The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

Volume 41, Issue 1 • Fall 2015

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:

  Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:

  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.

  Chapters or sections of books:

  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:

  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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November 5th and 6th 2015
Koury Convention Center in Greensboro, NC.
***Principles to Actions* in Action**

Registration and Hotel Information
Current members can preregister for $65. Those wishing to stay overnight can book rooms at a reduced rate at the Sheraton Greensboro.

NCCTM Leadership Seminar

November 4th 2015
Koury Convention Center in Greensboro, NC.

Presenters
Diane Briars, President of the National Council of Teachers of Mathematics; Jon Wray, Secondary Mathematics Instructional Facilitator for Howard County Public Schools and President of the Association of Maryland Mathematics Teachers Educators; and Daniel Brahier, faculty member at Bowling Green State University and a lead author for NCTM's *Principles to Actions*.

Visit http://ncctm.org for more information!

The Centroid: Celebrating 40 Years

Deborah Crocker and Holly Hirst, Editors

In celebration of 40 years of *The Centroid*, NCCTM is pleased to provide all members with a print edition. As you will see, the theme in this issue is that special point in the triangle that has given our journal its name. We hope that you enjoy this special print edition of *The Centroid*!

The Centroid: NCCTM's Center Point of Communication

Reported by John Kolb, First President of NCCTM and Centroid Founding Board Member, and Robert Jones, Centroid Founding Board Member

Forty-five years ago NCCTM was founded on a far-reaching vision for mathematics education in North Carolina. Its purpose was to unite all mathematics teachers, mathematics supervisors and mathematics teacher educators into one organization working together on one common cause: the advancement of the teaching and learning of mathematics.

Prior to the creation of NCCTM, African-American and white mathematics teacher organizations in the state were separate and functioned as small subgroups within their respective segregated state-wide general teacher organizations. Far sighted leaders in both the black and white mathematics teacher groups sought a way to leave the segregated past and come together in a new, jointly formed organization. Much of the early success of NCCTM came as a result of the dedication, resolve, and mutual respect displayed by the leaders of these organizations.

Before the establishment of NCCTM, meetings of mathematics teachers in North Carolina reached few teachers and usually focused upon high school teaching. Founding leaders in NCCTM diligently sought to attract and support teachers of mathematics across the broad spectrum of instructional levels: early childhood through college. With the new organization, emphasis was placed upon inclusion of elementary and early childhood teachers. These teachers were recognized as being mathematics teachers just as teachers in middle school and high school who teach only mathematics. NCCTM made a concerted effort to welcome mathematics teachers from the fast-growing Community College System. With the help of a grant from NCTM, organizational meetings were convened, a constitution written, elections held, and the first annual meeting conducted at Salem College in Winston-Salem.

Initially, NCCTM 's sole instruments for carrying out its mission of being the focal point of mathematics education were the annual meeting and the three spring regional conferences. Soon the organization added another means to support its members professionally, *The Centroid*. Just as the centroid of a triangle is a special point that in one sense is the "center" of the triangle, *The Centroid* is a special instrument of NCCTM that acts as a center point for communication and exchange of ideas in keeping with NCCTM's founding principles. Over these many years, *The Centroid* has grown in size and most importantly in quality and variety of articles, problem sections, and interest items. The editors are to be congratulated for the continued improvement over the years that have culminated with the present-day *Centroid* that now fulfills what was originally envisioned as a valuable membership benefit of NCCTM.

Congratulations and thank you to all who worked in any capacity to further the success of *The Centroid* this past 40 years.

President's Message

State President Ron Preston
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It is my pleasure to serve as your president of the North Carolina Council of Teachers of Mathematics (NCCTM); however, as president, I understand that I am just one person among many who work to make mathematics learning for our students a priority. The Board of Directors, committee chairs and members, conference speakers, volunteers for events like our math fairs and contests, NCCTM members, and those who participate in and benefit from NCCTM-sponsored events all have the mathematics learning of our students at heart. It is a privilege to work with a group of talented and caring people. I want to share with you some recent activity in our organization and point you to events in the near future.

Check out the redesigned NCCTM website (http://ncctm.org)! The Board of Directors, under the direction of then-president Debbie Crocker, made this redesign a priority and the work has paid off handsomely. Many thanks to Debbie for seeing this work to completion, even after the end of her presidency. Of course, she remains active as past-president of the organization.

There are two upcoming events that you will not want to miss. The Fall Leadership Seminar is Wednesday, 4 November at Koury Convention Center in Greensboro and will feature presentations from Diane Briars, President of the National Council of Teachers of Mathematics; Jon Wray, Secondary Mathematics Instructional Facilitator for Howard County Public Schools and President of the Association of Maryland Mathematics Teachers Educators; and Daniel Brahier, faculty member at Bowling Green State University and a lead author for NCTM's *Principles to Actions*.

The second of the two events is the NCCTM Fall 2015 Math Conference, North Carolina's premier mathematics conference, which will be held on Thursday and Friday, 5-6 November at the Koury Convention Center in Greensboro. Diane Briars, Jon Wray, and Daniel Brahier will provide keynote addresses at the conference, and will be joined as keynote speakers by Doug Clements and Tim Kanold. The theme for this year's conference picks up on the *Principles to Actions* book and is "*Principles to Actions* in Action." As an organization, we are indebted to the hard work of Conference Co-Chairs, Tracey Howell and Carol Seaman, and Program Co-Chairs, Lisa Carnell and Amy Travis. They, in turn, are quick to point out that the success of the conference depends on the approximately 300 speakers and 2000 conference attendees.

Although the fall conference is the crown jewel for NCCTM, I do not want you to forget the many spring events that our organization sponsors and to recognize that they require fall preparation and planning. Please encourage your students and make it convenient for them to participate in your regional Math Fair and Math Contest. Make it possible for your students to enter the logo contest. Start early to create excitement around these great events. Encourage students to participate in other mathematics clubs, activities, and contests. As NCCTM members, apply for a mini-grant or graduate scholarship. Plan to attend the state-wide Spring Leadership Seminar (8 April 2016) and your spring regional NCCTM conference.

Finally, I want to encourage you to become more involved in our great organization. If you are reading this and are not a member, consider joining. If you are a member, but participate in only one or two NCCTM activities, please consider plugging yourself in to additional events. Make this be the year that you write that *Centroid* article or present at the spring regional conference. Invite a fellow mathematics educator to join you at the conference, to get his/her students involved in the Math Fair, and to read *The Centroid*. Share your ideas with the president, the Board of Directors, the committee chairs. We have no limit on the number of members, the amount of talent, or the quantity of energy we can receive into our council!

The Centroid: A Brief History

Reported by Ralph Willis, Cullowhee, NC

The founding of NCCTM took place at a meeting held in October 1970 at the Nantahala Village Inn in Bryson City, North Carolina. Those in attendance established the following Goals of the North Carolina Council of Teachers of Mathematics:

1. To coordinate mathematics education in NC with that in the nation.
2. To maintain channels of communication between the various levels of mathematics instruction.
3. To stimulate local efforts to maintain excellence in mathematics education.
4. To maintain critical evaluation of mathematics program and curricula for grades K-12.

Among the specific Objectives and Activities designed to meet Goal 2 was: *Prepare and circulate a newsletter.*

1970-1975: The Newsletter
During the early years of the organization the membership was kept informed of activities and programs through the efforts of Bob Jones, Supervisor of Mathematics for the State's Department of Public Instruction (SDPI), and his staff who issued a Newsletter from his office in SDPI.

1975-77: The Centroid is Established
The first issue of *The Centroid* was published by NCCTM in the Spring of 1975, under the founding editorship of Joe Dodson, Coordinator of Mathematics in the Winston-Salem/Forsyth County Schools. What motivated Joe to become interested in and pursue such a venture?

Joe joined the faculty of the Mathematics Department at Western Carolina University as Director of Mathematics Education in 1970. During his first year, Joe and I initiated a newsletter entitled *The Abelian Grapevine*, which was distributed to 64 junior and senior high schools in the 16 western-most counties of North Carolina, reaching approximately 325 teachers of mathematics.

Its format evolved to feature a lead article regarding an issue in mathematics education and a problem of the month, in addition to being used as a vehicle to distribute information regarding activities such as WCU Mathematics Departmental course and program offerings, the Mathematics Contest, and regional and state NCCTM meetings. Joe left WCU in 1974, and based on his experience with *The Abelian Grapevine* I believe that Joe came to see a need for this more journal-like type of format to distribute across the entire state of North Carolina.

The initial Editorial Board consisted of Katye Sowell, Kate Morrow, Geneva Maney, Hunter Ballew, John Kolb, John Ogle, Margo Perkins, Linda Harlan, Virginia Newell, and Bob Jones. After Joe Dodson edited the second issue, he became the Business Manager for NCCTM, and the third, fourth, and fifth issues were edited by Bill Palmer of Catawba College and Harold Williford of Pfeiffer College.

The goals of *The Centroid* were established as and continue to be:

1. To serve as an official communication between the Board of Directors of NCCTM and the membership.
2. To inform the membership of important events in the field of mathematics education.
3. To inform the mathematics education community of successful programs and practices which are in operation.
4. To serve as a forum for professional ideas, proposals, and opinions.
5. To inform the membership of significant and applicable findings from research.

The Centroid has always held to the principle of printing news and articles of interest to teachers of students of all ages and levels of ability. The early issues featured special sections such as: The President's Corner, News from Regional Presidents, Research Speaks, You May Wish to Read About, Have You Read, From Around The State, Notes From the State Department of Public Instruction, Problem Section, Problem Solving, and Its Elementary. Over the years, other special sections have been added: Math in the Middle, Teaching with Technology, The History Corner, and Algebra for Everyone.

Editing the Centroid: Serving NCCTM

In the Fall of 1977, the editorship was moved to North Carolina State University under the leadership of John Kolb, Bill Waters, and Larry Watson, with each serving a year as editor. Bill Smith and Hunter Ballew at UNC–Chapel Hill served as editors for the next two years, and in the Fall of 1982 responsibility for the publication of *The Centroid* was moved to East Carolina University where John Daniels, Katharine Hodgin, Robert Joyner, Katye Sowell, and Bill Spickerman served as editors.

In the Fall of 1986, *The Centroid* moved to Appalachian State University under the editorship of Bill Paul, Theresa Early, Jim Smith, Wade Macey, Anita Kitchens, Mark Harris, and Bill McGalliard. *The Centroid* moved to Western Carolina University with Harold Williford as editor in August 1995. With the Fall 1998 issue, Kathy Ivey of WCU became the editor. While at WCU, the Editorial Board, in addition to the two editors named, consisted of Julia Barnes, Joe Klerlein, Nick Norgaard, Terry Rose, Scott Sportsman, Charles Wallis, and Ralph Willis.

With the Winter 2001 issue, the editorship returned to Appalachian State University, with Gregory Foley and Deborah Crocker as co-editors. Brian Felkel joined Gregory and Deborah for the Spring 2003 issue. Then from Fall 2003 through Spring 2008, Holly Hirst and Brain Felkel were the co-editors. Since Fall 2008, Deborah Crocker and Holly Hirst have served as the Co-editors.

What is in a Name - Centroid?

I would be remiss if I did not relate something of mathematics to this brief history of *The Centroid*. To my knowledge, there is nothing in the written record to explain why and by whom the name *The Centroid* was chosen. Thus, I offer the following connections as to why the word “centroid” is an appropriate name for our journal. First a few definitions:

- Altitude: A line from the vertex of a triangle that is perpendicular to the opposite side.
- Median: A line from the vertex of a triangle to the midpoint of the opposite side.
- Midline: A line connecting the midpoints of two sides of a triangle.
- Medial Triangle: The triangle formed by connecting the midpoints of the sides of a triangle.
- Orthocenter: The point of intersection of the altitudes of a triangle
- Circumcenter: The point of intersection of the perpendicular bisectors of the sides of a triangle

A few of the properties of triangles involving the centroid are:

1. The medians of a triangle meet at a point called the centroid, which is two thirds of the distance from each vertex to the midpoint of the opposite side.
2. A triangle and its medial triangle have the same centroid.
3. The medial triangle of the medial triangle and the medial triangle have the same centroid. (And so on as successive medial triangles are formed.)
4. The lines joining the centroid to the vertices of a triangle divide the triangle into three parts having equal area.
5. The orthocenter, the centroid, and the circumcenter are collinear.
6. The centroid is two-thirds of the distance from the orthocenter to the circumcenter.

The listing above could be continued, with many more properties involving the centroid. I relate this seemingly endless list with all those who have been a member of NCCTM and the myriad relationships and networks that membership has afforded. When a triangle with its medians, intersecting at the centroid, is superimposed on the

shape of North Carolina, I visualize a concentration of all the minds of the NCCTM membership as they pursue the furtherance of mathematics education both as individuals in their own classrooms and as a collective group in NCCTM membership.

Another relationship that property four above could allow one to conjure up has to do with the fact that the state wide NCCTM is organized into three equal sub-councils that have their own officers and sponsor individual activities such as math fairs and single day and multiple day meetings. When a triangle is cut from paper and the centroid used as a balance point, the triangle will balance on the head of a pin. Abstracting from this notion, I visualize our organization of mathematics educators all across the state in a concerted frontal assault making daily gains in the advancement of mathematics literacy.

A Centroid Challenge to Think About
Of the centroid, orthocenter, and circumcenter, which point is always in the interior of the triangle? Can you explain why?

NCTM Annual Meeting

Building a Bridge to Student Success
April 13–16, 2016 • San Francisco

Join more than 9,000 of your mathematics education peers at the premier math education event of the year. Registration and travel information will be available in August.

Examine the innovative ideas that can improve the quality of learning for every student.

- Insights into implementation and assessment of the Common Core State Standards for Mathematics
- Best practices directly from experts in mathematics education
- New ideas for integrating mathematics into other disciplines and supporting student learners

Get even more. Attending the NCTM Annual Meeting is also an important opportunity to collaborate and expand your professional network, and to learn about the latest teaching aids, lesson resources, and math activities in the Exhibit Hall.

Web Resources

Have you seen the following websites? There are some good ideas out there for use in K-12 math classrooms!

http://www.egfi-k12.org/
The American Society for Engineering Education (ASEE) posts interesting class activities and lesson plans.
http://www.homeadvisor.com/r/educators-guide-to-building-and-construction/#.VeW8mc7O_hl
The Home Advisor website posts lessons on a variety of topics related to construction.

Minimizing the Sum of the Squares of the Distances to the Vertices of a Triangle

Alfinio Flores, University of Delaware, Newark, DE

The author discusses the point in a triangle that yields the minimum sum of squares of the distances to the vertices, and outlines activities that guide students to investigating the concept from different perspectives, leading to connections to the centroid of a triangle.

The author provides downloadable Geiger files to aid in the interactive investigation.

The point that minimizes the sum of the distances to the vertices of a triangle is called the Fermat point of the triangle. Several approaches exist to discover that point and to prove that for triangles whose largest angle is less than 120˚, the point can be found by forming angles of 120˚ (Park & Flores, 2015). Similarly, there is an interesting point in the triangle that minimizes the sum of the squares of the distances to the vertices of a triangle. This is a well known point in the triangle, but surprisingly, this minimizing property is not usually mentioned when the point and its properties are studied in geometry. It is fairly easy to use tools from calculus and algebra to find by hand the point that minimizes the sum of squares of the distances to the vertices. In contrast, finding the Fermat point using such tools requires the use of powerful software but with human guidance (Palacios-Vélez, Pedraza-Oropeza, & Escobar-Villagran, 2015).

In the first part of this article we present activities for high school students and future mathematics teachers related to the point that minimizes the sum of the squares of the distances from a given point to the three vertices of a triangle from different perspectives, including the use of interactive software, to provide opportunities for students to develop a rich and connected understanding. In the second part, we provide some possible solutions.

Student Activities and Explorations

1) *Empirical exploration:* Use the interactive file available online at http://tube.geogebra.org/student/m745187 to approximate the location of the point that minimizes the sum of the squares of the distances to the vertices of a triangle (Figure 1).

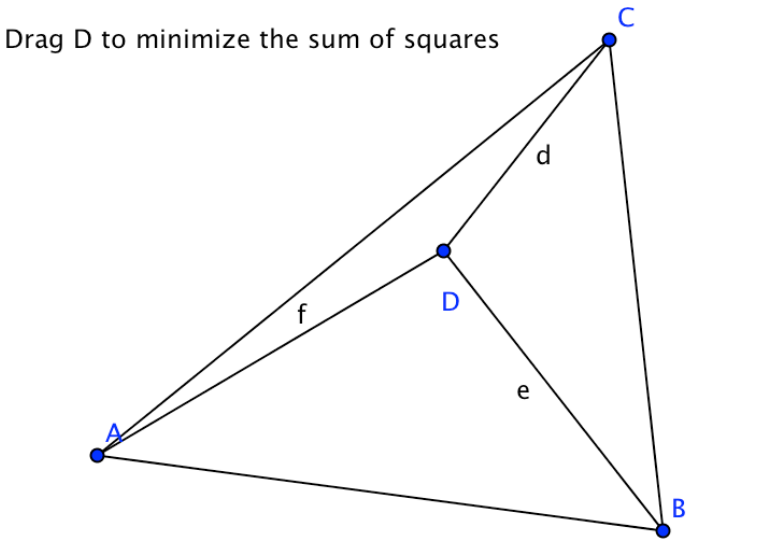


Figure 1: Exploring the sum $d^2 + e^2 + f^2$

Once you have found the approximate location, check the box in the lower part (Figure 2) to show extensions of the segments connecting the point with the vertices and their intersections with opposite sides. When the sum of the distances is minimal, observe the intersection of the dotted lines with the sides. What can you say about points E, F, and G? Conjecture about the location of the point that minimizes the sum of the squares of the distances to the vertices of the triangle. State your conjecture.

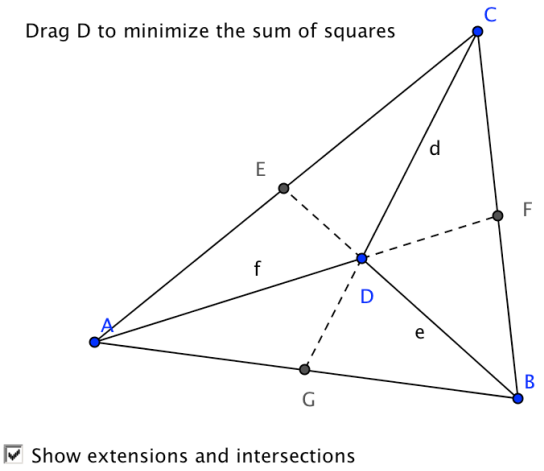


Figure 2. Minimizing the sum $d^2 + e^2 + f^2$

In what follows, we will use coordinates for the vertices of the triangle. Without loss of generality, and to make algebraic expressions simpler, we can put one of the vertices of the triangle on the origin, and another one on the x axis, with coordinates $(b, 0)$. The third vertex will have coordinates (c, d).

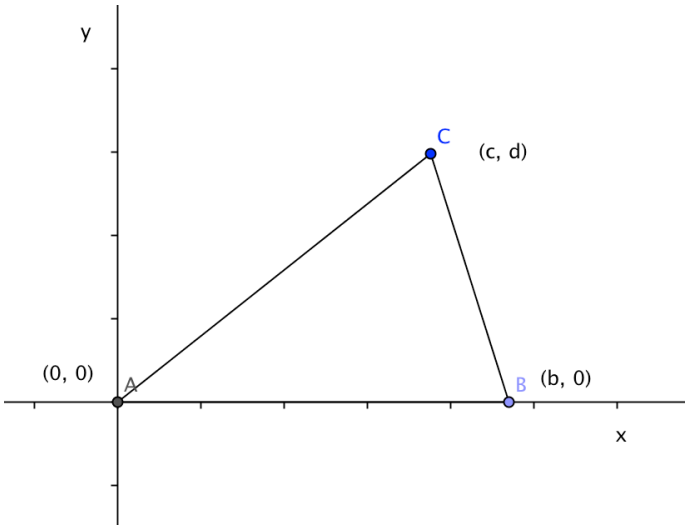


Figure 3. Coordinates of vertices

2) *Finding the point in the triangle that minimizes the sum of the squares to the vertices of a triangle:*

a) For each point (x, y) on the plane, express the sum of the squares of the distances to the vertices of the triangle as a function.

b) Find the minimum for this function in two different ways (you can use partial derivatives with respect to x and y, or you can use the fact that the mean of a set of data minimizes the sum of squares of the differences of the data).

c) Show that the coordinates obtained in part b) correspond to the point of intersection of the medians of the triangle.

3) *Graphing the function for the sum of squares in 3D*: From looking at the equation of the function obtained in 2a), can you say what type of quadratic surface is represented by its graph? Use a 3-dimensional graphing program to graph the function for particular values of b, c, and d, for example, $b = 1, c = .7, d = .8$. What is the shape of the surface? What can you say about its principal axis and its minimal point?

4) *Proving that the level curves of the function in 2a) are circles centered on the centroid*:

a) Use the interactive GeoGebra file at http://tube.geogebra.org/student/m745235 to find the constant value of the function obtained in 2a) for a circle of radius r centered on the centroid. Express the constant quantity in terms of the minimal sum of squares of distances and the radius of the circle.

b) Express the minimal sum of distances in terms of the coordinates of the centroid.

c) Use the equation of the function to make the sum of the distances of point (x, y) to the vertices of a triangle equal to the constant found in part a).

d) Show that the equation you found in part c) corresponds to a circle centered on the centroid with radius r.

Solutions

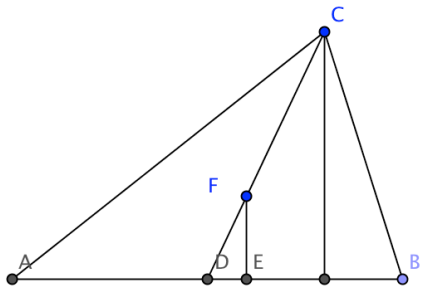
1) When the sum of the squares of the distances is minimal, the intersections of the dotted lines with the sides seem to be at the midpoint of the sides. The conjecture is that the point that minimizes the sum of the squares of the distances to the vertices is the intersection of the medians, which is the centroid!

2)

a) The sum of squares of the distances is $z = x^2 + y^2 + (x - b)^2 + y^2 + (x - c)^2 + (y - d)^2$.

b) Using the partial derivatives: $\frac{\partial z}{\partial x} = 2x + 2(x - b) + 2(x - c)$. Setting the equation equal to 0 and solving we get $x = \frac{b+c}{3}$. The mean average of the x coordinates of the vertices is $\frac{0+b+c}{3}$. $\frac{\partial z}{\partial y} = 2y + 2y + 2(y - d)$. Setting the equation equal to 0 and solving we get $y = \frac{d}{3}$. The average of the three y coordinates is $\frac{0+0+d}{3}$.

c) One median passes through the origin and the point $\left(\frac{b+c}{2}, \frac{d}{2}\right)$. Its equation is $y - \frac{d}{2} = \frac{d}{b+c}\left(x - \frac{b+c}{2}\right)$ or $y = \frac{d}{b+c}x$. Another median passes through $(b, 0)$ and $\left(\frac{c}{2}, \frac{d}{2}\right)$. Its equation is $y = \frac{d}{c-2b}(x - b)$. From these simultaneous equations we have $(c - 2b)x = (b + c)(x - b)$, which can be reduced to $0 = 3bx - b^2 - bc$ or $x = \frac{b+c}{3}$. Substituting this value in the first equation we have $y = \frac{d}{3}$, so the solution is $\left(\frac{b+c}{3}, \frac{d}{3}\right)$. An alternative method uses the fact that the medians in a triangle cut each other in the ratio 2 to 1 and is suggested by Figure 4.



$$\frac{b}{2} + \frac{c - b/2}{3}$$

Figure 4. x coordinate of the centroid

3) Figure 5 shows the graph of $z = x^2 + y^2 + (x - b)^2 + y^2 + (x - c)^2 + (y - d)^2$ for $b = 1, c = .7, d = .8$ using two 3-dimensional graphing programs (Graphing Calculator Program and Mathematica).

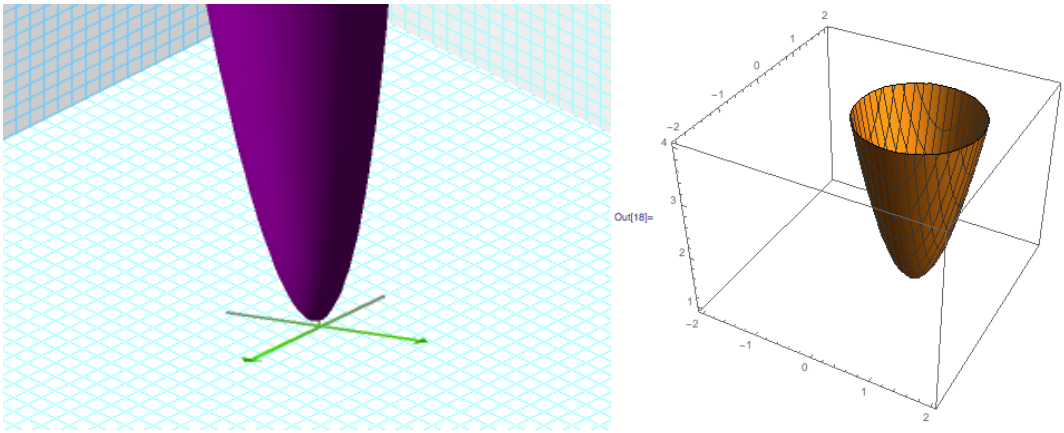


Figure 5. The graph of the function representing the sum of the squares of the distances

In Mathematica, the graph was obtained by typing the following commands.

```
c := .7
d := .8
Plot3D[x^2 + y^2 + (x - 1)^2 + y^2 + (x - c)^2 + (y - d)^2, {x, -2, 2}, {y, -2, 2},
 RegionFunction → Function[{x, y, z}, x^2 + y^2 + (x - 1)^2 + y^2 + (x - c)^2 + (y - d)^2 ≤ 4],
 BoxRatios → Automatic]
```

The graph seems to be a paraboloid of revolution. Its principal axis of revolution is parallel to the z axis. Its minimal point is off the origin (both x and y coordinates are positive), and above the xy plane.

4)

a) Students can use GeoGebra to construct a dynamic figure similar to Figure 6 (or use the dynamic figure available at http://tube.geogebra.org/student/m745235). F is the centroid of the circle and H is a point on the circle.

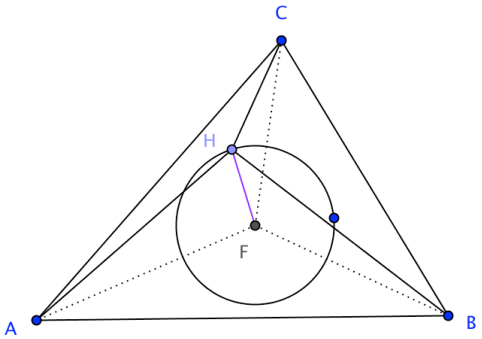


Figure 6. Distances from a point on the circle to the vertices

Students can use GeoGebra to measure the distances from the centroid to each of the vertices, square them, and add the squares of the distances $n = j^2 + l^2 + m^2$. Students can also measure the distances from H to each of the vertices, square them, and add them $k = g^2 + h^2 + i^2$. By dragging H around the circle students see that this value is constant for any point on the circle. Because we are dealing with squares of distances, it makes sense to compare the difference of these two sums with $r^2, q = (k - n) / r^2$. Students see that this value q is constant for any circle, and equal to 3. From here we can write $k = n + 3r^2$.

b) The sum of the squares of distances from the centroid to the vertices of the triangle is given by:

$$s = \left(\frac{b+c}{3}\right)^2 + \left(\frac{d}{3}\right)^2 + \left(\frac{b+c}{3} - b\right)^2 + \left(\frac{d}{3}\right)^2 + \left(\frac{b+c}{3} - c\right)^2 + \left(\frac{d}{3} - d\right)^2$$

$$s = \frac{1}{9}((b+c)^2 + d^2 + (c-2b)^2 + d^2 + (b-2c)^2 + (2d)^2).$$

Expanding, collecting similar terms, and simplifying we obtain $s = \frac{2}{3}(b^2 - bc + c^2 + d^2)$. Using the result from part a), the sum of the squares of the distances from a point (x, y) to the vertices is equal to a constant $s + 3r^2$:

$$x^2 + y^2 + (x-b)^2 + y^2 + (x-c)^2 + (y-d)^2 = \frac{2}{3}(b^2 - bc + c^2 + d^2) + 3r^2$$

Expanding, collecting similar terms, and simplifying we obtain:

$$x^2 - 2\frac{b}{3}x - 2\frac{c}{3}x + 2\frac{b}{3}\cdot\frac{c}{3} + \frac{1}{9}b^2 + \frac{1}{9}c^2 + y^2 - 2\frac{d}{3}y + \frac{1}{9}d^2 = r^2$$

Regrouping:

$$x^2 - 2\left(\frac{b}{3} + \frac{c}{3}\right)x + \frac{1}{9}(b^2 + 2bc + c^2) + y^2 + 2\frac{d}{3}y + \frac{1}{9}d^2$$

This can be written as $\left(x - \frac{b}{3} - \frac{c}{3}\right)^2 + \left(y - \frac{d}{3}\right)^2 = r^2$, which is the equation of a circle with center $\left(\frac{b+c}{3}, \frac{d}{3}\right)$ and radius r. Students can also use the 3D graphing program to see that the level curves are indeed circles (Figure 7).

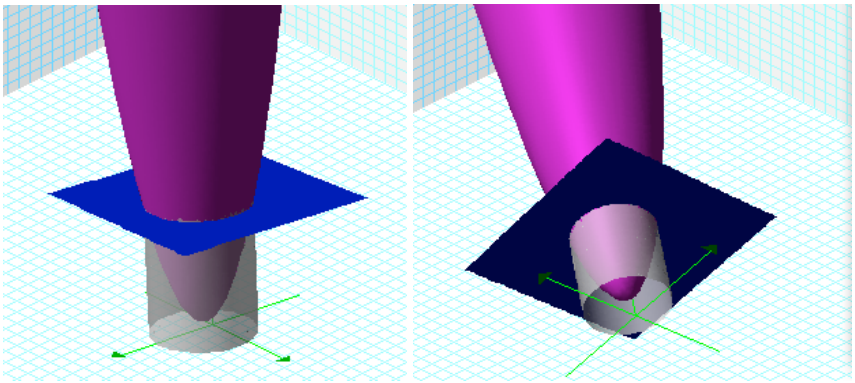


Figure 7. Level curves as shown in the 3D graphing program

Final remarks

The centroid is quite an important point in the triangle. Many of its properties are studied in geometry, but often with no connections to other parts of mathematics. By studying the minimizing property of the centroid with respect to the sum of the squares of the distances to the vertices of a triangle, students have the opportunity to link together concepts used in different parts of mathematics. The centroid is the center of balance of the triangle, the same way that the arithmetic mean is the center of balance of a set of data (Flores, 2008). The fact that the sum of squares is minimized at the arithmetic mean and at the centroid is closely connected to their balancing properties.

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Lying with Statistics: Promoting Numeracy with an Exciting Project

Samuel Luke Tunstall, Michigan State University, East Lansing, MI

The author describes an activity developed for middle school students using Darrell Huff's classic book, How to Lie with Statistics.

The activity can be adapted to a range of grade levels, provided the students have an understanding of the concepts of mean, median, and percent.

Making the ascent to the peak of Bloom's Taxonomy with a class is any teacher's dream; however, like climbing Mount Everest, such a trek is difficult for instructors at any level. Using the book *How to Lie with Statistics* can make that climb easier when teaching basic statistics. This article details an exciting related project for implementation in both secondary and post-secondary environments. Students create their own news piece that is infused with misleading statistics; they then evaluate others' articles to find as much deception as possible. Its flexibility manifests in that it will engage struggling or unmotivated students, as well as push those at the top of their class. One can assign it as an individual or group project in both face-to-face and online delivery formats. We begin with a brief discussion of the text and its place within the K-14 curriculum, then we give one version of the assignment along with potential variations.

The Book and Its Relation to Numeracy

Students of all ages are inundated with quantitative arguments each day. Although we live in the age of big data, the infiltration of numbers into our lives – both deceptive and in good faith – is nothing new. Darrell Huff discussed such deception in spectacular detail in his 1954 classic *How to Lie with Statistics*. This book has been reprinted numerous times in the years since. With a length of 144 pages, the text is easily readable; a background in the concepts of mean, median, and percent change are the only meaningful prerequisites. One of its most friendly features is its use of illustrations (by Irving Geis). Indeed, those familiar with Roald Dahl novels will find themselves right at home. In each chapter, Huff presents a method for using statistics. He infuses the discussion with examples which – although written in 1954 – still resonate with readers today. As an example, in the first chapter, Huff discusses the sampling techniques behind statistics reported in the media. The example he uses relates to self-reporting in sample surveys and the bias that it brings in; such volunteer samples are routinely quoted in media sources today (e.g., CNN viewer polls). Students would likely have no problem connecting the chapter to their own lives.

A wonderful benefit of the book is that it fosters students' numeracy, which is generally defined as one's ability to understand elementary mathematics and make quantitative arguments in day-to-day life (SIGMAA, 2004). It is suggested that middle-school is a time for the fundamentals of quantitative literacy (QL) to take root, and that such roots should be cultivated throughout high-school and college (Hallett, 2003). As such, an assignment associated with this text is meaningful for students at any of those levels. Moreover, justifying a project with this text is easily done, as the statistical concepts it addresses align well with both NCTM's *Principles and Standards for School*

Mathematics (NCTM, 2000) and the more recent Common Core State Standards Initiative (CCSSI, 2010). Both of these sets of standards pepper statistics throughout the K-12 curriculum. With all of this in mind, the assignment, which was used for middle-school students at a summer camp, is shown below. Since adapting this to a traditional school setting is simple, variations of this assignment are only briefly mentioned.

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The Classroom Activity: Lying with Statistics

Materials:

- A class set of the book *How to Lie with Statistics*, or internet access to read the book (free of charge) at https://archive.org/details/HowToLieWithStatistics
- Large post-it notes (or posters) for each group
- Computer lab (or digital device) access for student research later in the assignment

Classroom Setup: Students begin by reading certain chapters of the text based on an assigned group number (there are four distinct reading assignments). They then form groups so that each group has collectively read the text. This group will then work together for the remainder of the assignment.

The task:

1. (1 hour in class or as homework) Individually, begin by reading the introduction and taking brief notes on what you learn. Then, based on your group number, read those specific chapters while taking notes on the methods discussed on how to lie with statistics; note you will have to explain these methods to others, so be sure you have a solid grasp of the material – if not, just ask for help!
2. (20-30 minutes) After an hour has passed, scramble so that each group has collectively read the book. You'll then have 20-30 minutes to discuss what your chapters were about. At the end of this task, you should have a completed note-sheet summarizing the book.
3. (1-2 hours) Next, the group's assignment is to create – on a large post-it note or poster board – a media article that misuses statistics to convey a point. The content of the article is up to the group; examples of possible topics are listed below. You will have roughly one to two hours to do research on this topic online and create the poster. Use the internet to find information that you can use to twist a news article in favor of the point you wish to make.
    a) Be sure the information you use is accurate before twisting it in your favor.
    b) Include as many of the techniques from the book as you can.
    c) These posters will be hung in the class (or school hallway), so be creative and neat!
4. (30-45 minutes) Your final group task is to present the posters to the class for critique and admiration. On your own, while watching the presentations, identify as many techniques as you can.

Reading Assignment:

- Group 1: Read chapters 1 and 2
- Group 2: Read chapters 3, 4, and 5
- Group 3: Read chapters 6, 7, and 8
- Group 4: Read chapters 9 and 10

Example Topics:

- Genetically modified organism (GMO)s – good, bad, or too early to tell?
- Obesity in the United States – where are we heading, and what's to blame?
- Charter schools – friend or foe?
- Videogames and violence – do we have cause for concern?
- Gun violence in America– is there a problem, and who is to blame?

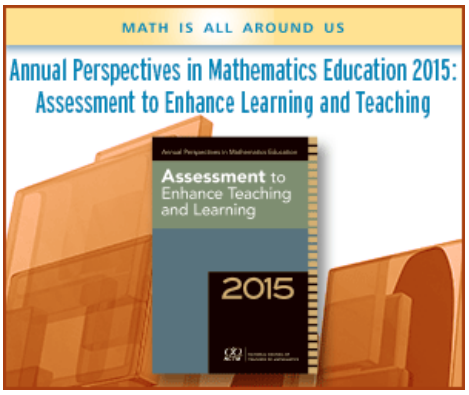
Variations and Tips:

- This project does not have to be done entirely in class; students could work individually the entire time or meet in groups outside of class. The presentation aspect is important, so class time should be devoted to that.
- One could easily execute this assignment in an online class, either in groups or individually.
- If one's budget does not allow for posters (or large post-it notes), a PowerPoint slide conveying the same information would likely suffice.
- For the last step, you may want to have students write down the techniques they see in each group's poster. If students are required to turn this in, they may be more engaged in the presentations.

New from NCTM

Using Assessment Effectively to Improve Mathematics Instruction for All Students

Assessment is one of the most discussed topics in mathematics education today. Issues drawing particular attention include the proper role of formative assessment, the need to show accountability through large-scale assessment, and the use of assessment to measure student learning of the content and practices in evolving new standards.



The 2015 issue of *Annual Perspectives in Mathematics Education* (APME) approaches assessment from a wide variety of perspectives. Its 21 chapters, written by leading mathematics educators and researchers, are grouped into four sections:

- Assessment in Action
- Design of Assessment Tools and Strategies
- Professional Learning to Enhance Classroom Assessment
- Assessment as Reasoning from Evidence

Interested in North Carolina Add-on Elementary Mathematics Licensure?

Catherine Schwartz. East Carolina University, Greenville, NC

This article provides a brief overview the North Carolina Elementary Mathematics Add-On Licensure (Emails) Program, established in 2009.

The Program is comprised of six courses created around high leverage practices and important mathematical content.

In 2009, the North Carolina State Board of Education charged the University of North Carolina System to create and maintain a program of courses to prepare elementary school mathematics leaders who would receive an add-on license. The development of this program of courses responds to national reports that have called for the placement of mathematics leaders in elementary schools in the U.S., e.g., *No Common Denominator* (National Council on Teacher Quality, 2008) and *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (NMAP, 2008).

A planning team of faculty from seven universities, representatives from the NC Department of Public Instruction, and LEA representatives worked collaboratively to establish and pilot the program from 2009-2011. Currently, courses are offered statewide in multiple formats (face-to-face, hybrid, online, blended online), depending on the university with which teachers are completing the program.

The Elementary Add-on Licensure (EMAoL) program of study includes six graduate-level courses focused on the teaching and learning of mathematics at the elementary school level. Each course was designed around a high-leverage practice (Ball, Sleep, Boerst, & Bass, 2009), a primary mathematics content area focus (approximately 80%), and a secondary mathematics focus (approximately 20%). The primary mathematics focus serves as the context for exploring the high-leverage practice, and the secondary mathematics focus serves to exemplify how the high-leverage practice can be applied across content domains (Rachlin, 2013). The following chart provides an overview of the six courses.

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High-Leverage Teaching Practice	Mathematics Content
Selecting, Designing, and Using Mathematical Tasks	**Primary**: Number Systems and Operations **Secondary**: Number Theory and Rational Numbers
Understanding and Applying Knowledge of Learning Trajectories	**Primary**: Rational Number and Operations **Secondary**: Measurement
Orchestrating Classroom Interactions	**Primary**: Measurement **Secondary**: Data Analysis
Fostering Reasoning through Discourse and Questioning	**Primary**: Algebraic Thinking **Secondary**: Number Systems and Operations
Assessing Student Knowledge	**Primary**: Geometry and Spatial Visualization **Secondary**: Early Number Concepts
Helping Teachers Develop as School-based Leaders	**Primary**: Mathematical Modeling **Secondary**: Connecting Number to Algebra

Many university programs will allow all or part of the coursework to be applied to a master's degree in Mathematics Education. For additional information on how to apply for courses at one of the seven approved UNC universities contact the university's EMAoL Coordinator:

Appalachian State University (Kathleen Lynch-Davis, lynchrk@appstate.edu)
East Carolina University (Katie Schwartz, schwartzca@ecu.edu)
North Carolina State University (Temple Walkowiak, temple_walkowiak@ncsu.edu)
UNC-Charlotte (Drew Polly, drew.polly@uncc.edu)
UNC-Chapel Hill (Susan Friel, sfriel@email.unc.edu)
UNC-Greensboro (Kerri Richardson, kerri_richardson@uncg.edu)
UNC-Wilmington (Shelby Morge, morges@uncw.edu)

Here's what teachers and math coaches who have earned the Elementary Mathematics Add–on License are saying:

> *Digging deeper is one thing I can say the EMAoL program and the CCSS have in common. For this reason, the coursework of the EMAoL has increased my knowledge of the content. I have found that my ability to present "rich" lessons to my students is birthed out of my own depth of understanding of the standards I am teaching.*
>
> *Throughout the coursework, the question for me became: "How do I implement these practices? What should it look like in action?" Thankfully, each of the courses completed during this program focused on research-based instructional strategies that help support student learning.*
>
> *Looking at where the students are coming from and going to has enabled me to present the material at my grade level in a more effective manner. I feel more confident in my teaching of math this year than I have in the past.*
>
> *Being exposed to information about cognitively demanding tasks has caused me to examine the tasks I present and has benefited my students because they are now participating in more tasks that promote higher-level thinking. Learning about math talks and questioning has increased the productive conversation during my math lessons. I feel my lessons and class discussions focus more on the math and are more beneficial for my students.*
>
> *Being taught years ago in a procedure based way left me clueless to many of the "whys?" I am supposed to be having my students explore. This program has done what most people don't have the opportunity to do!!!! That is have a "do over" in my own my learning to be able to present standards in a conceptual "doing math" way to my students!*

Albert Einstein once said, "The world as we have created it is a process of our thinking. It cannot be changed without changing our thinking." These courses have done just that. They have widened my view of what math instruction looks like and deepened my own knowledge of the content required of the K-5 curriculum. I could not be happier to have taken these courses, especially as teachers look to me now to help support them through this change to "deeper," not broader, standards.

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Mini-grant Report

Discovery Education's Math Techbook in the Middle School Classroom

Katie Martin, New Hanover County Schools, Wilmington, NC

The author describes the results of her mini-grant funding, used to help teachers investigate and implement the Math Techbook *curriculum using iPads in either a 1:1 or small group environment.*

The goal for my mini-grant from NCCTM was to conduct a pilot in my school district implementing Discovery Education's *Math Techbook* (2015), a digital curriculum that is meant to supplement a teacher's core instruction. My teachers would utilize the tools within the *Techbook* to facilitate student learning and increase engagement while adhering to the three shifts of the Common Core State Standards (2010). The ultimate goal would be increased outcomes on the End of Grade Tests. This project was of great importance to my teachers because they are faced with a lack of textbook funding to support the purchase of materials that align with the rigor of the Common Core State Standards Initiative. Our district is currently successfully implementing Discovery Education's *Science Techbook* and has already seen increased student outcomes. My thought was that this implementation would be a natural progression for teachers and students already familiar with the platform.

The components in the *Math Techbook* align with the three aspects of rigor in the Common Core: conceptual understanding, procedural skill and fluency, and application. Each lesson balances these three aspects in their discovery, practice, and application components. Teachers have lessons laid out for them in a format that is easily implemented with students in a whole group, or even in individual stations with a few devices, such as laptops or iPads. Additionally, teachers have the ability to store lessons and activities within the system that can be shared school or district-wide. The system allows teachers to set up classes and assign differentiated instructional activities to students that may need additional support in topics. Students have access to the content from anywhere that there is internet access. Without textbooks to send home, some teachers are relying solely on digital resources anyway.

Project Implementation

Twenty-eight teachers from five middle schools received one day of training on how to implement *Techbook* into their classrooms. They were provided time to collaborate with their grade-level counterparts to determine which lessons they would implement in the upcoming months. Given that only two of the five schools that participated are 1:1 with iPads, we knew that teachers would need to be creative and utilize learning stations with the small number of devices they have in their classrooms. This was more of a barrier than I imagined it would be because many teachers felt overwhelmed in utilizing a new technology tool on top of small-group instruction. During the semester, Discovery Education provided three follow-up webinars teachers could attend to dive deeper into the lessons and some of the other features of *Math Techbook*. They also included Q & A at the end of each session. This professional development was a crucial part to this project.

Outcomes

Overall, teachers that wholeheartedly participated in the pilot saw some great successes with *Math Techbook*. Two teachers dropped out of the pilot

because they felt that they needed remediation material for lower level students that wasn't available in *Math Techbook* (their curricula starts at 6th grade, but doesn't dip into 5th); however, survey results revealed that most participants' students enjoyed utilizing *Techbook* for guided practice and to reinforce previously learned concepts. They also found that the questions were challenging. This, in part, was due to the fact that students weren't always simply answering multiple choice questions like they were used to. The prompts and open-ended questions in this program require students to think critically and provide explanations for their answers.

Teachers also reported that they liked using the lesson openers and videos as well as the application pieces because often times they struggle with answering the "when will I ever use this" question. Other components of *Math Techbook* were the writing prompts and builder tools that teachers were able to utilize with students as they created EOG reviews and completed activities that went along with the concepts within each unit.

Next Steps and Advice

Our district will likely be moving forward with *Math Techbook* in some capacity in the upcoming year, thanks to this pilot! I do have a few suggestions for teachers thinking about purchasing this for their school. First, if teachers are not in a 1:1 situation, provide specific training on small-group instruction and how to effectively manage learning stations in the classroom with *Math Techbook*. I did not realize initially that teachers needed additional training in this area. Secondly, have patience! I've really learned that any time you implement a new technology tool, you cannot expect to get buy-in over night. Teachers that have been teaching the same way for many years will likely have a hard time changing their classroom from a teacher-centered to a more student-centered environment. Utilizing a few lessons in *Math Techbook* and giving a few assignments to your classes during each unit is a great start.

With the pockets of success I did see during the pilot, I found that many teachers are really beginning to realize that the same tired lecturing and drill and kill are no longer effective strategies for making our students into better problem-solvers—Nor have these strategies proven to be effective in obtaining or maintaining the math stills that are required of our students to be globally competitive. It is likely that we will no longer be purchasing hard copies of textbooks in the future, so I'm really glad I was able to give my teachers the opportunity to try on a digital curriculum. Nothing will replace a good teacher in a classroom, but if you are looking for an excellent resource to supplement and enhance what you are doing, *Math Techbook* just may be for you.

References

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Discovery Education. (2015). *Introducing Discovery Education's Math Techbook: North Carolina focus* [online]. Retrieved from http://www.discoveryeducation.com/what-we-offer/techbook-digital-textbooks/math/north-carolina.cfm

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website, http://ncctm.org. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Sandra Childrey, schildrey@wcpss.net, with questions.

The State of Mathematics Standards in North Carolina

Reported by Jennifer Curtis, North Carolina Department of Public Instruction, Raleigh, NC

Essential standards, Standard Course of Study, Academic Standards Review Commission, ACRE, ABCs, Common Core… What are all these things and how do they all fit together. Or do they? How did we get here from there? Where did these things come from? Did the General Assembly repeal our current standards? Are we reviewing our standards? It is difficult to make sense of these questions in isolation, yet, when looking at the history of public education in North Carolina, the accomplishments and the changes, a picture emerges. Ripened through celebrations and challenges, there is a long-standing, rich history of work and a passion for teaching and learning in our state.

Recently, when asked to provide a history of education standards in North Carolina, it seemed like a straightforward request. Through careful research using documents like *The History of the North Carolina State Board of Education* (NC SBE, 2001) it quickly became obvious that our current Standard Course of Study (SCOS) was born out of many variations evolving from laws, acts, and policies that are directly tied to the culture and times. Understanding how we arrived at our current SCOS is better done after a brief historical review of K-12 public education.

The Evolution of Public K-12 Education in North Carolina

Beginning with the State Constitution of 1776, North Carolina has had an established record of public schools tied to funding from the legislature. Though public education developed slowly and followed a bumpy road from its inception to the post-civil war era, the General Assembly (GA) in North Carolina has long been involved in the states' development of public education. Archibald Murphey, who has been referred to as North Carolina's Father of Public Education, called upon the GA to establish a plan for a public school fund and an elected State Board to manage it. In 1825, the Literary Fund was created and by 1846, every county in North Carolina had at least one public school. 1852 saw the first Office of General Superintendent of Common Schools with an appointed leader, Calvin Wiley. Many iterations of funding took place before the Civil War caused an upheaval in public schools, resulting in a depleted funding structure, an abolished State Superintendent's Office, and little hope for public schools to remain open. Tax-supported public schools came into existence afterwards along with local boards of education.

In 1868, the first official State Board of Education (SBE), a foreshadowing of today's structure, was put in place under the State Constitution adopted at that time. The next year saw the SBE being assigned additional duties that included determining a course of study, selecting textbooks, locating resources, and establishing guidelines for appointing/qualifying teachers. A case could be made for this being the beginning of our first Standard Course of Study and a first textbook adoption process, as well as licensing and certification processes.

In 1927, the State Board of Equalization was established and charged with overseeing the collection of property taxes for the general government use including use of funds for local education. In addition, the Board was responsible for approving all local school system budgets. A few years later, during the Great Depression, education in North Carolina once again suffered a difficult setback. County governments struggled to fund schools and many grappled to stay open. At this time, the General Assembly in North Carolina took over funding of public schools via the School Machinery Act. Out with the Board of Equalization and in with the new State School Commission which, along with the State Board of Education, established the uniform salary schedule for all teachers and principals, a predecessor of a system still in place. The State School Commission was abolished in 1942 when the State Board of Education was given more obligations by means of a State constitutional amendment. This change resulted in the State Board of Education gaining control of school finances and the establishment of a Controller's office.

The 1950s and 1960s were wrought with upheaval and challenges as well as historic firsts. Our Community College system was established and funded. The Pearsall Plan enabled vouchers and allowed local control of schools during

this decade; however, vouchers were used to pay for private school enrollment if a child was placed in an integrated public school, and in 1964, the Federal Civil Rights Act corrected this problem by declaring the Pearsall Plan unconstitutional. The State Board of Education once again was granted authority over local school divisions.

During the 1970s, principals and other administrators began 12-month contracts, the country's first statewide full-day Kindergarten was established, and there was a teacher and an instructional assistant in every K-3 classroom in the state (another national first). A comprehensive testing program for high school students along with those in grades 1, 2, 3, 6, and 9 was used as an accountability measure for students. Then in 1980 the North Carolina School of Science and Math, the first of its kind in the country, opened for exceptionally gifted students.

This brief overview lays the groundwork for understanding the interwoven nature of standards, accountability, and governance of education, all of which play a part in our current Standard Course of Study for K-12 education in NC. Fast-moving reforms over the next fifty years forever raised our expectations of student and teacher performance.

The Evolution of Accountability

Reforms were gaining momentum in North Carolina and so there followed a rise in the expectations for graduates in North Carolina. The General Assembly called for an increase in requirements in 1983 and in 1985 and commanded the SBE to adopt a *Basic Education Program* (BEP). By 1989, there was a mandatory accreditation program that held schools accountable in exchange for more flexibility in local school decisions. North Carolina became an innovator in school accountability in the mid-1990s when the state's education and political leaders developed the *ABCs of Public Education*, a school accountability model. Senate Bill 16, ratified by the General Assembly in 1995, directed the SBE examine the structure of public schools, increase local flexibility, and improve student performance. According to SB 16, DPI was also to reduce, eliminate, or reorganize itself as an agency (NC General Assembly, n.d.).

The *ABCs of Public Education* included renewed emphasis on reading, writing, and mathematics, higher standards and EOG testing in several grades, elementary through high school. The 1998 edition of NC DPI's procedures manual for developing and implementing North Carolina's Standard Course of Study describes the general premise behind the NC SCOS itself. It is described as a legal document directly related to the Basic Education Plan (BEP) mandated by Senate Bill 155C-81. The definition of the SCOS is the program of course work that must be available to all public school students in the state. In 1998, the language of the procedures manual specifically mentioned that the revised SCOS has "moved from a detailed, prescriptive curriculum guide to a more flexible guide to instruction, emphasizing what students should know and be able to do as they progress through various levels of proficiency and ultimately exit from high school. The revised curriculum focuses on themes and concepts rather than isolated facts. It emphasizes thinking skills and problem solving more than the memorization and recall of information."

The word *curriculum* had been used to explain what students should be learning. The language of 115C-81 at that point in time stated that course requirements and descriptions shall be similar in format to previous materials contained in the SCOS and that standards for student performance and promotion based on the mastery competencies be contained in the document. Furthering this shift in use of the word *curriculum*, the standards were quickly becoming the new guideline. Senate Bill 352, in 1999, spoke to the NC Standards and Accountability Commission being replaced by a Committee on Standards and Accountability. Further amendments to the BEP ensued and next came the development of stronger content standards that were rigorous, specific, and measurable. These standards for courses in grades 9-12 were now to be aligned with *National Assessment of Educational Progress* (NAEP) after Senate Bill 272, *An Act to Enact the Excellent Schools Act*, was ratified.

The next decade brought more involvement from community and business partners. In September 2006, the State Board of Education adopted a mission statement that every public school student will graduate from high school globally competitive for work and postsecondary education, and prepared for life in the 21st century. Educators, parents and lawmakers had pressed for several years to make changes to the curriculum and accountability systems. The Blue Ribbon Commission on Testing and Accountability was convened to study the accountability issues and

make recommendations, some very specific to mathematics. In January 2008, the Blue Ribbon Commission presented a report to the State Board of Education that recommended improvements in the current system of testing and accountability and steps toward a next generation of standards, assessments, and accountability for North Carolina's public schools. Following extensive input from the Blue Ribbon Commission, the State Board of Education crafted the *Framework for Change*: a set of 27 recommendations to dramatically change the scope of the Standard Course of Study, assessments, and accountability (DPI Accountability and Curriculum Reform Effort, n.d.). One of the Commission recommendations was to suspend the regular cycle of reviewing standards and immediately enter a comprehensive review of content standards. Another called for developing End of Course Assessments for the integrated math courses that would include performance-based and authentic, real-world tasks. Development was set to begin immediately so that assessments would be ready to use in the 2010-11 school year.

The review of standards began in 2008 and continued through 2009. At the same time, directors at the National Governor's Association were contacting states and soliciting input on the CCSS-M draft. While the CCSS-M were not complete at this time, the review allowed North Carolina to compare the work that it had been doing, i.e., writing *Essential Standards in Mathematics*. The 2003 standards still in use were very different in format and expectations. These 2003 standards had evolved from what used to be called curriculum to "a set of mathematical competencies for each grade and high school course to ensure rigorous student academic performance standards that are uniform across the state. It is not meant to be an instructional manual. It does not provide strategies for teaching or lesson plans." There are still references in this version to "curriculum."

As the Essential Standards model was implemented, the focus shifted further away from *curriculum* to *standards*. This allowed for more local control over the "how" of teaching. Schools, teachers, and districts could determine the most appropriate methods, strategies and overall pedagogy according to the needs of students. Quickly disappearing were the one size fits all methods of memorizing algorithms and repeated practice for procedural fluency. During this review and revision cycle, the *Essential Standards in Mathematics* were going to be the skills, understandings, and learning experiences that all students must master and/or complete at each grade level or course in order to move to the next level of learning, lessening the chance that critical knowledge was overlooked. They would be a focal point for professional development, teacher education programs, and supporting documents.

Enter the CCSS-M review of drafts. There were many similarities between the drafts of the Essential Standards and the CCSS-M. The Common Core also created opportunities for collaboration across states, integration of standards K-12, and, possibly, equal comparisons across states whereby North Carolina would have an additional measure of success. After much review and consensus among teachers, higher education partners, education leaders, and DPI, the decision was made to present the CCSS-M to the State Board of Education for adoption.

A three-phase plan was adopted by the SBE in 2010: 2010-11 would be a Get Ready phase where districts and charter schools could attend Summer Institutes to prepare for the shift to CCSS in mathematics and ELA and Essential Standards in other content areas. During the 2011-12, additional professional development was planned and the "local option" was added to the high school choices. Districts could either implement Math I, II, and III all at once or phase in over three years, beginning in 2012. Districts could also choose to implement Algebra I Common Core and phase in the next two courses, Common Core Geometry and Common Core Algebra II annually. The third option was to implement Math I the first year and keep 2003 integrated standards for the next 2-3 courses, but phase in with the new cohort of students (DPI Common Core State and Essential Standards, n.d.).

This seemed a bit much to keep up with when considering the impact on students and teachers who might move between districts and for test development purposes. Therefore, the High School Math Task Force was convened late in 2012 and charged with determining whether or not high schools should all be on the same pathway of course offerings. If so, what should it be? After much deliberation and feedback from teachers and leaders across the state, DPI leadership recommended that the SBE adopt the integrated path of high school math courses (Math I, II, and Math III). These courses were based on the Common Core standards integrated across five conceptual categories

and represent our current high school portion of the SCOS in Mathematics. The entering freshman cohort of students in 2012-13 would be the first class of students who had potentially completed Math I, II and III upon graduation.

Periodic Review of Standards

The Department of Public Instruction is also charged with reviewing and determining if standards should be revised on a five-year cycle. Therefore, during the 2014-2015 school year DPI began collecting data to inform the review/revision process by opening an educator survey and a parent and community survey. Additionally, focus groups were conducted in all eight regions of North Carolina along with math leaders at the 2015 Spring NCCTM leadership conference. The focus groups consisted of teachers from every district in the state collaborating together across grade bands to give feedback on the standards including suggestions for revisions.

Separately, in 2014, Senate Bill 812, Session Law 2014-78, called for a convening of an Academic Standards Review Commission under Section 2.(c). The Bill reads, in part:

> The Commission shall: Conduct a comprehensive review of all English Language Arts and Mathematics standards that were adopted by the State Board of Education under G.S. 115C-12(9c) and propose modifications to ensure that those standards meet all of the following criteria: a. Increase students' level of academic achievement; b. Meet and reflect North Carolina's priorities; c. Are age-level and developmentally appropriate; d. Are understandable to parents and teachers; e. Are among the highest standards in the nation.

The results of the DPI educator survey were shared with the ASRC to help inform its external review of the mathematics and English Language Arts standards. At its August 2015 meeting, the ASRC shared interim committee reports, and in December 2015 the Commission will finalize its recommendations to the State Board of Education. The Department of Public Instruction is awaiting the final results and recommendations from the ASRC as it constitutes an important stakeholder group. The review and revision process is a careful, intentional method outlined by SBE policy GCS-F-012 titled Policy delineating the Standard Course of Study Curriculum Development Process. The policy states the DPI "shall convene a review committee to determine if revisions are needed in a SCOS area. By using data, research, and surveys, the committee recommends whether revision should take place. If the committee recommends substantive revision, the State Board of Education shall review the recommendations and implications for textbook selection and adoption and any necessary revisions [of] state assessments." The policy also states "…standards and objectives must be developed in consultation with teachers, administrators, parents, students, institutions of higher education, and business/industry."

The Department of Public Instruction remains committed to gathering input from all stakeholder groups and will continue to do so. The current pause in the review cycle is to allow the ASRC to finish its work and share its recommendations with the SBE in December. At this point it is difficult to point to a specific date of when the revision work will be completed. Until then, the current NC SCOS including the Mathematics standards, will remain in place.

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2015 NCCTM Logo Contest Winners

Reported by Courtney Howlett, Mount Airy High School, Mount Airy, NC

The Mathematics Logo Contest is held each spring and submissions are accepted in K-2, 3-5, 6-8, and 9-12 categories. The NCCTM Board selects the winning logo at its Spring meeting.

The 2015 winning logo, pictured, will be available on shirts at the NCCTM State meeting in October.

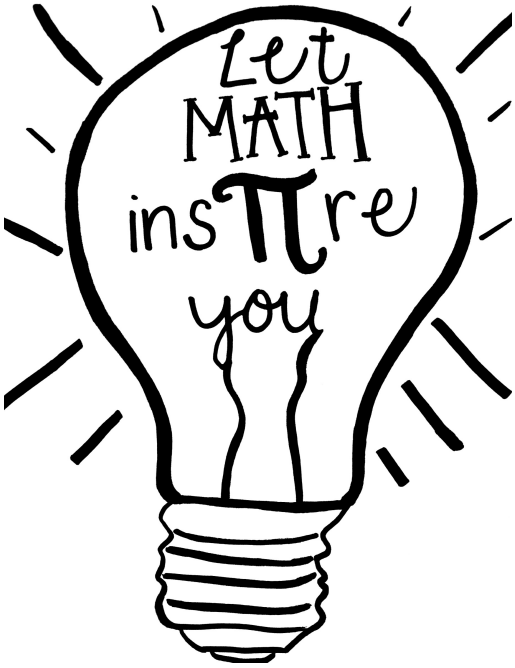
State Winner
Emma Kate Clark
Eastern Region 9-12 Winner

Finalists

Eastern Region
Sky Mendoza: 3-5 Winner
Claire Storum: 6-8 Winner
Emma Kate Clark: 9-12 Winner

Central Region
Diana TeJeda Silva: K-2 Winner
Haley Hedrick: 3-5 Winner
Killian Carpenter: 6-8 Winner
Bailey Jackson: 9-12 Winner (State Alternate)

Western Region
Wyatt Warren: K-2 Winner
Harrison Thomas: 3-5 Winner
Emma Sheets: 6-8 Winner
Mary Grace Oglesby: 9-12 Winner



The following students were selected as alternates:
Eastern Region
Jacob Beamon: 3-5 Alternate
Brandon Burgiss: 6-8 Alternate
Hunter Hairston: 9-12 Alternate

Central Region
Skylee Smith: K-2 Alternate
Anna Shedden: 3-5 Alternate
Jaclyn Simmons: 6-8 Alternate
Dalila Nicholson: 9-12 Alternate

Western Region
Genevieve Shutt: 3-5 Alternate
Anna Ritter: 6-8 Alternate
Faith Morgan: 9-12 Alternate

2015 State Math Fair Winners

Reported by Betty Long, Appalachian State University, Boone, NC

NCCTM sponsors three regional Math Fairs each spring, and the best projects presented at these regional Fairs qualify for the State Math Fair. This year's State Fair was held at the North Carolina School of Science and Mathematics on 8 May 2015. The following students were selected for top honors in each division.

Primary Division, Grades K-2

1st Place: "Westward Ho!"; Caden Bringewatt; Bringewatt Learning Community, Davidson, NC
2nd Place: "Mom! No Bath for Me Today"; Ashwin Varadarajan; Bellamy Elementary School, Wilmington, NC
3rd Place: "Causation, Correlation and Cold: Does Winter Weather Cause School Absences Due to Cold?"; Sarrah Kitchell; Valle Crucis Elementary School, Sugar Grove, NC

Honorable Mentions: "Eating is Fun"; Ainsley Kiser; Coddle Creek Elementary School, Mooresville, NC
"Is 250 Yards Enough?"; Cody Surface; E. K. Powe Elementary School, Durham, NC

Elementary Division, Grades 3-5

1st Place: "Urban and Rural Voting"; Reif Snyder; Woodland Heights Elementary School, Mooresville, NC
2nd Place: "Pascal's Triangle"; Sreenidhi Elayaperumal; Cox Mill Elementary School, Concord, NC
3rd Place: "Will I Have to Swim to College?"; Varun Varadarajan; Bellamy Elementary School, Wilmington, NC

Honorable Mentions: "Bat Moves"; Jeannette Graham and Kevin Laur; Balfour Elementary School, Asheboro, NC
"Superbowl Correlations"; Isaac Barsoum; Davidson Elementary School, Davidson, NC
"Feeding Our Planet"; Sam Schneider; Dr. John Codington Elementary School, Wilmington, NC

Middle School Division, Grades 6-8

1st Place: "Pencils, Pens and Profit: Using Linear Programming to Maximize Profit at Our School Store"; Valerie Kitchell; Valle Crucis School, Sugar Grove, NC
2nd Place: "Solving for the Belt Length of a Fixed Pulley System"; Ethan Brewer; E. B. Aycock Middle School, Greenville, NC
3rd Place: "Comparing the Numbers of Blades on a Wind Turbine to Energy Production"; Keenan Wasklewicz, E. B. Aycock Middle School, Greenville, NC

Honorable Mention: "Fact or Friction?"; Toby Bryson; Hope Middle School, Greenville, NC

High School Division, Grades 9-12

1st Place: "What's Wrong With Wall Street?"; Sumani Nunna; Cox Mill High School, Concord, NC
2nd Place: "Calculating Crimes"; Lindy Bustabad; Clinton High School, Clinton, NC

Problems to Ponder

Holly Hirst, Appalachian State University, Boone, NC

Fall 2015 Problems

Grades K–2: Ms. Davenport has 24 students in her class. If she arranges 24 desks into 6 rows, how many students will be in each row of her classroom?

Grades 3–5: Tanisha wants to purchase a new iPad for $499. She will use a $50 gift certificate, and also $100 in money she had saved up. For her birthday, her dad gave her one-half of the remaining money she would need to get the iPad. How much more money does Tanisha need to save to make her purchase?

Grades 6–8: A picture that is 18 inches tall is to be hung on an 8 foot tall wall in Tanya's home. She wants the center of the picture to be 5 feet 2 inches off the floor. She needs to put the fastener on the wall at the top of the picture. How far from the ceiling should the top of the picture be?

Directions for submitting solutions:

1. Students: NEATLY print the following at the top of each solution page:
   - Your first name (we will not publish last names)
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page. ***Students who submit correct solutions will be recognized by their first names only in the next issue of The Centroid.*** We will also publish one or two especially creative or well-written solutions from those submitted. If you would rather not have your name published, please so indicate on your submission.

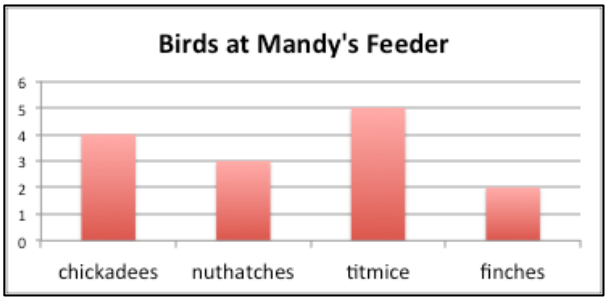
Proper acknowledgement is contingent on legible information and solutions. Send solutions by 20 December 2015 to:

Problems to Ponder, c/o Dr. Holly Hirst
Mathematical Sciences
BOX 32069 Appalachian State University
Boone, NC 28608

A submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

Spring 2015 Problem Solutions

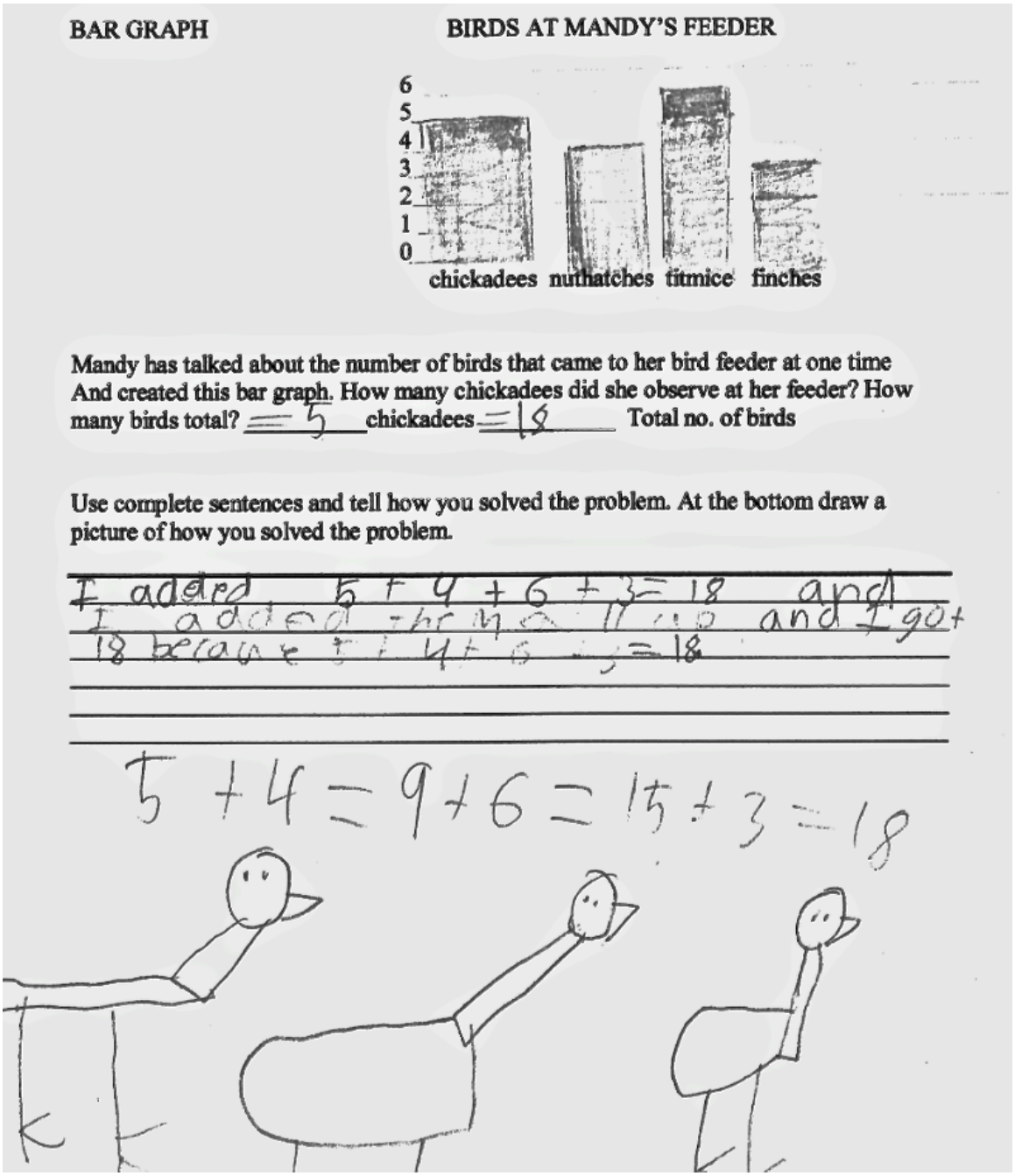
Grades K–2: Mandy has tallied the number of birds that come to her bird feeder at one time and created this bar graph. How many chickadees did she observe at her feeder? How many birds total?



The correct solution to this problem is 4 chickadees (the height of the first bar) and 14 birds total (adding all four heights).

Correct solutions were submitted by 8 students from The Ravencroft School (Teachers: Mrs. Byrne and Mrs. Bednarz). Note: The bar graph the Ravencroft students worked with was a little different from the one in *The*

Centroid, as can be seen from the image below. For the bar graph given to these students, the correct answer was 5 chickadees and 18 birds total.



Grades 3–5: Keshia is shopping for new clothes and finds some clothes she likes on sale. The sale is advertised as 30% off the retail price, and she has found the following garments with retail prices given. For which combinations of items does she have enough money if she has a total of $75.00 to spend?

$45 pair of jeans; $30 red blouse; $28 pink sweater; $39 sneakers; $25 sandals; $22 skirt.

Note: This problem stumped everyone! There were two common mistakes: (1) Some students stopped when they found one combination of two items that came to less than $75 rather than checking for several different combinations with more than two items; (2) Some students did not first take the 30% reduction in the price before looking for combinations.

The correct solution: First find the sale prices by subtracting 30% of the price from each item:

	Jeans	*blouse*	*sweater*	*sneakers*	*sandals*	*skirt*
price	*$45*	*$30*	*$28*	*$39*	*$25*	*$22*
sale price	*45-0.3(45) = $31.50*	*30-0.3(30) = $21*	*28-0.3(28) = $19.60*	*39-0.3(39) = $27.30*	*25-0.3(25) = $17.50*	*22-0.3(22) = $15.40*

Then look for combinations that can add up to less than $75. There are many pairs of items and sets of three items that sum to less than $75. There is one set of four items that totals less than $75: blouse, sweater, sandals, and skirt: 21 + 19.60 + 17.50 + 15.40 = $73.50.

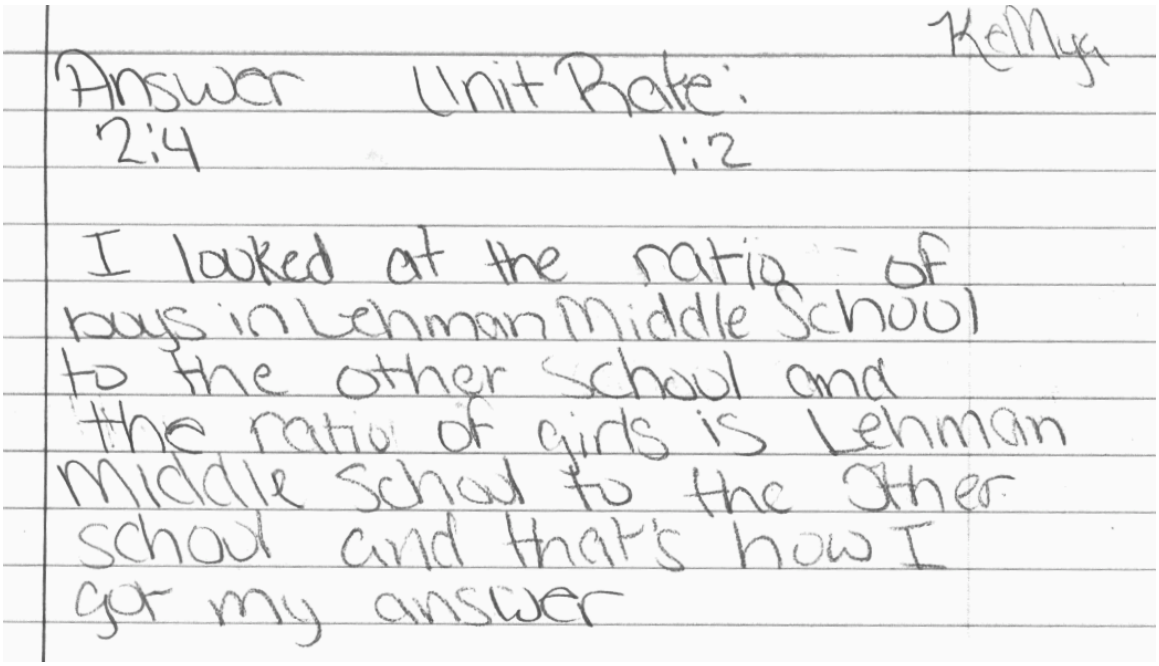
Students who found at least one set of items that totaled less than $75:

- *Ahoskie Elementary School:* Mrs. Dail: Jairus; Ms. Ellis: Amanda, Ashanti, Marion, Rodasia, Toniyah; Mrs. Hines: Robert, Dawn, Jada, Karmyla, Seth, Shyanne, Terkira; Ms. Sitnic: Carlos, Julian, Makayla, Sha'Brea, Ty'Quan; Mrs. Watford: Abigail, A'Nyla, Christopher, Damaria, D'Ayshia, Jayla, Ja'Kiera, JerKayla, Jua'Sha, Khadijah, Kimora, Naomi, Quadasia, Shanell

Grades 6–8: Two middle schools have the same number of students. The ratio of boys in Lehman Middle School to boys in Dallas Middle School is 2:1 and the ratio of girls in Lehman Middle School to girls in Dallas Middle School is 4:5. Find the ratio of boys in Lehman Middle School to girls in Lehman Middle School.

Correct solutions were submitted by:

- *Ahoskie Elementary School:* Mrs. Bowser: Bryan, Jada, KeMya, Mya, Tiffany, Tye; Mr. Hobbs: Demetrius



Note: None of the students who solved this correctly were clear on their reasoning in their description. Can you carefully explain why 1:2 is correct? (It is harder than it seems!)

Trust Fund Scholarships

$600 scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants an scholarships" link. The nomination form can be obtained from the "awards" area of the NCCTM Website (http://ncctm.org). More information can be obtained from: Janice Richardson, richards@elon.edu.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, http://ncctm.org. More information can be obtained from: Todd Abel, AbelTA@appstate.edu.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Lee V. Stiff, lee_stiff@ncsu.edu.

Donating to the NCCTM Trust Fund

If you wish to memorialize or honor someone important to you through a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

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contact information can be found at ncctm.org

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