## The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

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The Centroid
s the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. The Centroid is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.
Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- A Teacher's Story,
- History Corner,
- Teaching with Technology,
- It's Elementary!
- Math in the Middle, and
- Algebra for Everyone.


## Guidelines for Authors

Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in Microsoft Word or rich text file format. To allow for blind review, the author's name and contact information should appear only on a separate title page.

## Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the Publication Manual of the American Psychological Association.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

Books and reports:
Bruner, J. S. (1977). The process of education (2nd ed.). Cambridge, MA: Harvard University Press.
National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.
Journal articles:
Perry, B. K. (2000). Patterns for giving change and using mental mathematics. Teaching Children Mathematics, 7, 196-199.
Chapters or sections of books:
Ron, P. (1998). My family taught me this way. In L. J. Morrow \& M. J. Kenney (Eds.), The teaching and learning of algorithms in school mathematics: 1998 yearbook (pp. 115-119). Reston, VA: National Council of Teachers of Mathematics.
Websites:
North Carolina Department of Public Instruction. (1999). North Carolina standard course of study: Mathematics, grade 3. Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

# The Centroid 

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NCCTM's Annual State Math Conference

# NCCTM Leadership Seminar 

October 26, 2016
Koury Convention Center in Greensboro, NC
Leaping Forward: Teaching \& Learning, Equity, Curriculum, and Assessment

## Conference Keynote Speakers

Dan Meyer, Zachary Champagne, Robert Berry, Jennifer Wilson, Susan Empson, and Ellen Whitesides

## Leadership Seminar Presenters

Dan Meyer, Zachary Champagne, Jere Confrey, and the mathematics team from the Department of Public Instruction

## Preregistration Information

Conference: $\$ 65$ for NCCTM members; $\$ 105$ for non-members Leadership Seminar: \$65 for NCCTM members; $\$ 75$ for nonmembers
Hotel Information
Rooms can be booked at the Sheraton Greensboro - Conference Hotel at a discounted rate using the online form.

Visit http://ncctm.org to preregister and for more information!

# President's Message 

State President Ron Preston<br>East Carolina University, Greenville, NC<br>prestonr@ecu.edu

What good news do you have to share? Do you have students who have made impressive gains in their mathematical knowledge? Are there teachers in your school who have developed some engaging and powerful lesson plans? Have teachers from your district participated in (or are preparing to participate in) revision of the North Carolina Standard Course of Study in Mathematics? Have students from your school excelled in mathematics competitions? Have teachers and administrators in your district participated in rewarding professional development? Has a professional learning community in your school developed useful assessments? Has your school hosted a mathematics night that helped parents better understand the mathematics curriculum? Do you have mathematics educators sharing their expertise with others, say, at the NCCTM Conference in October?

Now why begin my president's message with a series of questions about positive mathematics happenings in your classrooms, schools, and districts? I believe that we need to do better job of telling positive stories about the teaching and learning of mathematics in our North Carolina schools. Why are these stories so important? Clearly, we live in a world that focuses on negative stories and those include stories about the state of mathematics education in our schools. Witness some of the headlines from a recent web search about the Common Core:

- Common Core Testing Makes Children Vomit, Wet Their Pants: N.Y. Principals
- Even a Dad with an Electronics Engineering Degree Can't Figure out Common Core Math!
- Top Ten Things Parents Hate About Common Core
- The Ten Dumbest Common Core Problems

Now we also live in a world where a few complaining parents or grandparents can get the ear of a central office, legislator, or segment of the media. We cannot do anything about those in the media who seemingly prefer to pay attention to the negative over the positive, nor can we do anything about the appetite of media consumers who dwell on the negative. But we can do more to advance positive messages. Perhaps the best defense against the next attack on mathematics education in NC is a good offense. What can we do to highlight good teaching in our classrooms? Can we invite the media to take a look? Can we get a legislator to visit a school? Can we write a legislator with a positive anecdote about student performance? The best offense may be telling positive stories about mathematics teaching and learning before a crisis emerges.

Another way to celebrate success is to nominate someone for an award. It might be a nomination of someone for the Rankin Award or someone or some group for the Innovator Award. It might be recognizing a teacher of the year in your district. Writing strong nomination materials for someone who exhibits quality teaching requires work on our part, but we need to recognize such excellence.

Now I have good news for you. NCCTM is sponsoring two outstanding professional development events this fall. The Fall Leadership Seminar is Wednesday, 26 October at the Koury Convention Center in Greensboro. I invite you to participate in this event that features Dan Meyer, Zachary Champagne, Jere Confrey, and the mathematics team from the Department of Public Instruction.

Our 46th Annual Fall Conference is Thursday and Friday, 27-28 October, also at the Koury Convention Center. Our program chairs (Jenna Regan, Kim Solomon, Kwaku Adu-Gyamfi) and conference chairs (Drew Polly and Stefanie Buckner) have been working hard to prepare an excellent conference with the theme: "Leaping Forward: Teaching \& Learning, Equity, Curriculum, and Assessment." Highlighting the event are over 300 workshops and sessions, mostly from NC teachers who know what works for NC students. There will also be keynote addresses from Dan Meyer, Zachary Champagne, Robert Berry, Jennifer Wilson, Susan Empson, and Ellen Whitesides. Featured speakers
include Maria Blanton, Mike Bossé, Jere Confrey, and Jennifer Curtis. There will be a room full of exhibitors, an awards celebration, chances to network, and much more.

I look forward to seeing you in Greensboro at the end of October! But before then, I want you to share some good mathematics news with those in your sphere of influence.

# North Carolina Recipients Presidential Awards for Excellence in Mathematics and Science Teaching 

Reported by Kitty Rutherford, NC Department of Public Instruction
Two North Carolina teachers were recently named Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) recipients - the highest honor bestowed by the United States government specifically for K-12 mathematics and science teaching.


The 2014 Elementary Mathematics recipient is Kayonna Pitchford (pictured on the left), previously with Stoney Point Elementary, Cumberland County Schools, and now with The University of North Carolina at Pembroke.

The 2015 Secondary Mathematics recipient is: Lauren Baucom (pictured on the right), Forest Hills High School, Union County Schools.


In making the announcement, President Obama said, "The recipients of this award are integral to ensuring our students are equipped with critical thinking and problem-solving skills that are vital to our Nation's success. As the United States continues to lead the way in the innovation that is shaping our future, these excellent teachers are preparing students from all corners of the country with the science, technology, engineering, and mathematics skills that help keep us on the cutting-edge."

State Superintendent June Atkinson congratulated the recipients saying, "North Carolina is fortunate to have such exceptional mathematics and science teachers in our public school classrooms. Students who graduate high school with a strong math and science foundation will be well-positioned for the high-demand careers leading our $21^{\text {st }}$ century economy."

The PAEMST program, which is administered by the National Science Foundation (NSF) on behalf of the White House Office of Science and Technology Policy, recognizes outstanding K-12 science and mathematics teachers from across the country for their contributions to the teaching and learning of mathematics and science.

A panel of distinguished scientists, mathematicians, and educators select the winners following an initial selection process at the state level. Nomination years alternate between teachers in the kindergarten through 6th grade level, and those teaching 7th through 12th grades. These recipients represent two nomination years, one of teachers in kindergarten through 6th grade classrooms, and the other in 7th through 12th grade classrooms.

Each recipient will receive a citation signed by President Obama and a $\$ 10,000$ award from NSF to be used at their discretion.

# 2016 NCCTM Logo Contest Winners 

Reported by Courtney Howlett, Mount Airy High School, Mount Airy, NC

The Mathematics Logo Contest is held each spring and submissions are accepted in K-2, 3-5, 6-8, and 9-12 categories. The NCCTM Board selects the winning logo at its Spring meeting. The 2016 winning logo, pictured, will be available on shirts at the NCCTM State meeting in October.

## State Winner and Eastern 9-12 Winner <br> Elise Herring, Middle Creek High <br> Teacher: Katie Taylor

## Other Finalists and Alternates:

- Eastern 3-5 Winner: Kristen Williams, Contentnea Savannah, Teacher: Amy White
- Eastern 6-8 Winner: Delaney Guidi, Magellan Charter, Teacher: Wanda Sutton
- Central K-2 Winner: Day Richardson, Seagrove Elementary, Teacher: Karlyn Sugg
- Central 3-5 Winner: Mackenzie Martin, Seagrove Elementary, Teacher: Mary Smith
- Central 6-8 Winner: Kylie Cristobal, Luther Nick Jeralds Middle, Teacher: Phyllis Cannon
- Central 9-12 Winner: Hannah Eldara, Asheboro High, Teacher: Tammy Applegate
- Western 3-5 Winner: Jack Stewart, North Brook Elementary, Teacher: Denise Smith
- Western 6-8 Winner: William Hurt, C.D. Owen Middle, Teacher: Patti Lloyd

- Western 9-12 Winner: Sara Flynn, Christ the King Catholic High, Teacher: Kimberly Antolini
- Eastern 3-5 Alternate, NaRyah Gore, Richland Elementary, Teacher: Tracy McIntyre
- Eastern 6-8 Alternate, Ashtyn Cook, The Magellan Charter, Teacher: Wanda Sutton
- Eastern 9-12 Alternate, Hailey Dunlow, John A. Holmes High, Teacher: Ray Mock
- Central K-2 Alternate, Olivia Bowles, Seagrove Elementary, Teacher: Donna Dalke
- Central 3-5 Alternate, Samantha Heglar, Trindale Elementary, Christy Hutchins
- Central 6-8 Alternate, Emma Aquino, Mac Williams Middle, Yvonne Wheeler
- Central 9-12 Alternate, Maggie Redding, Asheboro High, Tammy Applegate
- Western 3-5 Alternate, Laura Willis, Union Elementary, Denise Smith
- Western 6-8 Alternate, Ruby Navarrete, C.D. Owen Middle, Patti Lloyd
- Western 9-12 Alternate, Ellie Scanlon, Charlotte Catholic High, Carol Huss


# Using Financial Literacy to Teach Math Concepts 

Amber Mellon, Appalachian State University, Boone, NC

So many financial dreams are thwarted by the failure to act upon good intentions. -- Suze Orman

A January 17, 2013 Time Magazine article puts some numbers on the face of the financial literacy problem among young adults in the United States. That article cites the following statistics: Rent.com surveyed 1,000 renters and found that $75 \%$ of the $18-24$ year olds who responded overspend their paycheck every month; more than $20 \%$ said that they overspend their paycheck by $\$ 100$. An Ohio State University study (Grabmeier, 2013) states that young adults have more credit card debt and pay it off more slowly than any other age group.

Over the years of tutoring and teaching mathematics, I have often been asked by my students how they will use the math they learned in my class in their real lives. Financial Literacy is an excellent application of mathematics that concerns everyone, and given the statistics from the Time article, young adults appear to be especially in need. Financial literacy can be defined as having accurate information to make financial decisions in order to achieve financial goals. I created worksheets on 16 topics that affect students: student loans, investments, planning for retirement, comparing prices, taxes, budgets, personal credit, renting versus owning a home, credit cards, balancing a checkbook, emergency funds, owning versus leasing a car, simple savings, mortgages, life insurance, and interest rates. The worksheets are designed to be an introduction to the subject in simple, everyday language. Included in the worksheet are worked out examples of math problems within the subject area, as well as several real life examples for the students to consider.

During the summer and fall of 2013 , four undergraduate mathematics classes at Appalachian State University field-tested the worksheets to determine the effectiveness of the worksheets on increasing student understanding of finance concepts and student performance on associated math problems. At the beginning and end of the semester, each student in these classes was given a fifteen question, multiple choice test created from exercise problems in the worksheets. The main lesson learned through this testing was: The classes that actively discussed the worksheets during class time showed improvements in the post-test scores, while having the students read through and complete them for homework did not yield improvements.

I have used the worksheets many times now with college freshmen. Compared to the standard materials from the course texts, these are the lessons that my students engage in the most. They ask more questions, take more careful notes, and do better on the tests. My students have told me that they are excited about learning about financial literacy as they can see the impact in their lives. One student told me she was using her Christmas money to open her mortgage savings account. I see them learning how to use formulas, fractions, percentages, and algebra.

The woksheets use skills from arithmetic and algebra that would make them appropriate for use in middle and high school math classes as well. Example worksheets on Comparing Prices and on Mortgages follow this article. The Comparing Prices worksheet gives students practice with number sense, units, and arithmetic. The more advanced Mortgage worksheet gives students practice with the same concepts and adds work with a more complicated algebraic formula. For more information on the worksheets, see the text Math For Life: A Mathematical Introduction to Personal Finance (Mellon, 2015).

## References

Grabmeier, J. (2013, January 14). Credit card debt: Younger people borrow more heavily and repay more slowly, study finds. In Research and innovation communications. Retrieved from Research News at The Ohio State University, http://researchnews.osu.edu/archive/creditrepayment.htm
Mellon, A. (2015). Math for life: A mathematical introduction to personal finance. Dubuque, lowa: Kendall Hunt. White, M. C. (2013, January 17). Today's young adults will never pay off their credit card debts. Retrieved from TIME.com, http://business.time.com/2013/01/17/todays-young-adults-will-never-pay-off-their-credit-carddebts/

## Comparing Prices Worksheet

Most of our financial decisions are daily, little decisions that can make big differences. Comparing prices is an easy way to help live within your budget. Prices vary from store to store and from brand to brand. If you take the time to notice the differences, the differences can be significant. We'll talk about comparative shopping, buying in large quantities, using coupons, and using unit prices.

Comparative shopping is pretty easy. If you are looking for a specific item, research the item online and notice for what price different stores are selling the item. Grocery stores release ads every week, so take time to see which store offers the lowest price for what you need. Also, for many items there are different brands of the same item. For example, there are several companies that sell canned vegetables. Often, the generic is cheaper and tastes the same. Try a can of store brand corn; you may not notice a difference in taste, but you will see it in your budget. For example, when looking for canned corn, the Food Lion brand may be $89 \phi$ per can, where a can of Del Monte corn may be $\$ 1.15$. That is a $26 \phi$ difference per can. If you buy 2 cans of corn per week, over the course of a year, that would be a $(26 \phi) \times 52$ weeks $=\$ 27.04$ difference in corn alone.

Buying in large quantities can also help you out. When you walk down the aisles of a grocery store, it is hard not to notice multipacks of many items- soups, yogurt, and corn bread among others. Have you ever noticed the price difference between a single item and the multipack? It's simple-divide the total price of the multipack by the number of items in the multipack. Often the mulitpack is cheaper; however, if you are not going to be using all of the items in the multipack, you may be wasting money, so keep that in mind. So, if you eat yogurt, you might want to look at the multipack. If individual cups are $80 \phi$ each, but an 8 pack is $\$ 4.00$, each yogurt cup in the 8 pack would be $50 \phi$. So, if you eat one yogurt a day, the difference would be $30 \phi \times 365$ days $=\$ 109.50$ savings just on yogurt.

Using coupons can also help you save money. Often you can find coupons in your local paper. They are available everywhere- from manufacturers' websites, in the stores, and on various coupon sites online. There are also several smart phone apps that contain coupons. Before you make a purchase, check out retailmenot.com for coupon codes for thousands of online retail stores. In addition, many grocery stores double and triple manufacturers' coupons. For example, suppose that your favorite cereal is normally priced at $\$ 4.00$ per box, but it is on sale this week for $\$ 2.50$ and you have a $50 \phi$ coupon that your store will double. So, by buying the cereal on sale with a coupon, you will save ( $\$ 4.00-\$ 2.50)+(2 x 50 \phi)$, or $\$ 2.50$. One important note: Only use coupons on an item that you would have bought anyway or you could be wasting money.

Noticing unit prices can help you stay within your budget. A gallon of milk is usually cheaper than a half-gallon. Some larger food packages can be broken down into single meal portions, stored in freezer bags, and frozen. A 10 pack of paper towels only takes up a little more room in the pantry and will save you time in not having to go to the grocery store when you run out of paper towels. For example, a half-gallon of milk is $\$ 2.65$, but a gallon of milk is $\$ 3.59$. Therefore, if you bought milk by the gallon instead of the half gallon, it would save you ( $2 \times \$ 2.65$ ) - $\$ 3.59$, or $\$ 1.79$ per gallon. If your family drinks 3 gallons a week, over the course of the year you would save 3 gallons $\times \$ 1.79$ $x 52$ weeks, or $\$ 279.24$ per year in the cost of milk. When you buy in bulk, be sure to store food in ways it will not spoil, such as freezing extra meat.

It may take some extra time when you are shopping to really compare prices. But, it will be worth it when you add up all the savings.

Let's practice! Explain your answers.

1. Which is a better price: one kiwifruit for $55 \phi$ or 3 -pack of kiwifruit for $\$ 1.55$ ?
2. Which is a better deal: 20-pack of gum for $75 \phi$ or a package of 10 -packs of gum each containing 10 sticks for \$5?
3. Which is a better buy: a box of corn bread for $55 \phi$ or a pack of 6 boxes for $\$ 3$ ?
4. You noticed that the store brand 12 -pack of cans of your favorite drink is on sale at your local grocery store for $\$ 2$. You try it and like it. So, you go back and buy another three 12-packs. How much money will you save over the course of a month if you drink one can per day at work instead of spending $\$ 1.25$ at the vending machine, assuming there are 20 working days in a month? How much would you save in a year?
5. You need yogurt and you notice that 6 -ounce single serving of yogurt is $60 \phi$ where a 24 -ounce container of yogurt is $\$ 2$. You have containers at home that you could divide the larger one into. Which container should you get? Why?
6. Which is a better buy: a 5 -pound bag of apples for $\$ 5.95$ or loose apples on sale for $\$ 1.15$ a pound?
7. You need Italian sausage for spaghetti tonight. You notice two different brands in the grocery store: a 12ounce package for $\$ 2.98$ that you have 2 identical coupons for $50 \phi \mathrm{o}_{\text {_ }}$ or a 24 -ounce package for $\$ 3.98$. Which one is a better deal? Note that this grocery store doubles coupons up to $50 \phi$.
8. You need a dessert for this meal and decide on ice cream. Which one is the least expensive one to buy: Package $A$ is 1 quart for $\$ 1.98$ or package $B$ is $\$ 5.22$ for a gallon?
9. You need Italian seasoning for your spaghetti. You notice that your store offers bulk bin spices. From the bulk bin, Italian seasoning is $20 \phi$ an ounce. The store also sells 12 -ounce containers for $\$ 3.98$. Which one should you get?
10. You need a cleaner to clean up after dinner. You noticed that there is a store brand cleaner for $\$ 2.64$ for 22 ounces, and the brand name cleaner is 32 ounces for $\$ 3.84$. Which one are you going to buy and why?

## Mortgage Worksheet

A mortgage is a loan from a bank or credit union to finance the purchase of a home. For average Americans, a mortgage is the largest loan they will have in their lives, their largest bill each month, and maybe their largest investment. Applying for a mortgage can be a complicated and lengthy process. There are lots of decisions to be made. Let's go through some of them.

The length of the mortgage. Most mortgages in the United States are 30 years or 360 payments. It is also possible to get a 15-, 20-, or 25 -year mortgage. This is one of the largest determining factors of your mortgage payment. The length of the mortgage also helps to determine to your interest rate as the shorter the term of your mortgage, the lower the interest rate typically is.

Interest rates. When shopping for a mortgage, you will notice the terms "fixed rate" loans versus "adjustable rate" loans. A fixed rate loan has an interest rate that is the same for the entire length of the mortgage. If the average
mortgage rates fall during the course of the mortgage, the mortgage must be refinanced if the borrower wishes to take advantage of the lower rate, which costs money and time. An adjustable rate loan has an interest rate that will change during the length of the loan based on market conditions. Based on the conditions specified in the mortgage, your interest rate will change, either up or down, and so may your payment change. There is typically a cap on how much the rate can change and how often it can change.

How is a mortgage interest rate determined? Your credit score plays a large role in a mortgage interest rate. Market conditions determine an interest rate. It is also possible to lower a mortgage interest rate by what is known as "buying discount points." A discount point is $1 \%$ of a mortgage loan. By prepaying $1 \%$ of a mortgage, you can lower the interest rate of a mortgage by $1 / 4$ to $1 / 2$ of $1 \%$. The person paying the points receives the benefit of lowering the interest rate while the financial institution benefits by having this prepaid interest.

Fees. When your mortgage is created, there are several fees that will be part of the loan. This includes an appraisal of the house to make sure the house is worth what you offered for it, a title search fee, and attorney's fees. Before you sign paperwork for your loan, you will receive a Good Faith Estimate listing all of these fees. See www.hud.gov/offices/hsg/rmra/res/gfestimate.pdf for more information.

Insurance and taxes. As long as a bank holds your mortgage, the lender will make sure that your property taxes are paid and you hold homeowner insurance on your house. To do this, many mortgage holders will collect $1 / 12$ of the total amount of your estimated taxes and property insurance each month in an account called escrow. For example, if your homeowners insurance is $\$ 800$ a year and your property taxes are $\$ 1,000$ a year, the lender would add (1/12) $(\$ 1,000+\$ 800)=\$ 150$ to your mortgage payment. You may select your homeowner insurance company, but you must inform your lender of any changes. The lender will then pay your taxes and homeowner insurance. Your mortgage payment may change from year to year if there are changes in these two amounts.

Down payments. Your lender will require you to make a down payment on your mortgage. Ideally, it should be at least $20 \%$ of the appraised value of a house as the down payment on a mortgage in either equity on the house or as a cash down payment. Therefore, the loan to value of the house ratio (LTV) should be less than $80 \%$ when the mortgage is originated or started. If you are not ready to make that down payment, your bank may charge you private mortgage insurance (PMI). PMI protects the lender if you default on the loan and will be added to your monthly mortgage payment. Your PMI payment each monthly would be one-twelfth of one-half of $1 \%$ of your mortgage balance at the time of the loan. For example, if you put $10 \%$ down on a $\$ 100,000$ house you would like to buy, your monthly PMI payment would be $112(0.005 \times 90,000)=\$ 37.50$. By law, your PMI will automatically be canceled once you mortgage balance is under $78 \%$ of the value of the home, but you can ask to have it cancelled once you are under $80 \%$ of the value of the home by contacting the lender.

The monthly payment. The actual mortgage payment follows a loan payment formula:

$$
R=(P(r / n)) /\left(1-(1+r / n)^{\wedge}(-n t)\right)
$$

where $R$ is your payment, $P$ is the total amount borrowed, $r$ is the interest rate, $t$ is the number of years of the loan is being repaid, and $n$ is the number of times the loan is compounded a year. We will assume for these exercises that you make monthly payments and the interest is compounded monthly. So, if you borrowed $\$ 150,000$ for your $\$ 200,000$ house at a $3 \%$ rate for 30 -year fixed mortgage, your mortgage payment would be:

$$
R=(\$ 150,000(0.03 / 12)) /\left(1-(1+0.03 / 12)^{\wedge}(-360)\right)=\$ 632.41
$$

You would not be required to pay PMI as your LTV would be $150,000 / 200,000$ or $75 \%$. Remember that when calculating your final monthly payment you need to add your tax and insurance payments.

Let's practice! Show your calculations.

1. You are interested in buying a home that has been appraised for $\$ 150,000$. You can make a $\$ 20,000$ down payment. If you are approved for a mortgage, what would be your PMI each month?
2. (Continued from number 1) Your yearly homeowner's insurance bill is $\$ 600$ and your yearly property taxes are $\$ 1,200$. How much would that add to your monthly mortgage payment?
3. You have your Good Faith Estimate and after your down payment and your fees, your mortgage balance is $\$ 175,000$. You are approved for a 30 -year loan at a $4 \%$ rate. Your mortgage provider says your escrow balance will be $\$ 150$ a month and you do not need PMI. What is your monthly mortgage payment?
4. You are offered a $4 \%$ mortgage. You would like to buy down your rate to $3.5 \%$. Your mortgage balance is $\$ 100,000$. How much in points would you have to pay?

## Trust Fund Scholarships: Now $\boldsymbol{s} 1000$

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

> This year the Trust Fund Committee is pleased to announce that the amount that can be requested to help with the cost of graduate coursework is now $\$ 1000$.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from Janice Richardson, richards@elon.edu.

## Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett<br>North Carolina Council of Teachers of Mathematics<br>P. O. Box 33313<br>Raleigh, NC 27636

# 2016 State Math Fair Winners 

Reported by Betty Long, Appalachian State University, Boone, NC

NCCTM sponsors three regional Math Fairs each spring, and the best projects presented at these regional Fairs qualify for the State Math Fair. This year's State Fair was held at the North Carolina School of Science and Mathematics on 13 May 2016. The following students were selected for top honors in each division.

## Primary Division, Grades K-2

| 1st Place: | "Tsum Tsum Math"; Cooper Beecham; Coddle Creek Elementary, Mooresville |
| :--- | :--- |
| 2nd Place: | "A Dancer's Flexibility"; Tess Kiser; Coddle Creek Elementary, Mooresville |
| 3rd Place: | "Store Brand vs. Name Brand"; Ainsley Kiser; Coddle Creek Elementary, Mooresville |
| Honorable |  |
| Mentions: "Distributions"; Cody Surface; E. K. Powe Elementary, Durham |  |
|  | " 60,000 Vitamins for Charity"; Daniel Satterfield; Wells Elementary, Wilson |
|  | "Where in the World Are Our Grandparents?"; Seth Funai; Ridgewood Elementary, Winterville |

## Elementary Division, Grades 3-4

1st Place: "Tesla Coast to Coast"; Oliver Hines; Sandy Ridge Elementary, Durham
2nd Place: "My PVC Purple Phone"; Sam Morgan; Codington Elementary, Wilmington
3rd Place: "Monarch Butterfly: Where are You?"; Ashwin Varadarajan; H. C. Bellamy Elementary, Wilmington

## Honorable

Mentions: "Light Bulb Battle: Halogen vs. LED"; Rika Parui and Nika Parui; Lake Norman Elementary, Mooresville "Open Fire! Does Angle Affect Distance of a Nerf Dart"; Ben Roylance; A. B. Combs Elementary, Raleigh
"Small Farm Math"; Madilynn Rose Miller; Hendersonville Elementary, Henderson

## Intermediate Division, Grades 5-6

1st Place: "A Roll of the Dice"; Ethan Connolly; Greene County Intermediate, Snow Hill
2nd Place: "The Math in Model Rocketry"; Aiden Fogleman; Cane Creek Middle, Fletcher
3rd Place: "Making Cases for New Bases"; Jessica Orr; Lake Norman Elementary, Mooresville
Honorable
Mentions: "Landon's Dream Treehouse"; Landon Kelly; Mountain View Elementary, Hickory
"High to the Sky"; Tripp Markland; Freedom Trail Elementary, Elk Park

## Middle School Division, Grades 7-8

1st Place: "Sustainable State Index: How Does Your State Measure Up?"; Valerie Kitchell; Valle Crucis Middle, Sugar Grove
2nd Place: "Chain Reaction"; Gabriel Aldous; Camden Middle, Camden
3rd Place: "U.S. Agriculture Production"; Nate Worley; Clyde A. Erwin Middle, Asheville
Honorable
Mentions: "Curving the Wi-Fi Issue"; Reagan Bustabad; Sampson Middle, Clinton
"Lunch Trays: They All Add Up"; Zoey Locklear; Sampson Middle, Clinton

## High School Division, Grades 9-12

1st Place: "Rippling Rates"; Lindy Bustabad and Katey Yang; Clinton High, Clinton
Honorable
Mention: "The Golden Ratio in Flowers"; Cordelia Gilligan and Marina Perez Mazariegos; School for Creative Studies, Durham

# Flipped Mathematics Classrooms for the Inflexible 

Amy Whicker, Northwest Middle School, Winston-Salem, NC

The author discusses how she implemented the flipped classroom instructional method and the lessons learned during the process.

As teachers we are always asked to be flexible - to be ready for an unexpected fire drill, to welcome new students no matter the time of year, and of course to be ready for repeated intercom interruptions to call a student for a forgotten lunch box. When it comes to instruction, however, we have little flexibility. We have a set number of instructional days and a list of topics to teach to a diverse group of students, some of who are easily distracted by these interruptions or have learning challenges to overcome. The pressures of these tests and the desire as an educator to impart knowledge creates a feeling of white-knuckled inflexibility when it comes to instructional time.

Because of this perceived inflexibility, it was with some hesitancy that I decided to try an instructional method called "flipped classroom" for my seventh grade mathematics classes. In this article, I would like to share what I gained from this year of instructional flexibility, including (1) how it made me more aware of passive versus active learning in my classroom, (2) the importance of immediate assessment, and (3) the benefits of having an archive of video lessons. All three of these lessons were learned in relation to working with the unique audience of middle school students. Working with pre-adolescents means you need to be willing to ask them to be mature academically while also realizing some are not ready to be independent learners and may need more accountability before reaching high school. Understanding how to adapt the ideas of the flipped classroom for the unique maturing middle school student has helped make these lessons relevant for my current and future years as an educator.

## Defining and Implementing the Flipped Classroom

While there are several models of the "flipped classroom," I chose to use a model recommended by colleagues that provided online tutorials and handouts, authored by Dr. Lodge McCammon of the FIZZ Institute (2016). The basic premise of this model is that students preview the lesson material at home by way of watching a teacher-created video, freeing class time for investigations and applications of concepts. This method also eliminates (or at least diminishes) the need for the typical classroom format of a teacherdelivered lesson with students listening passively or copying notes. This method appealed to me as a mathematics teacher because of the amount of information we must present to students and the desire for a more interactive note-taking process. In addition, while the ideas of the flipped classroom began small in a Colorado high school, the amount of sustained teacher implementation of this method, particularly in math classrooms, suggest it is more than just a "fad" (Raths, 2013).

Along with colleagues who were up for the challenge, the year's experiment began with planning each video on paper, agreeing with each other about what concepts we should cover in our preview and which concepts should be left for student discovery in class. We limited our material to what could be
covered on six lecture white boards, approximately 3 by 2 feet each in size. We then video-taped each other, using the basic white boards and a flip-video camera. The boards are put in sequential order and slid across the chalk tray as they are completed. For more training on how to easily plan and film videos, view training modules (McCammon, 2016).

## Lesson 1: Passive verses Active Learning

To ensure easy and consistent access, I created a YouTube channel that housed all of our classroom videos. To introduce the idea to my students, we began with watching the first few lessons together to set the expectations for video note-taking that would theoretically continue at home for the rest of the school year. I modeled note-taking strategies such as underlining important concepts, determining when to abbreviate, and pausing the video when necessary. Parents were also given a newsletter with information regarding our video lessons and the benefits of this instructional method. Those without internet were asked to bring a flash drive that would allow me to save the videos for them, and those without computers were asked to arrange a time before or after school. Once the time came for students to watch the video at home and come prepared for class with their notes and questions, any middle school teacher (or any teacher for that matter) can imagine what happened. There was not $100 \%$ participation. So, as a teacher, you have to decide what your reaction should be - just give the students who didn't come prepared a zero and hope they somehow get the information through their classwork or make a way for them to see the video that you labored to prepare during their class time.

While experimenting with both reactions in my classroom, I learned my first lesson of this experiment. A unique benefit of the flipped classroom videos is the opportunity for students to be active note-takers. In allowing students to take video notes in the back of my classroom on personal laptops, I watched a very simple phenomena. Students were stopping and starting the video at their own pace, rewinding if necessary, and seemed to be dramatically more engaged in the material compared to a live lecture. The video note-taking on individual laptops had actually resolved several frustrations I had with live lectures. I didn't have to wait for the slower writers to finish before moving on to the next screen. I didn't even have to repeat the lesson for students who were out of the room or absent. The video method, while not perfect, gave me a much more individualized, active method for presenting information to my students. This has made me much more conscious of when and how I present new lessons to my students. If I can think of a way to give the student control of the pace of lesson delivery, whether through video or another method, I always try to make it happen. As I further researched teacher experiences with flipped classrooms, I found that giving students who don't watch the videos at home this opportunity in class also prevents penalizing those that are prepared with video notes and ready to practice a concept. Those that have watched the video can now spend class time applying their notes to math practice with instructor help and will also feel that their time watching the video at home was valued by the teacher (Raths, 2013).

## Lesson 2: Importance of Immediate Assessment

The second lesson I learned from the flipped classroom model is the need for immediate assessment on any type of lesson presentation, whether video or live. The concept of note-taking is relatively new for middle school students, and therefore can sometimes turn into a hyper-focus on getting everything copied perfectly as opposed to ensuring the content is understood. To keep the focus on the content, I began using Google Forms as a means of immediate assessment. Students completed a few questions based on the video, some that were basic comprehension and some that required them to apply a concept to a new math problem. I could then download the results in an Excel spreadsheet format to see who sent what response. Figures 1 and 2 provide an example of a Google Form and Excel spreadsheet results from a typical video lesson.

The incorporation of the Google Forms after the videos proved to have both benefits and challenges. As with any assessment, it is beneficial to recognize common misconceptions, but also difficult to know how to provide meaningful feedback. During my experimental year of flipping, I can't say I always knew what to do with the results immediately, and mostly settled for whole group feedback. The concept of making sure each lesson I deliver is interactive and has built-in assessment has carried over to successive school years, however. When doing a live
lecture now, I may stop and ask a question about what has been presented so far using the clickers, just like students would do if watching the video and answering a Google Form question. Helping students who are new to note-taking remain focused on the content has been invaluable to my students and to my teaching practice.

## Lesson 3: The Benefits of Video Archive

The last lesson learned from the "flipped classroom" model is the benefit of having an archive of video lessons. Following this year of instructional flexibility, I decided that I did not want to completely abandon the ideas of the "flipped classroom," but I also knew that I wanted to improve the way in which I taught some of the material and utilized the videos. Because of this, I developed a hybrid version of the flipped classroom that utilized the videos in many different ways - (1) as a whole class video lecture if I wanted to ensure a concise presentation, (2) as a follow-up activity at individual laptops for early finishers, (3) remediation for students who did not grasp the concept in a live lecture, and (4) as an online resource for parents or students who were frequently absent. These are four different purposes that an online archive of video lessons has served as I adapted the "flipped classroom" model for my middle school classroom. These benefits alone would be a reason I would encourage any teacher to experiment with video-taping a set of lessons.

[^0]Describe how we found the probability of landing in the shaded region in the third slide.


What is the probability of getting bankrupt twice in a row?

- $1 / 576$
- 1/1000

1/2000

Figure 1. Sample Google Form

## Looking Forward: Google Forms

In the pursuit of continual improvement, I am currently experimenting with updated ways of using Google Forms results. One weakness of using Google Forms is not knowing what to do with the results. Should I print out the results and grade it like a regular assignment or would that be a waste of paper? Should I use class time to review commonly missed questions? How should I remediate for a student who missed every question? While these are the same questions all teachers have after giving any assignment, utilizing technology as intended can make providing feedback easier. One of those


Figure 2. Google Form Results in Excel
methods is utilizing Google Classroom which allows teachers to type feedback individually to students or the whole class based on their Google Forms results, eliminating the need for paper or any wasted class time. Another adaptation of Google Forms is to differentiate based on correct answers. Google Forms can be created in such a way that remedial questions can appear for students who are struggling and challenge questions can appear for those that are answering correctly. Figure 3 shows an opening integers question that could be used to determine if a student has grasped the foundational skill of adding two negative values. Figure 4 shows a follow-up challenge question if a student answered correctly that would require them to generalize and think critically. Figure 5 shows a remedial question if a student answers incorrectly, prompting them to relate the opening question back to a concrete money example. This method has proved to be the most effective for student learning and allows students to reflect on their own understanding without immediate teacher feedback.

## Conclusion

As an educator, it may seem like flipping your classroom wouldn't be worth your limited time or efforts in teaching your diverse group. I will admit that this was my initial thought as I reflected on that year, fearful that maybe my efforts were miss-directed. However, just as with our students, lessons are learned through trying, some failure, adapting, practicing again, and reflecting. I do not believe these three lessons would have shaped my values as an educator had I not tried flipping first hand. My current and future students will benefit from these three lessons that I

# Integers Challenge 

*Required

What is the sum of $(-8)$ and $(-3)$ ? *

- -5
- 5
- -11
- 11

Continue :

Figure 3. Opening Question

## Integers Challenge

## Integers Challenge - Level 2

The sum of any two negative numbers is....
O always positive

- always negative
- sometimes positive
- sometimes negative

ع. Back
Continue \%

Figure 4. Challenge if Opening Question is Answered Correctly consider to have been worth every minute. See Ms. Whicker's YouTube Channel and updates: www.youtube.com/channel/UCkGk3yd_1CIBQQ Lh_wKj4PQ

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Raths, D. (2013). Nine video tips for a better flipped classroom. T.H.E. Journal, November 2013, 12-18.

## Integers Challenge

## Integers Challenge - Level 1

Let's think about that question again. If I borrow 8 dollars ( -8 ) and then borrow another 3 dollars $(-3)$, what integer represents how much I owe?

- $-\$ 5$
- $\$ 5$
-     - $\$ 11$
- $\$ 11$
« Back Submit

Figure 5. Remediation if Opening Question is Answered Incorrectly

# Using Linking Cubes to Understand Prime Factorization 

Diana L. Moss, Appalachian State University, Boone, NC Stephanie Vega, University of Nevada-Reno, Reno, NV<br>Teruni Lamberg, University of Nevada-Reno, Reno, NV

The authors discuss the importance of student understanding of factorization and prime numbers and how to use linking cubes to help students visualize prime factorization.


#### Abstract

Although students in middle grades have learned about factoring, they lack conceptual understanding of prime and composite numbers, which might inhibit their understanding of prime factorization (Zazkis \& Campbell, 1996). Prime factorization, the ability to decompose numbers through multiplication using primes, is a foundational skill because it relates to several mathematical topics, including divisibility, fractions, square roots, simplifying, and algebra. The NCTM standards (National Council of Teachers of Mathematics, 2000) emphasize that students in grades 6-8 should be able to use "factors, multiples, prime factorization, and relatively prime numbers to solve problems." The Common Core State Standards for Mathematics (CCSSM; National Governors Association, 2010) recommend introducing students to factors in fourth grade and being able to fluently compute with multi-digit numbers to find common factors by sixth grade. This article considers how linking cubes and factor trees can be used to promote understanding of prime and composite numbers by exploring divisibility and prime factorization.

Prime numbers are defined as building blocks of natural numbers with factors of one and itself (Zazkis \& Liljedahl, 2004). A composite number is any natural number greater than one that is not a prime number. Figure 1 illustrates a difficulty students have when solving problems using traditional methods of prime factorization (Kurz \& Garcia, 2010) without understanding prime and composite numbers.


In Figure 1, a student wrote an incorrect answer to the factoring problem. The student had correctly identified the factors, but only indicated the numbers in the longest branch as the answer to the problem. The student ignored, or did not notice, the other numbers in the shorter branches and did not relate the prime factorization back to the original number. The student made an attempt to take apart the two-digit numbers into two factors but was not successful in using the factor tree to create the prime factorization. This indicates that the student was procedurally applying a rule to solve the problem and did not conceptually understand the mathematics.


Figure 1. Example of Student Misconception of Prime Factorization

As practicing teachers, we asked ourselves: how do we make meaning of prime factorization in order to build understanding? We, along with other practicing teachers, explored this concept in a graduate class by examining how to factor the number 36. Many started drawing a factor tree and found the prime factorization by factoring the numbers until prime numbers were reached. However, we were unable to articulate the underlying mathematics and reasoning of how the factor tree is used to show multiplication and the relationships between the numbers on the factor tree. It seemed that even our own knowledge of prime factorization lacked conceptual understanding.

Therefore, we explored this concept using linking cubes in order to model prime factorization. The purpose in using linking cubes was to explore prime factorization as quantities that can be manipulated in order to make sense of the factor tree. We began drawing on our previous knowledge of building rectangular arrays with composite numbers, such as eight and nine, with linking cubes. We discovered that it was easier to build the array and explore the meaning of prime factorization by dealing with a smaller number. From this foundation, our graduate class constructed meaning for "prime factorization." In other words, we answered the question, "What does it mean when $8=2 \times 2 \times 2$ ?" We decomposed the eight cubes into two groups of two groups of two cubes (Figure 2). This method related to the factor tree where eight can be decomposed through multiplication into $2 \times 4$. We justified how eight represented two groups of two groups that contained two cubes. In doing so, we were able to make connections between the arrays and the factor tree.


Figure 2. Model of $8=2 \times 2 \times 2$
We discovered by our exploration that the cubes were only helpful if students can visualize and articulate the factoring process as the "size of groups" and the "number of blocks" in each group. Some of our classmates, practicing teachers, discussed that one misconception that can take place is that students might multiply all groups of two by one another, which results in a model of 16 instead of eight (Figure 3). The discussion was expanded to ensure that eight cubes could be recreated from the two groups of two groups of two cubes by counting the individual linking cubes.

## Teaching Prime Factorization through Investigation

To link these understandings with our teaching practice, we collaborated to recreate this investigation with eighth grade
 students. We designed a lesson to help our students gain a conceptual understanding of prime factorization rather than providing them with a procedural approach. Similar to Kurz and Garcia's (2010) findings, most lessons on this topic use factor trees and are typically taught without tools for understanding. The rationale for our lesson maintains that students must be afforded opportunities to study whole numbers and encouraged to use representational tools to help them visualize the prime decomposition of numbers
(Kurz \& Garcia, 2010; Zazkis \& Liliedahl, 2004). We wanted our students to experience a hands-on approach in the same manner that we did. The activities that we planned were designed to bring about rich mathematical discourse.

## Preparation

To begin, we designed a pre-assessment of students' prior knowledge of factors, prime numbers and composite numbers. We asked students to define and provide examples of each of these concepts. We analyzed their responses to understand their ways of thinking about factors, prime numbers, and composite numbers. The responses confirmed our notion that students had previous experience with factorization through procedural knowledge (divisibility rules, multiplication, memorized definitions). An example of how a pair of students wrote their own definition for factor, prime number and composite number is shown in Figure 4. The students described prime and composite numbers by their previous procedural knowledge of division. The students' definition (Figure 4) for prime number is partially correct, but is true of all numbers, not exclusively prime numbers. The students' definition of a composite number is also partially correct, but it does not explicitly state that a composite number can also be divided by itself and one evenly. It is also interesting that these students wrote, "family of numbers" for a factor. When asked about this term, they recalled that they had learned it from another teacher's lesson on factoring in fifth grade.


Figure 4. Student Definitions
To connect their past procedural knowledge to a conceptual understanding of prime and composite numbers, we asked students to use linking cubes to show which numbers, one through eight, are prime, composite, or neither. Students built two-dimensional area models and labeled the models to include how many cubes in each array (arrangement of blocks in rows and columns) and how many arrays (Figure 5). Area models were used to bridge their previous experiences with multiplication through area and to extend to future work with algebra tiles. They checked duplicates of the same array and described similarities and differences between models. Students discussed how some numbers have only one array representation and some have more than one array representation. During their discussion, a student pointed out, "One, two, three, and five can make only one group." In response, another stated, "Composite ones have more than one array." This appeared to be the precursor to the discovery of prime numbers as building blocks for composite numbers. Students noticed that the labels they wrote were also factors of the given number.

The students determined that numbers with only one array, with factors of one and itself were prime numbers. Numbers with more than one array representation and two or more factors besides one and itself were composite. To connect back to their previous knowledge, students revised or confirmed their definitions. Some students clarified their misconceptions, discovering that not all odd numbers are prime and concluding that all even numbers besides two are composite. Additional numbers were used to test their definitions of prime numbers and composite numbers. The use of linking cubes promoted mathematical discussions and assisted students in developing mathematical understandings about these concepts.


Figure 5. Student Work with Labels

## Posing the Task

To capture students' curiosity and engage them in mathematical thinking about prime factors, students worked in pairs to write a multiplication sentence for 12 using only prime numbers. Knowing that students need to model concepts concretely, we designed the task to include justification by using linking cubes or pictorial representations. All students agreed that the decomposition of 12 using only prime numbers was $12=2 \times 2 \times 3$. In response to the question "How can you show that $2 \times 2 \times 3$ is in fact 12?", students represented 12 by constructing two groups each with two groups of three cubes (Figure 6).

Students decomposed or broke down four into prime numbers as a group.


Figure 6. Prime Factorization of 12 with cubes. One student shared with the class, "It's $2 \times 2 \times 1$, because you have two groups of two cubes and you have one of these groups." A student responded by critiquing the argument of his peer, "Yes, but I think it is $2 \times 2$. One is neither prime nor composite and should not be in the sentence." The student replied, "Oh yeah! It's only primes." This demonstrates students' ability to use this newly developed knowledge, that one is neither prime nor composite, and apply it to prime decomposition.

At the conclusion of this lesson, the teacher introduced factor tree notation as a way to help students represent their thinking when finding prime factorizations (Figure 7). After the teacher wrote the factor tree for 12, she asked students to analyze how the tree represents the model that they had made with cubes. One student described the first branches of two and six. She explained how she could see two groups of six cubes in the multilink model. She said in each group of six, there were two groups of three cubes. Another student added that the teacher circle the prime numbers. They all agreed that the circled numbers or primes were used in the prime factorization.


Figure 7. Prime Factorization of 12 With a Factor Tree

In pairs, students factored $8,13,15,16$, and 20 using factor trees and multilink cubes successfully. Students demonstrated facility with connecting the multilink cubes to factor trees. One student exclaimed, "Prime numbers are the shortest factor trees-just the number!" Challenge numbers, including 72, 156, and 2015 were also posed to allow students to decompose numbers abstractly. Some students began to connect divisibility rules to prime factorization for efficiency because they identified how division could be utilized to find factors. In response to the prompt "Tell me about your factor tree of 72 with two branches of 36 ," one student in the pair responded, "Well, 36 and 36 makes two groups of 36 ." Krysti, his partner replied, "No David, think of the blocks, it is two groups of 36. The branches are two and 36 ." While the pair clearly understood that 72 is equivalent to two groups of 36 , David inaccurately represented two groups of 36 is in his factor tree notation by using addends rather than groups or
multiples. Krysti accurately used factor tree notation by visualizing the size of the groups and the number of blocks. The discussion between this pair demonstrates the critical need for meaningful experiences that include communication and representation centered on mathematical concepts.

## Final Thoughts

All of us, the practicing teachers and eighth grade students, developed new understandings of prime factorization. Specifically, we discovered that prime factorization involves dividing a quantity into arrays and using multilink cubes to model prime factorization is helpful. Similar to other approaches for prime factorization, this method aims to show how prime numbers are the building blocks of composite numbers. This article differs from the semi-concrete approach of assigning colored blocks to a numerical value to represent prime factorization designed by Burkhart (2009), because it introduces a concrete method of prime factorization, the use of cubes to decompose a quantity into its smallest groups in order to deepen understanding of the factor tree. This approach challenges students to perform division to find factors of a given number until prime factorization is achieved.

Students needed to make explicit connections between the problem context, model, and the meaning of numbers through discussion. The conceptualization of how the factor tree is the representation of the number of groups and the quantity in each group for a given number made learning factors meaningful for the students. Students made additional connections from the concrete model to the factor tree. Through their ability to perform prime factorization with a model, students decomposed the number into groups via branches and determined that the smallest group was represented as a stub of each branch, a number whose factors are only one and itself-a prime.

This investigation prompted students make the mathematical connection that the product of the prime factorization is the original number. In the student example in the beginning of this article (See Figure 1), a number was factored into primes correctly but not all factors were included in the prime factorization. Students must understand the relationship between factoring to decompose a number and the number's prime factorization.

Our preconceived notions that composite numbers and prime numbers are disjointed sets were re-conceptualized to an understanding that prime numbers and composite numbers relate to one another and contribute to the structure of whole numbers. Additional opportunities to deepen understanding of prime factorization might dispel the common misconception that decomposition into primes means small numbers. Further investigations, similar to these, might help students better understand how factors and multiples relate to division and fractions.

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## Fall 2016 Problems

Grades K-2: Put the numbers $2,3,5$, and 7 in the boxes so that when added the sum is as large as possible.

Grades 3-5: A triangle with a perimeter equal to 30 inches has sides in a ratio of 2:3:5. What is the length of the longest side?


Grades 6-8: A company produces fire crackers. During a recent test of the quality of the fire crackers, 90 firecrackers were lit and 3 failed to explode. If the company produces 1500 fire crackers each day, how many are expected to fail on average in each daily batch?

## Directions for submitting solutions:

1. Students: NEATLY print the following at the top of each solution page:

- Your first name (we will not publish last names)
- Your teacher's name
- Your grade
- Your school

2. Submit one problem per page. Students who submit correct solutions will be recognized by their first names only in the next issue of The Centroid. We will also publish one or two especially creative or well-written solutions from those submitted. If you would rather not have your name published, please so indicate on your submission.

Proper acknowledgement is contingent on legible information and solutions. Send solutions by 20 December 2016 to:
Problems to Ponder, c/o Dr. Holly Hirst
Mathematical Sciences
BOX 32069 Appalachian State University
Boone, NC 28608

A submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

## Spring 2016 Problem Solutions

Grades K-2: Tom wants to purchase a really nice pair of ear buds for his iPod. Tom has already saved up \$33, and he can earn $\$ 5$ each week for doing his chores. If he needs $\$ 69.99$ to get his ear buds, how many weeks of chores will he need to do to have the money?

Editor's Note: Quite a few students said 7 weeks, which would not be enough money.
Correct submissions were received from:

- The Ravencroft School: Four 2nd Grade students taught by Mrs. Crawford and Mrs. Byrne. Below is an example of a correct solution from The Ravencroft School.


Grades 3-5: Tena and Jai'Lisa are playing a game using the grid of squares and rectangles shown to the right. The winner is the girl who claims the most area. Each girl can choose a strategy from the list below. Which would be the best winning strategies to choose and which would be the worst? Explain why.

- Claim odd numbers
- Claim even numbers
- Claim all squares
- Claim all rectangles (and not squares)

| 1 | 3 | 4 | 3 |
| :--- | :--- | :--- | :--- |
| 2 | 4 |  |  |
| 1 |  | 2 | 3 |

Correct submissions were received from:

- Ahoskie Elementary School: Five $4^{\text {th }}$ Grade students from classes taught by Ms. Hall, and two $5^{\text {th }}$ Grade students taught by Ms. Cudd and Mr. Kilsheimer: Brian, Clara, Gabrielle, Layla, Malay, Nailah, Tre'Quell
- The Ravencroft School: Eight $4^{\text {th }}$ Grade students from classes taught by Mrs. Childrey, Mrs. Keane, and Ms. Byrne.

The most complete explanation came from a student at The Ravencroft School:


Draw your'picture here to explain the best stragedies:


Editor's Note: Quite a few students looked at the numbers in the squares and rectangles instead of the areas of the squares and rectangles! In addition, there were students who counted the number of squares and rectangles instead of looking at areas. It just goes to show that it pays to read problems carefully!

Grades 6-8: A cube has a total surface area equal to 150 square feet when all faces of the cube are included in the measurement. What is the volume of this cube in cubic inches?

Editor's Note: More than forty $5^{\text {th }}$ and $6^{\text {th }}$ grade students from Ahoskie Elementary submitted their final answer as 1500 cubic inches. They all had the right reasoning until the last step: 150 square feet implies that each face was 50 square feet in area. So each face was a 5 by 5 foot square. Thus the volume in cubic feet is $125(5 \times 5 \times 5)$. Next, though, students multiplied 125 by 12 once rather than by multiplying by 12 three times, one for each foot in the cubic feet unit. Don't feel bad! My college students usually make this mistake, even math majors! We all need to work more with units!

Only one correct solution was received, from a $6^{\text {th }}$ Grade student taught by Mrs. Bowser: William.


## Puzzle Page

Holly Hirst, Appalachian State University, Boone, NC
Are you looking for interesting math problems to give your students for class warm-ups or extra credit? Check out the Math Is Fun website (MathlsFun.com). Here are a few sample problems. Can you solve these problems? To find the solutions, go to the puzzle page listed in the reference.

## Fraction

Can you arrange the numerals 1 to $9(1,2,3,4,5,6,7,8$ and 9$)$ in a single fraction that equals exactly $1 / 3$ (one third)? An example that doesn't work: $7192 / 38456=0.187$

Reference: Pierce, R. (18 Feb 2014). Fraction puzzle. Math Is Fun. Retrieved from
http://www.mathsisfun.com/puzzles/fraction.html

## Five Pirates

5 pirates of different ages have a treasure of 100 gold coins. On their ship, they decide to split the coins using this scheme: The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it. If $50 \%$ or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain. As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?

Reference: Pierce, R. (29 Jan 2012). 5 pirates puzzle. Math Is Fun. Retrieved from http://www.mathsisfun.com/puzzles/5-pirates.html

## Nine Digit Number

What 9 -digit number has the following features:

- It has all 9 digits from 1 to 9
- It is evenly divisible by 6 and 7
- Each time it is rounded (starting with units, then tens, hundreds, etc) it rounds in an alternate pattern (up, down, up ...), until after rounding 8 times the final number is 500000000
- After rounding four times the sum of the digits is 24

Reference: Pierce, R. (1 Sep 2016). 9-digit number puzzle. Math Is Fun. Retrieved from http://www.mathsisfun.com/puzzles/9-digit-number.html

## A Weighty Problem

I have ten boxes that I want to pack into crates. Each crate can carry a maximum of 25 kg . But I only have three crates, and the total weight of the boxes is 75 kg :
$15 \mathrm{~kg}, 13 \mathrm{~kg}, 11 \mathrm{~kg}, 10 \mathrm{~kg}, 9 \mathrm{~kg}, 8 \mathrm{~kg}, 4 \mathrm{~kg}, 2 \mathrm{~kg}, 2 \mathrm{~kg}, 1 \mathrm{~kg}$
How can I pack these boxes into the crates?
Reference: Pierce, R. (5 Oct 2012). A weighty problem puzzle". Math Is Fun. Retrieved from http://www.mathsisfun.com/puzzles/a-weighty-problem.html

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## Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an ongoing benefit from the grant. In recent years, grants averaged just less than $\$ 800$.

The application is available on the NCCTM website, http://ncctm.org. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Sandra Childrey, schildrey@wcpss.net, with questions.

## Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, http://ncctm.org. More information can be obtained from: Todd Abel, AbelTA@appstate.edu.

## Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Lee V. Stiff, lee_stiff@ncsu.edu.

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Follow the "Membership Information" link on the ncctm.org website, or go directly to:
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[^0]:    What is the probability of getting two tails in row when flipping a coin? *
    (Look at the tree diagram in the second slide for a clue.)
    ( $1 / 2$
    $1 / 4$
    $1 / 8$
    $0=$

