” link on the ncctm.org website, or go directly to:
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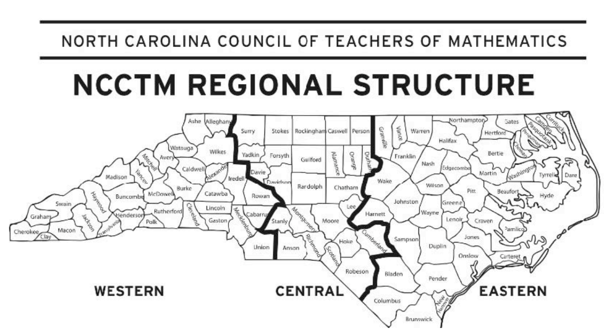

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