The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

Volume 44, Issue 1 • Fall 2018

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements

We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts

We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors

Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures, and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:

  Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:

  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, 7, 196–199.

  Chapters or sections of books:

  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:

  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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November 1 and 2, 2018
NCCTM Leadership Seminar
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Conference Keynote Speakers
Susan O'Connell, Sandra Linder, Kathy Richardson, Julie McNamara, Graham Fletcher, Pam Harris, Steve Leinwand, Scott Hendrickson, Andres Ruzo, and NC2ML

Leadership Seminar Presenters
Graham Fletcher, Pam Harris, and Steve Leinwand

Preregistration Information
Conference: $65 for NCCTM members; $105 for non-members
Leadership Seminar: $65 for NCCTM members; $75 for non-members

Hotel Information
Rooms can be booked at the Sheraton Greensboro – Conference Hotel at a discounted rate using the online form.

Visit www.ncctm.org to preregister and for more information!

President's Message

State President Julie Kolb
Meredith College, Raleigh, NC
kolbjuli@meredith.edu

It is hard to believe that we're already in "back to school" mode! For as long as I have been teaching, amazingly I am beginning my 37th year in the classroom, this is still an exciting time of the year. I look forward to new students, new ideas and activities, and collaborating with my colleagues at Meredith and from across the state of North Carolina. I sincerely hope that you share in my anticipation of another fun school year.

The 2018 Fall Leadership Seminar and Conference present a unique opportunity for mathematics educators to connect and collaborate. These events will take place at the Koury Convention Center in Greensboro; the leadership seminar is scheduled for October 31 and will be followed by the fall conference on November 1 and 2. The theme for this year's conference is Cultivating Coherence and Connections. Featured speakers for the Leadership Seminar include Graham Fletcher, Pam Harris, and Steve Leinwand. Please explore the website www.ncctm.org for additional information about the conference.

The Spring 2019 Leadership Seminar will be the last statewide spring meeting for the foreseeable future. This seminar, scheduled for March 22, 2019, at the Radisson in High Point, features Lee Stiff as the keynote speaker and includes break-out sessions with a variety of speakers related to the TOOLS Project, Teaching Circles, and NC2ML. If you have not had the privilege of hearing about the work of these groups across the state, you will want to make sure to put this event on your calendar. Plans are being made for the Spring 2019 regional conferences.
In addition to the conferences, please continue to Celebrate Math and take advantage of the opportunities to participate in and contribute your expertise to our math fairs, contests, mini-grants, scholarships, award and grant opportunities. Take the time to read the Centroid – this publication is now available to anyone who visits our website. Thank you to everyone who makes each of these opportunities available to the students and teachers of North Carolina – the list is long!

While vacationing with family in Maryland, a relative mentioned that she was reading a book about planning for her retirement. Her husband complained that she kept dropping the "F" bomb – Fibonacci! She explained that to determine the amount of money she would need to invest to maintain a certain income level in retirement, the book advocated the use Fibonacci analysis. Were you aware that Fibonacci analysis is used to identify price targets for investments? This discussion evolved into a more general conversation about Fibonacci, the Fibonacci sequence, and natural logarithms – this at a party at the lake! Well, of course, there were other individuals in attendance who were clearly uncomfortable with this conversation. I'm certain that you have witnessed the look of absolute terror on the faces of those individuals at the mention of mathematics. They looked as if they wanted to sink into their chairs – they had no desire to listen to nor engage in the conversation. It's disheartening that so many people have this negative attitude toward mathematics.

As stated in the mission of our organization, we must continue to advocate for excellence in mathematics teaching and learning for all. We must do all we can to dispel the myth that some people simply do not possess the "math gene"! It is my hope that the effective implementation of the NC mathematics standards will assist in the creation of a mindset that mathematics is accessible to everyone. To inspire and enable you to advocate for "Math for Everyone," I encourage you to read two books: *Teach Students How to Learn* by Saundra Yancy McGuire and *Mathematical Mindsets* by Jo Boaler. If you haven't read them, they'll add to your excitement about beginning another school year; if you have, please share them with a colleague.

Make it a great year and I hope to see you at the conference.

-- *Julie*

Bayesian Reasoning in a High School Statistics Course

Samantha Everett, Ross Gosky, Appalachian State University, Boone, NC

The authors introduce two activities that can be used in a high school statistics course to explore basic Bayesian inference.

The vast majority of statistics courses taught in the United States are fundamentally based within the frequentist view of probability. The frequentist perspective defines the probability of an event as its long-term frequency of occurrence. Introductory textbooks such as Utts and Heckard (2014) briefly touch upon the idea subjective probability, which reflects an individual's certainty that a particular event will occur. Subjective probabilities can be informed by long-run frequency, but are open to individual interpretation. Bayes' Rule is also covered in many introductory statistics courses, in which tree diagrams are often used to illustrate the rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' Rule allows the directional reversal of a conditional probability when total probabilities of each base event are known. A common explanation of the rule shows how the original probability of an event *A*, often termed a *prior probability*, is updated to a *posterior probability* once the occurrence of another event *B* is known, $P(A|B)$. Therefore, Bayes' Rule describes how prior probabilities can change as new information becomes known. When the logic of Bayes' Rule is combined with subjective probability, students can begin to use the Bayesian approach to make statistical inferences.

While uncommon, early introductions of Bayesian reasoning within a statistics curriculum have been practiced. Bolstad (2002) advocated that a statistical Bayesian course be offered to undergraduate college students. Bolstad promoted the Bayesian approach as an alternative to a traditional statistics course, and that such a curriculum would pursue the same goals of a standard introductory statistics course. Stewart and Stewart (2014) used ventriloquism and debates as an innovative method to expose students to Bayesian inference. Page and Satake (2017) used the Bayes' Factors methodology as an accessible approach to undergraduate students. However, not all literature recommends introducing Bayesian statistics in a first-level course (Moore, 1997).

In this paper we will introduce two activities that integrate high school students' knowledge of statistical inference, subjective probability, and Bayes' Theorem to conduct basic Bayesian statistical inferences. These activities do not require a significant time investment, but are best presented after students have had an introduction to hypothesis testing and general statistical inference. The goal of these activities is not to promote early course adoption of Bayesian inference, but rather to introduce students to its fundamental concepts and demonstrate a new school of thought.

Introducing the Ideas of Bayesian Inference through Subjective Probability

The frequentist definition of probability, requiring an inherently repeatable process, is commonly accepted in statistics. In many situations, a repeatable process is easy to comprehend, with examples like flips of a coin, measurements of height from randomly chosen members of a population, or controlled administration of a drug to many patients with an illness. However, adherence to this definition lends itself to two complications. Firstly, some events are very specific and not inherently repeatable. For example, assume that someone forgets his or her mother's birthday. What is the probability that their mom is upset? That is a reasonable question; yet the answer from a frequentist position is elusive, due to the difficulty of conceptualizing and/or conducting a repeatable process. Should the repeatable process be based upon all moms who have had their birthdays forgotten? Or upon repeated failings to remember this specific mom's birthday? A subjective interpretation would allow one to express a degree of belief that *our* mom is angry with *us* for having forgotten *this* birthday. Subjective probability could factor everything else we know about our mom over the course of our lives, not just forgotten birthdays. Subjective probability does not require formalization and could be based on intuition.

A second complication with frequentist probability is its inability to represent uncertainty based upon lack of knowledge of the outcome of an event, unless that probability is 0% or 100%. In contrast, the subjective definition of probability specifically incorporates one's current knowledge. For example, consider someone claiming they are 95% certain that James Madison was president after Thomas Jefferson. Clearly, the outcome of this event is an established fact, and is not based upon a repeatable process. Without accounting for this individual's realm of knowledge, a frequentist perspective declares this event to have a 100% or 0% probability. But the individual's uncertainty in the truth of this statement could be a subjective probability. As he or she gains more knowledge, the initially stated probability can be revised accordingly.

Before data is even available, Bayes' Rule allows one to make a subjective probability statement about the unknown population parameters. This is the *prior probability* in Bayesian statistical inference. Once new information becomes available, the original probability statement about the parameter is updated accordingly. This is the *posterior probability* in Bayesian statistical inference.

The activities we will use to illustrate these processes should be accessible to all students familiar with Bayes' Rule, conditional probability, and statistical inference.

A Card Drawing Example to Illustrate Reallocation of Beliefs through Bayes' Theorem

This example is an adaptation from Bolstad (2004), modified to use a set of four cards with two possible colors. Four cards from a common deck of playing cards would work well, as each card is either red or black. At the discretion of the teacher, students can work individually or in groups. The teacher should choose how many red cards to put in the deck (0 to 4), and not reveal this information to the students. If we denote R as the number of red cards in the deck, then R can be 0, 1, 2, 3, or 4. As R is fixed, but unknown to the students, it will be the population parameter.

At the first step of the exercise, students express their prior probability for each of the possible values of R. The rules of probability require these probabilities to sum to 1 and be non-negative; so many different probability assignments are possible. Students may struggle at this stage, but some suggestions from the teacher may help clarify their beliefs. If the students truly have no prior opinion about R, they can express this with equal probabilities of 0.2 on each of the five possible outcomes. Alternatively, the students may believe that the teacher has stacked the deck in one direction or another, and can express that opinion with unequal probabilities at the different possible values of R. It is fine, and even preferable, for different students to have varying prior probability distributions.

At the second step of the exercise, each student or group selects two random cards, without replacement, from the teacher's deck. This will be their observed data. The conditional probability of the observed color sequence can be calculated for each possible value of R. For example, if the number of red cards in the deck (R) is 3, then probability of drawing two red cards is:

$$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$

After observing the new data, students should use Bayes' Rule and a tree diagram to revise their beliefs. The first step in this process involves calculating the total probability of the number of observed red cards, and then using conditional probability to calculate their revised beliefs of *R* based on the observed cards in their sample. These revised probabilities will be the posterior probability distribution of *R*.

Table 1 illustrates this process with an example: A student has no prior belief that any values of *R* are more likely than any others, so 20% probabilities are assigned to all five outcomes, as shown in the second column. For illustration, suppose both cards drawn from the deck are red. The conditional probability of this sequence, given the value of *R*, is shown in the third column of the table. The joint probability of the number of red cards and the observed outcome (two red cards) is calculated by multiplying answers within the second and third columns. Notice that these probabilities sum to ${}^{10}/_{30}$ or ${}^{1}/_{3}$, which is the total probability of selecting two red cards. The rightmost column then shows the posterior probability of each possible outcome of R. This is calculated as the conditional probability of each outcome of *R* given the data. For instance, in Table 1, we calculate:

$$P(R = 2 \mid two\ red\ cards\ drawn) = \frac{P(R = 2\ and\ two\ red\ cards\ drawn)}{P(two\ red\ cards\ drawn)} = \frac{1/30}{10/30} = \frac{1}{10}$$

Similar calculations can be done for the other possible values of *R*.

Notice that the posterior probabilities for *R* differ from the prior probabilities in this example; larger values of *R* tend to result in higher posterior probabilities. This is due to modifications made after the two red cards were drawn. Some scenarios, such as all black cards, became impossible while other scenarios, such as all red cards, became more plausible. The implication is that our posterior conclusions are governed by a combination of the data and our prior opinion about the parameter of interest.

Table 1. Bayesian Analysis with Equal Prior Probabilities

R	*Prior Probability Of R*	*Conditional Probability Of Drawing Two Red Cards*	*Total Probability Of # of Red Cards in Deck & Drawing Two Red Cards*	*Posterior Probability Of # of Red Cards in Deck*
0	$\frac{1}{5}$	0	0	$0 \div \frac{10}{30} = 0$
1	$\frac{1}{5}$	0	0	$0 \div \frac{10}{30} = 0$
2	$\frac{1}{5}$	$\frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$	$\frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$	$\frac{1}{30} \div \frac{10}{30} = \frac{1}{30}$
3	$\frac{1}{5}$	$\frac{3}{4} \times \frac{1}{3} = \frac{1}{2}$	$\frac{1}{5} \times \frac{1}{2} = \frac{3}{30}$	$\frac{3}{30} \div \frac{10}{30} = \frac{3}{10}$
4	$\frac{1}{5}$	$\frac{4}{4} \times \frac{3}{3} = 1$	$\frac{1}{5} \times 1 = \frac{6}{30}$	$\frac{3}{30} \div \frac{10}{30} = \frac{6}{10}$
Totals	**1**	**--**	**10/30**	**1**

The preceding example can also be performed if a student has a prior opinion that some values of *R* are more likely than others, such as in a case where the student believes the teacher is likely to stack the deck toward red or black. And, even if starting originally with equal prior probabilities, posterior probabilities from one experiment can be used as prior probabilities in a subsequent study. For example, if a participant were to repeat the experiment, his or her posterior probability may be used as the prior probability distribution in a subsequent experiment.

Paul the Octopus

While the previous example is useful to grasp the basic understanding of Bayesian methodology, a real-data example such as Paul the Octopus may be more memorable. Here, we will use data discussed in Smith (2015).

In the years 2008 and 2010, an octopus at a German aquarium named Paul was famously presented with two boxes of food. Prior to soccer matches with Germany's team, Paul was prompted to select one of the boxes. Each time, the boxes were decorated with the flag of Germany and whichever country it was competing against. It was publicized that whichever box Paul chose eat was his pick to win the upcoming match. This data is memorable, as in the years 2008 and 2010, Paul successfully selected the winner of 12 out of 14 matches. In this section, we will use this data to illustrate how students can examine a single situation from frequentist and Bayesian views of probability. In this example, we define π as the true probability that Paul will guess correctly.

From a frequentist perspective, if we tested $H_0: \pi \leq 0.5$ versus $H_A: \pi > 0.5$ using a binomial distribution, our *p*-value would be the probability of at least 12 correct guesses if $\pi = 0.5$. This would equal $p = 0.0065$. This small *p*-value would lead many people to reject H_0, prompting students to conclude that Paul had unique abilities and was not guessing; however, the results garnered by this frequentist methodology fail to incorporate the idea that an octopus with the ability to predict the winner of soccer matches are very implausible. The use of Bayesian practices to analyze this data will allow the implausibility of these events to be incorporated. Based on the subjective probabilities of π, the posterior probabilities may yield results more realistic in this scenario.

For our Bayesian analysis, we will allow for only two possible values of π to simplify the example for understanding. One possibility is that Paul's abilities were just chance ($\pi = 0.5$). The other possibility is that Paul had guessing abilities beyond simple chance ($\pi > 0.5$). For this example exercise, let's assume that if Paul can select the winner with high accuracy, his success rate must be 90%, or $\pi = 0.9$. In practice, the choice for determining the higher accuracy value of π can be decided by discussion among students. Once the two possible values of π are determined, students should state their prior probabilities for both possible values of π. While these prior probabilities can vary by student, most of them should assign higher prior probability on chance, or $\pi = 0.5$. Next, the data that Paul successfully predicted 12 out of 14 games is incorporated into the analysis. A binomial distribution based upon each of the two possible values of π should be used to calculate the probability of the observed data in each case.

Accounting for this new information, students should update their prior probabilities for each of the two options for π. Table 2 illustrates this process for a student who does not believe Paul has any special abilities. This is represented by a prior probability of 99% that Paul is guessing ($\pi = 0.5$) and a 1% prior probability that Paul can predict accurately ($\pi = 0.9$). The conditional probabilities of Paul's selection success are calculated using the binomial distribution, given the stated level of π. For example, if $\pi = 0.5$ and Paul's results are chance, then the binomial probability of 12 successes in 14 independent observations is:

$$\binom{14}{12} 0.5^{12}(1-0.5)^{14-12} = 0.0055$$

Table 2: Tabular Representation of Bayesian Analysis Using Paul the Octopus

π	***Prior Probability Of Event (π)***	***Conditional Probability Paul's Actual Performance (12 of 14 correct predictions)***	***Total Probability Of Event (π) and Paul's Actual Performance***	***Posterior Probability Of Event (π)***
Chance: (π = 0.5)	0.99 probability that $\pi = 0.5$	0.0055	0.0055 x 0.99 = 0.0055	**0.0055/0.00807 ≈ 0.682**
No Chance: (π = 0.9)	0.01 probability that $\pi = 0.9$	0.257	0.257 x 0.01 = 0.00257	**0.00257/0.00807 ≈ 0.318**
Totals	**1**	**--**	**0.00807**	**1**

From Table 2, we see that while the probability of Paul's ability to correctly pick soccer matches increases from 1% in the prior to 31.8% in the posterior, the belief that Paul is guessing ($\pi = 0.5$) still holds over 68% of weight in the posterior probability. Despite a prior opinion that Paul's success is simply chance, his interesting streak of success increases the credibility of the assertion that he is not guessing. However, between the two options, these posterior probabilities still favor the belief that Paul's results are just chance. This is different from the results of hypothesis test discussed earlier. With a frequentist approach, the small value would promote one to reject the hypothesis that Paul is guessing.

Conclusions

These activities introduce students to the fundamental ideas of Bayesian statistical inference. The ideas presented require a different concept of probability, but integrate the existing skills built within an introductory course to learn the basic ideas of Bayesian statistical methods in relatively simple and understandable situations. These activities require students to explore different interpretations of probability, including the frequentist definition, which can enhance students' overall understanding of statistical inference and decision-making. In a frequentist analysis of the Paul the Octopus data, the *p*-value is the same for every participant. In the Bayesian analysis, students with different prior probabilities will have different posterior probabilities, opening up the possibility for class discussion. Additionally, in each example, the data is easy to understand and memorable, either because it was generated in class, or because the data is somewhat surprising and memorable. Utilizing a Bayesian approach for these examples shows students how subjective probability is used to make decisions in a systematic manner, and students are introduced to a different school of thought about probability while reinforcing other class concepts through these examples.

References

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Upcoming NCCTM Regional Conferences

2018 Eastern Regional Mathematics Conference
Saturday, April 14, 2018 at East Carolina University

2018 Central Regional Mathematics Conference
Saturday, February 24, 2018 at UNC Greensboro School of Education

2018 Western Regional Mathematics Conference
Saturday, March 17, 2018 at Owen High School

Check www.ncctm.org for more information later this fall!

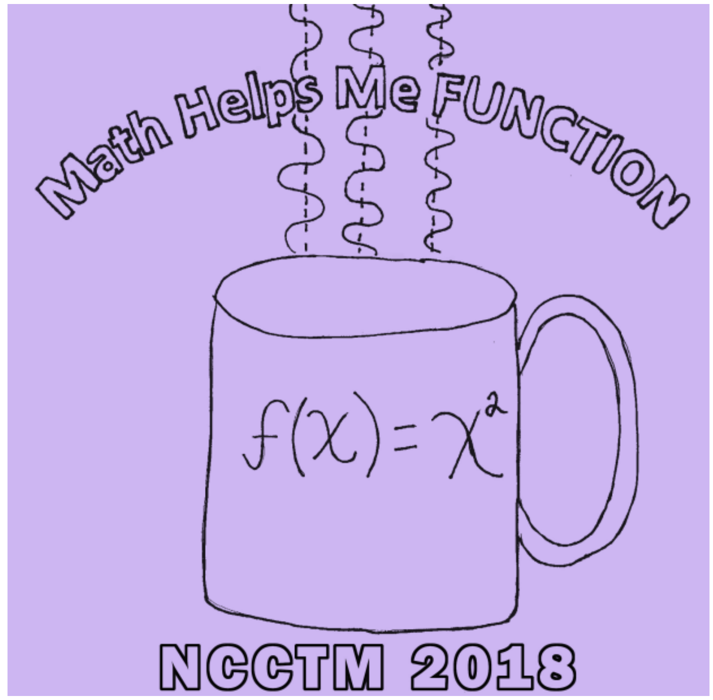
2018 NCCTM Logo Contest Winners

Reported by Anthony Finlen, Asheboro, NC

The Mathematics Logo Contest is held each spring. The NCCTM Board selects the winning logo at its Spring meeting. The 2018 winning logo, pictured, will be available on shirts at the NCCTM State meeting in October.

State Winner: Amelia Miller - 8th Grade - Sampson Middle School, Eastern Region

Other Finalists:



Eastern Region

Jenna Saldana - 10th Grade - Whiteville High School
Lilyana Wilson - 8th Grade - Sampson Middle School
Reagan King - 11th Grade - Ridgecroft School
K. Coloma - 7th Grade - Sampson Middle School
Angie Mejia - 8th Grade - Sampson Middle School
Gabbie Hayes – 10th Grade - Whiteville High School

Central Region

Jordan Snow - 6th Grade - Elkin Elementary
Karleigh Parker - 4th Grade - Seagrove Elementary
Lacie Cockerham - 6th Grade Elkin Elementary
Zoe Brim - 1st Grade - Seagrove Elementary
Mollee Maness - 1st Grade - Seagrove Elementary
Laynee Dennis - 6th Grade - Randleman Middle School
Smith Ray - 6th Grade - Elkin Elementary
Christina Rogers - 10th Grade - East Gaston High School
Daniel Kittrell - 4th Grade - Montlieu Academy of Technology
Thomas Hunt - 4th Grade - Seagrove Elementary
Caiden Garner - 5th Grade - Seagrove Elementary

Western Region

Jacob Key - 12th Grade - Ashe County High School
Julia Barrett - 10th Grade - Ashe County High School
Lyndsi Holman - 12th Grade - Ashe County High School
Rachel Reynolds - 10th Grade - Ashe County High School

Tessellations: Tiling the Plane

Patricia Marie Barrier, Holly Peters Hirst, Appalachian State University, Boone, NC

The authors present an activity that ties the geometry of regular polygons to number sense through building tilings of the plane.

Tessellations—tiling of a surface using geometric shapes with no gaps between the shapes—are all around us in everyday life. Most traditional floor and wall tiles are made from regular polygons, but many non-traditional tiles are possible. Working with regular polygons and two transformations—edge translation and edge half-turns—students can create many interesting tessellations. They can be guided through the basic ideas quickly and then allowed to explore tessellations independently. This article provides an outline for a stand-alone module that has been used effectively in a college quantitative literacy course and can easily be adapted to the school and college curriculum.

Place in the Curriculum

Geometric learning is strongly woven throughout North Carolina's educational curriculum, with explicit geometric curricula for K-12 students transitioning into college courses that offer the possibility for geometric learning more implicitly. The geometry concepts found in this module aligns with the North Carolina Mathematics Curriculum beginning at the first grade level and moving beyond. By first grade, students begin putting shapes together in order to see the way pieces connect. In fourth grade, students begin to identify angles, followed by better angle classification in seventh grade, and coupled with student comprehension of translations, rotations, and reflections in eighth grade. Geometry found in this module again appears at the high school level, in NC Math 2, "where students begin to apply transformational geometry learned in the middle grades to the study of functions. Geometric reasoning and proof are also emphasized in NC Math 2 as students focus on the study of triangles and their relationship to other planar figures" (North Carolina Department of Public Instruction, 2018). In the North Carolina Community College curriculum, courses such as MAT 110 (Math Measurement & Literacy) and MAT 143 (Quantitative Literacy) leave the door open for including material that can help further develop and bridge the geometric exploration from K-12 into adult learning (North Carolina Community College System, 2018).

The common goal for college students entering general education courses is for previous math skills to be solidified while learners gain the ability to express and think about quantitative ideas in a more fluent manner, often called *quantitative literacy*. As advocated by the National Council on Education and the Disciplines in its 2001 report: "Quantitative literacy is more a habit of mind, an approach to problems that employs and enhances both statistics and mathematics….Educators know all too well the common phenomenon of compartmentalization, when skills or ideas learned in one class are totally forgotten when they arise in a different context. This is an especially acute problem for school mathematics, in which the disconnect from meaningful contexts creates in many students a stunning absence of common number sense" (pp. 5-6). Geometric ideas combined with numerical reasoning could be easily forgotten in the context of a quantitative literacy curriculum at the college level. Very often, the easier academic approach is focus on algebraic models, ignoring the importance of inclusion of concepts across the math spectrum when developing quantitative literacy. This tessellation activity allows for a strong connection between geometric concepts and number sense.

Exploring Regular Tilings

Most tessellations that use regular polygons edge-to-edge on floors and walls are composed of tiles shaped like equilateral triangles, squares or regular hexagons. Clearly, these three shapes work well, because when they are set edge-to-edge their corners meet without any gaps between tiles (Fig. 1). How might we determine what other regular polygons could be used in this manner? How well the polygonal tiles fit together at a *vertex* (corner) will depend on the size of the interior angle for the polygon.

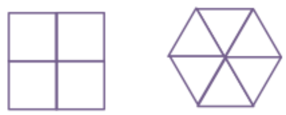


Figure 1. Common regular tiles that tessellate.

Most students know the interior angles for two regular polygons:

- Equilateral Triangle (3 sides): angle = 60° (there are three interior angles and they must add to 180°). These triangles tile the plane because six of them meet at a corner and $6 \times 60 = 360$, so the angles that meet at a corner fill an entire 360° rotation.
- Square (4 sides): angle = 90° (each corner forms a right angle). Squares tile the plane because four of them meet at a corner and $4 \times 90 = 360$.

What about other regular polygons? There are many ways to determine the interior angle of a regular polygon. Let's start with a general case: a regular *n*-gon (i.e., an *n*-sided regular polygon). If we walk around the figure, we should have rotated 360°. So if we walk along the edges of a regular *n*-gon starting at a corner and ending at the same corner facing in the original direction, the total number of degrees in all of the turns must add to 360°, a full rotation. Each of these turns is the same size as an *exterior angle of the regular n-gon* (Fig. 2).If there are *n* sides, then there are *n* equal exterior angles, which implies that each exterior angle is $\left(\frac{360}{n}\right)°$.

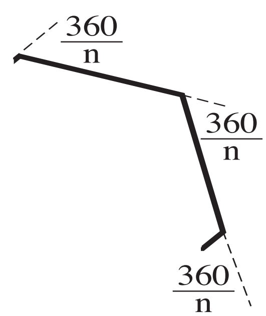


Figure 2. Exterior angles of a regular n-gon.

At each corner of the polygon, the interior and exterior angles together add up to 180°, so the interior angle for a regular *n*-gon is:

$$\left(180 - \frac{360}{n}\right)°$$

Physically walking around a regular polygon that has been marked out on the floor with painter's tape can help students to visualize this idea.

Using the formula for the interior angle, students can complete Table 1. Suggested follow-up questions for the students are: *What appears to be happening to the interior angle as the number of sides increases? How big do you think the interior angle can get if the table is continued?* A few hints help students to arrive at the idea that the interior angle approaches, but never reaches, 180°. If the angles are such that the corners can meet with no gaps, we can create a tessellation. Since rotating completely around once is equivalent to turning 360°, if the interior angle measurement evenly divides 360 we can use that regular polygon to tile the plane. By examining the information in Table 1, students can now determine which of those listed could tile the plane. Indeed, triangles, squares, and hexagons all work, but none of the other interior angle dimensions in the table evenly divide 360.

Table 1. The Exterior and Interior Angles of Regular Polygons

Shape	Number of Sides	Exterior Angle	Interior Angle
Triangle	3	360/3=120	60
Square	4	360/4=90	90
Pentagon	5	360/5=72	108
Hexagon	6	360/6=60	120
Heptagon	7	360/7=51 3/7	128 4/7
Octagon	8	360/8=45	135
Nonagon	9	360/9=40	140
Decagon	10	360/10=36	144
Hendecagon	11	360/11=32 8/11	147 3/11
Dodecagon	12	360/12=30	150

This leaves us with the question: *What about regular polygons with more than 12 sides?* Posing this question to students can yield some interesting discussions. One way to help them see that larger regular polygons won't tessellate is to list the all of the factor pairs of 360 (Fig. 3). Only three have factors in the list of interior angles in Table 1—triangle, square, and hexagon—and there is no reason to go beyond interior angles of 120°, since there is no larger factor of 360 other than 180 (an "interior angle" of 180° would be a straight line and so not a polygon).

$$1 \times 360, 2 \times 180, \mathbf{3 \times 120}, \mathbf{4 \times 90},$$
$$5 \times 72, \mathbf{6 \times 60}, 8 \times 45, 9 \times 40,$$
$$10 \times 36, 12 \times 30, 15 \times 24, 18 \times 20$$

Figure 3. Factor pairs of 360.

Tiling with Irregular Polygons

Suppose we create a polygon for which the interior angles are not all the same size. Which "irregular" polygons could tessellate? We will assume that the polygons are planar (no crossing sides; Fig. 4), to be sure the polygon could still be considered a 2-dimensional tile in the usual sense.

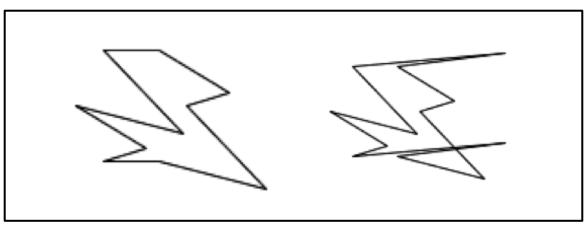


Figure 4. A planar (left) and non-planar (right) polygon.

Triangles and quadrilaterals always work as tiles, even when they are not regular. Asking students to explore this idea by cutting out random triangles and quadrilaterals can lead to some interesting discoveries. Marjorie Rice discovered that, while regular pentagons don't work, some irregular pentagons can tessellate (Sharon's Place, 2013). Perhaps most famously, Maurits C. Escher worked with regular and irregular tiles (M. C. Escher Company, 2018b). Both Rice and Escher used their tessellations to create beautiful images, some abstract and others representing natural objects.

Building Tiles and Tessellating

What can we do to build interesting images like those of Escher and Rice? Two easy-to-learn transformations that can generate beautiful tessellations are edge translations and edge half-turns (Fig. 5). Starting with any polygon that tiles the plane:

- *Edge translation:* Cut out a shape from one edge and translate it to a parallel edge, and the shapes will still fit together. The shapes will remain facing in the same direction.
- *Edge half-turns:* Cut out a shape from half of an edge and rotate it to the other half of the edge, and the shapes will still fit together. The shapes will need to be flipped to fit together. This translation does not require parallel edges and so works with triangles.

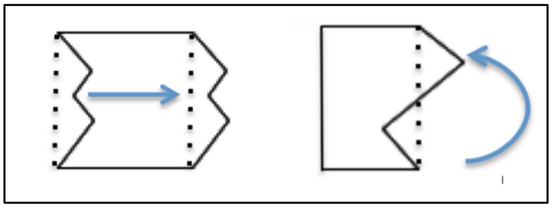


Figure 5. Edge translation (left) and half-turn (right).

Here is a simple example to share with students, which creates a chicken tessellation (Fig. 6).

1. Start with a square.
2. Cut a shape from the top and translate to the bottom for the leg.
3. Cut a shape from the right and translate to the left for the beak.

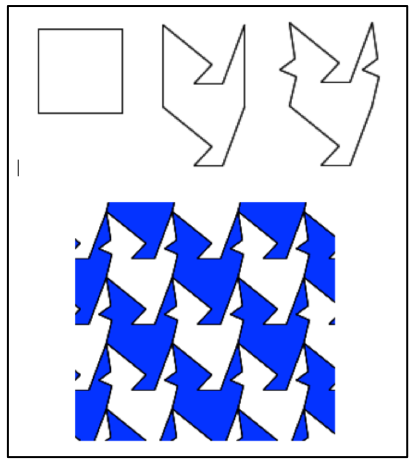


Figure 6. Chicken tessellation.

In the Classroom

We have used this material in our quantitative literacy course *Introduction to Mathematics*. After discussing the ideas outlined above with the students and having them explore some of the Rice and Escher illustrations, we have the students experiment with creating their own tiles using paper and scissors. Then for the main activity, the students are given directions on preparing posters to present to the class (Fig. 7), after which the class votes on the most interesting creation. Grades are based on completing the task correctly; creativity is rewarded—for us, either a small number of bonus points or gift certificates to the local coffee shop work well.

(1) Select a regular polygon that tessellates. Your polygon should be between three and four inches on a side. Create a new shape using at least two transformations (edge translations and/or edge half turns).

(2) Construct an 11x17 poster illustrating a tessellation using your figure. Be sure your figure repeats enough on the poster to show how the figure fits together in a repeat pattern. If you wish, you can embellish your poster with color and other accents to make your tessellation represent an object à la Escher. Feel free to be creative! The class will be voting on the most interesting tessellation.

(3) Bring cut outs of the original polygon and the resulting shape along with your poster to class and be prepared to explain what you did. You will have two minutes to describe your transformations and show your poster to the class. Each student will have a voting sheet to vote on the most interesting tessellation. Prizes will be awarded!

Figure 7. Directions for the tessellation project.

Results

Along with other colleagues in our department, we have used this module in our quantitative literacy course for many years. Our students really like this project because it allows them to merge their creative thinking with geometric reasoning and analytical communication in a hands-on way, emphasizing the beauty of math. They get excited about the opportunity to express themselves freely in a math course, and have produced some very creative posters. Students routinely comment on this project being one of their favorite learning activities. A few of the posters from the last several years are pictured in Figures 8 and 9.

Figure 8. Examples of student work.

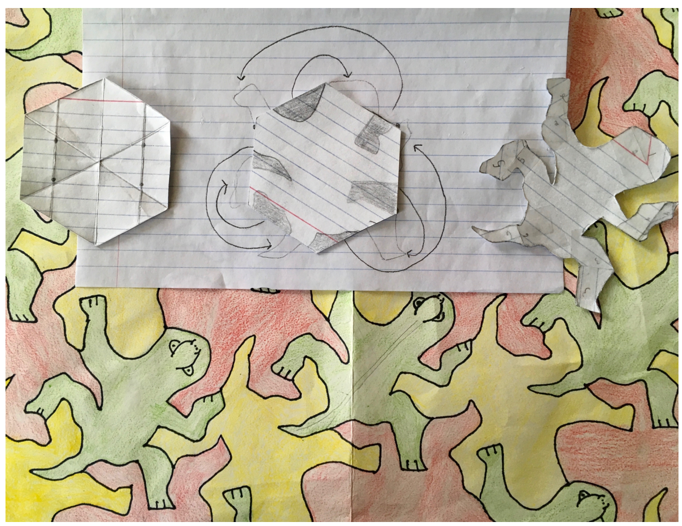


Figure 9. A student imitating Escher's Lizard No. 56 (M. C. Escher Company, 2018a).

We have also used a number of variations and extensions of this activity. There are other transformations that preserve tiling, and students are good at discovering them. Semi-regular tilings—those that use two or more regular polygon shapes in combination—are very interesting; Kepler showed there are only eight combinations (Jardine, n.d.). Online applets that allow experimentation without using paper can be used to create some beautiful images to post online (e.g., Shodor's *Tessellate!* Applet; Shodor Foundation, Inc., 2018).

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Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Rose Sinicrope, sinicroper@ecu.edu.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from Lee V. Stiff, lee_stiff@ncsu.edu.

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website, www.ncctm.org. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick at **jmccormick@rock.k12.nc.us** or Sandra Childrey at **schildrey@wcpss.net**, with questions.

ASU Math Walk: Promoting STEM using Creative Activities

Sharareh Nikbakht, Appalachian State University, Boone, NC

The author presents the STEM outdoor activity, ASU Math Walk, and describes how teachers could create a math walk at their own school.

Integration of science, technology, engineering, and mathematics (STEM) into the K-12 curriculum has been the focus of many research projects in recent years (e.g., Stohlmann, Moore, & Roehrig, 2012). This integration not only improves math and science literacy, but also enables students to become critical thinkers and future innovators (Craft & Capraro, 2017). Using real-world problems while teaching math subjects creates excitement among students and gets them engaged. Development of creative and interesting K-12 STEM activities can be challenging, and often teachers do not find the time, support, and resources needed to create such activities. This article provides details on a set of STEM activities that have been created based on the landmarks, natural resources, buildings, and statues located on the Appalachian State University campus, along with some guidelines for teachers who want to develop similar activities at their schools or other places they visit regularly with students. All of the activities and the solution manual have been organized in PDF files (Nikbakht, Moore, & Illenye, 2017), which can be accessed on the author's web site (appstate.edu/~nikbakhts) and downloaded by teachers to use with their students. Since most of the activities have all the required measurements included in them, those activities can be solved by students from anywhere, even before a visit to campus.

Although the activities are created using the landmarks of the Appalachian State University campus, the approach can be adapted by teachers to create similar activities to include landmarks that are of significance in their respective schools or region as well as places of interest such as a university they regularly visit. The purpose of all the activities is to promote STEM literacy and critical thinking among students.

Creating Activities

Every year many K12 students visit Appalachian's campus to attend outreach activities or benefit from our resources. We wanted to provide these students with an opportunity to solve different STEM problems as they walk through campus. We chose the landmarks used for the activities not only to represent the campus well, but also to provide activities for a range of grade levels. We paid particular attention to accessibility, diversity of geometric shapes, and applicability to math and science concepts when choosing sites. Some of the activities provide background information along with the data and measurements to create interest in students and help them gain knowledge. An example of such an activity is the wind turbine problem (Nikbakht, Moore, & Illenye, 2017). The activity not only provides background information about the wind turbine, but also includes a link to a web site with live data which tracks the energy generated by the turbine.

We included measurements in the descriptions of activities because it might not be practical for students to make those measurements. Furthermore, this allows students to save some time on the measurements and focus more on the solutions. As a result, they get a chance to go through more activities during their limited time on campus. There are some measurements that are within reach and can be verified by students using simple tools if they wish to do so. Teachers who decide to adopt this approach and design similar sets of problems in their own school should consider letting students do their own length and angle measurements.

As teachers look for sites in their area for developing STEM activities, they should consider the following questions:

1) Can concepts from biology, chemistry, physics, engineering or technology be tied to the site?
2) Is there interesting geometry at the site to investigate?
3) What are some of the measurements that students could do themselves? Are there any assumptions about the measurements that students need to consider?

The ASU Math Walk Handout

The handout for "ASU Math Walk" (Nikbakht, Moore, & Illenye, 2017) starts with the location of the fourteen structures used in the activities, which are marked on a campus map. There is a recommended path (the activity trail) illustrated on the map to help guide teachers on where the walking paths are located. The activity trail allows teachers to select locations on the trail that are on their path to their events or are closest to drop off or pick up locations. The handout includes information on preparing for the walk. For example, teachers and students are advised to bring some basic measurement tools such as a ruler with them. As students visit a location for an activity, they will be encouraged to solve the problem and have a group discussion about the solution before moving to the next one. The handout allows students to explore, have fun, and celebrate the beauty of STEM and its presence all around them.

When using the ASU Math Walk on our campus, we strongly recommend that teachers select the sites they want to visit in advance based on the level of their students and the path they are taking to their final destination on campus. It might be a good idea for the teachers to talk to their students as they walk to a site in order to generate some excitement about the problem they are going to solve. For example, before getting to the Duck Pond on ASU campus (see Example 2 below), teachers can start a conversation about the role of a retention pond and harmful chemicals that contaminant our creeks and rivers. Also, they can briefly mention the purpose and goal of the activity.

Example Activities

At the end of this article, we have included four activities that integrate math with a science concept to help teachers think about selecting their own sites for activities: Example 1 involves estimating the volume of "The Rock" in front of Kidd Brewer Stadium (volume and weight measurements); example 2 asks students to determine the amount of a contaminant in the famous "Duck Pond"(concentration of sediments and unit conversion); example 3 guides students through approximating the height of the statue of the school mascot "Yosef" at the cross section of Rivers Street and Stadium Drive (estimation, height and angle measurements); and example 4 asks students to calculate the volume enclosed by the art installation near Walker Hall (geometry and unit conversion).

Conclusion

As the examples indicate, each activity is designed to teach a concept by engaging students in developing the problems and working together to come up with solutions. The approach can be adopted by teachers to include landmarks that are specific to their locations. We believe that students enjoy the challenge of working together to solve the problems at each site, and hopefully get inspired to come up with ideas for new problems related to that site.

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Example ASU Math Walk Activities

Example 1: The Rock

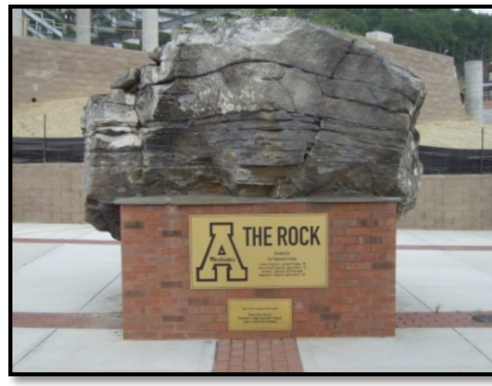


Figure 1. The Rock at Kidd Brewer Stadium.

Purpose: The purpose of this activity is to promote critical thinking by helping students practice their math and geometry skills.

Materials needed: Pencils, papers, calculators

Kidd Brewer Stadium's nickname is ***The Rock,*** but now there actually is a rock where students meet their friends for football games! While "The Rock" is impossible to measure very accurately with a tape measure, approximations for its dimensions are given in Figure 2. It is roughly 84" x 106" on top and has a front height of approximately 48" and a back height of approximately 64". We may consider shape of The Rock as a rectangular solid with a triangular wedge under it. It is made of granite, which has a density of approximately 2.70 g/cm^3.

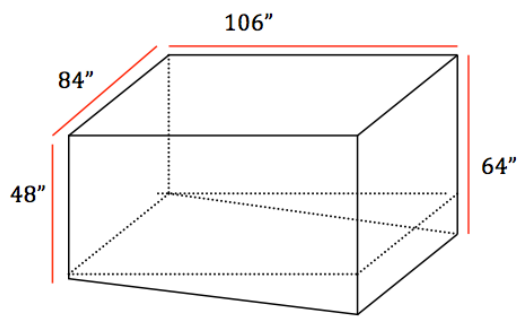


Figure 2. A rough diagram of The Rock's dimensions.

Task:

- Grades 6-8: What is the volume in cubic inches of "The Rock"?
- Grades 9-12: Using this information, what would you estimate "The Rock" to weigh in pounds? In kilograms? Note that 454 g = .454 kg = 1 lb and 2.54 cm = 1 inch.

Example 2: The Duck Pond

Figure 3: The Duck Pond.

Purpose: The purpose of this activity is to help students practice unit conversions and generate discussion questions about the role of a sediment retention pond that filters out debris.

Materials needed: Pencils, papers, calculators
Every student who desires to earn a chemistry degree from Appalachian State University (ASU) must complete a course in quantitative analysis. While in this course these students complete many experiments that deal with “Duck Pond and Boone Creek,” most of the students at ASU do not realize that the duck pond has a purpose other than aesthetics. Actually, it is a sediment retention pond that filters out debris and other harmful contaminants. One of the contaminants that is tested each semester is paraben levels. Parabens are found in many products such as soaps, lotions and cosmetic products. The levels are checked in both the Duck Pond and Boone Creek to determine the effectiveness of the retention pond. So, let’s do some math. The quantitative analysis class here at Appalachian State University just conducted a recent experiment (Spring of 2017) and found that there is 1 ppm of methylparaben in the duck pond and 0.3 ppm in Boone Creek. This shows that the Duck Pond is doing its job.

Task: If the duck pond contains 35,000 gallons of water, how many grams of methylparaben are in the duck pond? Some equivalencies you may need:

$$1 \text{ ppm} = 1 \text{ g/m}^3$$
$$1 \text{ mL} = 1 \text{ cm}^3$$
$$1 \text{ gallon} = 3.79 \text{ L}$$
$$1 \text{ m} = 100 \text{ cm}$$

Example 3: Measure Yosef!

Purpose: The purpose of this activity is to promote critical thinking by helping students practice their Math and geometry skills.

Materials needed: Pencils, papers, calculators, measuring tape

Task:

1) Go to the Yosef statue and see how you measure up. How many times taller do you think Yosef is than you? Write down your guess on the line before you continue. I think Yosef is ________ times taller than me.

Figure 4: The Yosef’s Statue. From the ground to Yosef’s belt is 80 inches.

2) Yosef is so tall we couldn't figure out a way to measure his whole height without using a ladder! Instead, we measured the length of his leg to his belt (the yellow line on the photo). If you measure from the ground to where your hip bends, that should be approximately 50-55% of your height. Of course, some people are "all leg" and some are "all body" while others are about half and half. Do you know where most of your height is?
3) Measure your ground-to-hip height and figure out what percentage that is of your height. To figure out the ratio of Yosef's height to yours, you can either compare his ground-to-hip height to yours or, you can compare Yosef's total height (using 55% as the percent his legs are of his total height) to your total height.
4) Calculate the following ratio:

$$\frac{\text{Yosef's ground} - \text{to} - \text{hip height}}{\text{Your ground} - \text{to} - \text{hip height}} \text{ OR } \frac{\text{Yosef's total height}}{\text{Your total height}} = \text{How many times taller Yosef is}$$

According to trackstarusa.com/long-stride-length/ the optimum stride length for someone running as fast as they can is 2.3-2.5 times their leg length for females and 2.5-2.7 times their leg length for males. What would your optimum stride length be? What would Yosef's be?

Example 4: The Volume of Art

Purpose: The purpose of this activity is to promote critical thinking by helping students practice their unit conversions and geometry skills.

Materials needed: Pencils, papers, calculators.

Many types of art have math that is involved in them, from design or from perspective. One such piece of art gives us the opportunity to refine some of our skills as mathematicians to understand volume. Figure 5 is a picture of an art display located on the east side of the Walker Hall.

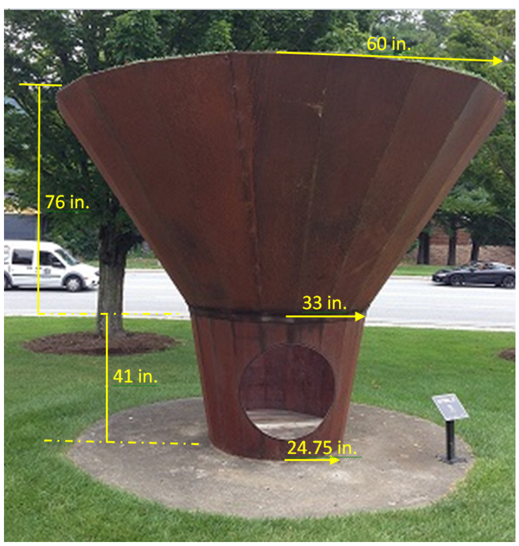
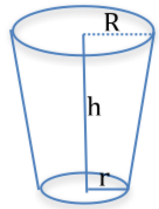


Figure 5: The art display at the east side of Walker Hall.

This piece has been displayed since June 2017. When looking at this piece of art we can see it is made up of multiple circular cones. The following formula is used to find the volume of a circular cone as pictured on the next page:



$$V = \frac{\pi}{3}h(R^2 + Rr + r^2)$$

Figure 6: The volume of a right circular cone.

Task:

1) Calculate the volume of this art piece in cubic inches, using the measurements given to you in Figure 5.
2) What is the volume of this shape in cubic feet?

Acknowledgements

The author wishes to thank the Department of Mathematical Sciences at Appalachian State University for support of the project. Special thanks go to Ms. Stephanie K. Moore and Mr. Kory Illenye for their contributions to the activities.

NCTM Announces New Grant for High School Mathematics Educators

Reston, Va. - September 12, 2018 - The National Council of Teachers of Mathematics (NCTM) has announced a new scholarship for educators currently teaching grades 9-12.

The scholarship, Advanced Mathematics Education Course Work Scholarship for Grades 9-12 Teachers, will provide financial professional learning support to improve competence in the teaching of mathematics by supporting students completing advanced course work in mathematics education. Advanced course work may include graduate courses or senior level undergraduate courses.

NCTM's Principles to Actions: Ensuring Mathematical Success for All suggests that creating effective classrooms and learning environments requires leaders committed to supporting sustained professional development that engages teachers in continual growth of their mathematical knowledge for teaching, pedagogical content knowledge, and knowledge of students as learners of mathematics.

The scholarship of $3,000 will be awarded in February 2019. NCTM is seeking applicants who are interested in enhancing their knowledge and competence in teaching mathematics and making an impact on their students' learning. The deadline for applications is November 2, 2018.

This scholarship is jointly supported by the Carol A. Edwards Fund and NCTM. For more than 55 years, Dr. Edwards's passion for mathematics education inspired mathematics students and teachers through mentorship and leadership.

For more information about applying or other funding opportunities, please visit www.nctm.org/Grants/.

2018 State Math Fair Winners

Reported by Betty Long, Appalachian State University, Boone, NC

NCCTM sponsors three regional Math Fairs each spring, and the best projects presented at these regional Fairs qualify for the State Math Fair. This year's State Fair was held at the North Carolina School of Science and Mathematics on 4 May 2018. The following students were selected for top honors in each division.

Primary Division, Grades K-2

1st Place: Camden Alford, "Marshmallow Math," Wrightsville Beach Elementary School, Wrightsville Beach
2nd Place: Della Ruth Koster & Loulie Harrison, "Door to Door Distance," Elm City Elementary School, Wilson
3rd Place: Levi Alderete, "Guess My Number," Woodland Heights Elementary School, Mooresville

Honorable Mentions:
Serah Ann Mobin, "My Birthday Planning Project," Coddle Creek Elementary School, Mooresville
Ema Garcia, "Math in Computational Science," Snow Hill Primary School, Snow Hill
Kaydence Moore, "Hoverboarding Kids vs. Adults," Poplar Springs Elementary School, King

Elementary Division, Grades 3-4

1st Place: Grace Young, "Probability: It's a Piece of Cake," Parkway Elementary School, Boone
2nd Place: Arjun Patel & Landon Bruck, "Ballumes," Woodland Heights Elementary School, Mooresville
3rd Place: Kate Bowling & Reese Wilson, "Plastic + Ocean = Problem," Woodland Heights Elementary School, Mooresville

Honorable Mentions:
Morgan Davis & Sylver Best, "Disney By the Numbers," Greene County Intermediate School, Snow Hill
Evan Crater, "Can You Get More Eggs for Peanuts?," Troutman Elementary School, Troutman

Intermediate Division, Grades 5-6

1st Place: Rylee Greene, "Moving at the Speed of Tennis," Greene County Intermediate School, Snow Hill
2nd Place: Avery Mays, "The Adventures of Starman," North Windy Ridge Intermediate School, Weaverville
3rd Place: "Easton Creech, "Down Set Hut," Greene County Intermediate School, Snow Hill

Honorable Mention:
Peyton Surridge, "Dolphin Facility," Mountain View Elementary School, Hickory

Middle School Division, Grades 7-8

1st Place: Jayden Brown & Akshar Patel, "Ahead of the Hurricane," J. N. Fries Magnet School, Concord
2nd Place: Rolando Hernandez & Jeannette Graham, "What Are the Odds?" South and North Asheboro Middle Schools, respectively, Asheboro
3rd Place: Lane Schafer, "Pick's Theorem," Lakeshore Middle School, Mooresville

Honorable Mentions:
Colin Hanes, "Effect of Spacing on Diffraction: A Double Slit Experiment," Carnage Magnet Middle School, Raleigh
Safiya Khambaty, "Math is Flipping Amazing!," Hope Middle School, Greenville
Michael Giurcanu, "Grids Meet Computer Software," Hope Middle School, Greenville

High School Division, Grades 9-12

1st Place: Arvin Singh, “Sharing Secrets,” Asheboro High School, Asheboro

2nd Place: Jayden S., “Creating a Hip-Hop Beat Using the Note to Number Method of a Fibonacci Sequence and Modular Arithmetic,” Stonewall Jackson Youth Development Center, Concord

3rd Place: Taten B., “Dream Design by Charlie Inc.,” Stonewall Jackson Youth Development Center, Concord

Honorable Mentions: Valerie Kitchell , “Predictive Policing: Exploring the Accuracy of Crime Indicator Models,” Watauga High School, Boone

Kaia Markert, “Alternative Representation of Quotients in Division of Polynomials,” Southwest Guilford High School, High Point

Applying for Trust Fund Scholarships

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

The Trust Fund Committee is pleased to announce that the amount that can be requested to help with the cost of graduate coursework is now $1000.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (www.ncctm.org). More information can be obtained from Janice Richardson, richards@elon.edu.

Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
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Problems2Ponder

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Starting in 2018, the Problems to Ponder column has received a face lift. The new Problems2Ponder will present problems similar to those students might encounter during elementary and middle school olympiad contests.

Student submissions are still welcome as are problem submissions from teachers. Please consider submitting a problem or a solution! Enjoy!

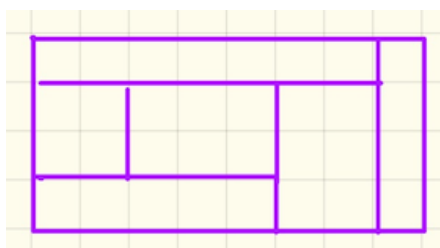
Problem submissions are welcome! If you have an idea for a problem to publish, please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of a typed or neatly written problem statement, along with a solution. Include your name and school affiliation so that we can credit you with the submission.

Solution submissions are welcome! In particular, if teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of *The Centroid*. Please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of the correct solution, with the name of the school, the grade level of the student, the name of the students (if permission is given to publish the students' names), and the name of the teacher.

Deadline for publication of problems or solutions in the Spring 2019 Centroid: January 1, 2019.

Fall 2018 P2P Problems

Problem A: A large rectangle is cut into smaller rectangles as pictured. How many rectangles of all sizes are in this diagram?



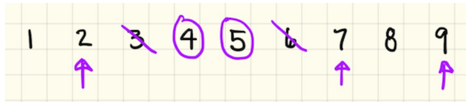
Problem B: If we have 21 cards, how many different ways can we organize them in 3 piles if the piles must all contain an odd number of cards?

Solutions will be posted in the next edition of *The Centroid*.

Spring 2018 P2P Problem Solutions

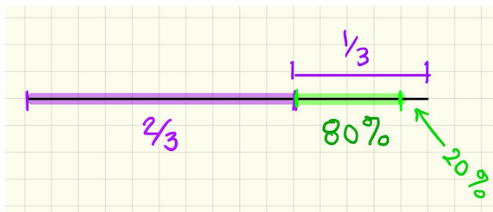
Problem A: Jerrod chose 5 numbers from the list 1, 2, 3, 4, 5, 6, 7, 8, 9. Two of the numbers he chose were 4 and 5. If these are the only two numbers chosen that differ by 1, what is the largest sum of the 5 chosen numbers?

> **Solution:** Because 4 and 5 are included and no other numbers can differ by 1, we cannot use 3 or 6. Based on that and choosing the highest possible numbers spaced at least 1 apart, the answer is 27 using 2, 4, 5, 7, and 9 as illustrated on the next page.



Problem B: On a trip to the local amusement park, Gwyn agreed to drive the first 2/3 of the total distance if Shan drove the rest of the way. After changing drivers, Gwyn fell asleep and when she woke up Shan had 20% of her part of the drive left. For what percent of the whole trip was Gwyn asleep?

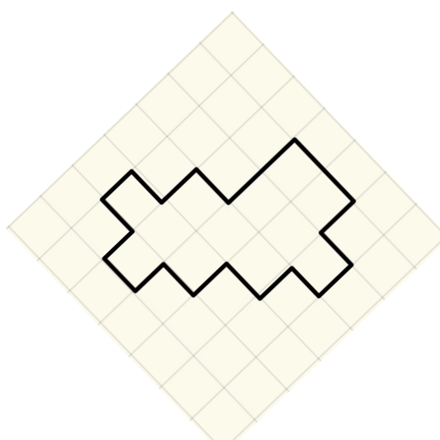
> **Solution:** Since Gwen agreed to drive 2/3 of the distance, Shan drove 1/3 of the distance. If Shan had 20% left when Gwen woke up, Gwen slept for 80% of Shan's part of the drive as illustrated below. Thus the answer is 80% of 1/3 of the drive or 26.7%



Puzzle

Martin Gardner, the great recreational math puzzle genius would have turned 104 on October 21, 2018. In honor of his birthday this fall, here is one of his classic puzzles to think about! For more about Martin Gardner, a solution to this puzzle, and a few more puzzles to think about, check out Alex Bellos' article in the Guardian referenced below!

The Crazy Cut Problem: Make one cut (or draw one line)—of course it need not be straight!—that will divide the figure into two identical parts.



Reference:

Bellos, A. (2014, October 21). Can you solve Martin Gardner's best puzzles? *The Guardian* [online]. Retrieved from www.theguardian.com/science/alexs-adventures-in-numberland/2014/oct/21/martin-gardner-mathematical-puzzles-birthday

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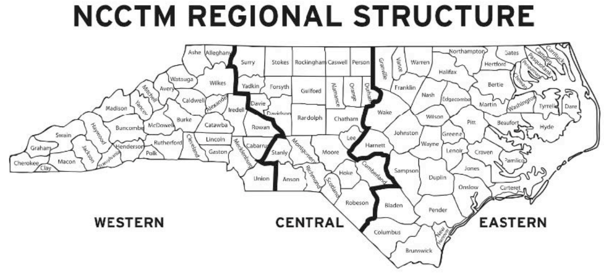

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