# The Centroid 

The Journal of the North Carolina Council of Teachers of Mathematics

## In this issue:

Using Leslie Models Built from Scientific Data to Illustrate Matrix Arithmetic

Zooming Through Number Talks: Considerations for Virtual Instruction

Games A-'Round’ the Classroom: Supporting Rounding Strategies in Third Grade

Problems to Ponder


The Centroid is the official journal of the North Carolina Council of Teachers of Mathemaics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics-pre-kindergarten through college levels. The Centroid is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

## Submission of News and Announcements

We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

## Submission of Manuscripts

We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- A Teacher's Story,
- History Corner,
- Teaching with Technology,
- It's Elementary!
- Math in the Middle, and
- Algebra for Everyone.


## Guidelines for Authors

Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Submit one electronic copy via e-mail attachment in Microsoft Word or rich text file format. To allow for blind review, the author's name and contact information should appear only on a separate title page.

Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

## Formatting Requirements

- Manuscripts should be single column, double-spaced with one-inch margins and should not exceed 10 pages.
- Tables and figures should be numbered consecutively with a title and included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the Publication Manual of the American Psychological Association.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article. Citations and references should APA $7^{\text {th }}$ edition. Reference examples:

Books and reports:
Bruner, J. S. (1977). The process of education (2nd ed.). Harvard University Press.
National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Journal articles:

Perry, B. K. (2000). Patterns for giving change and using mental mathematics. Teaching Children Mathematics, 7, 196-199.
Chapters or sections of books:
Ron, P. (1998). My family taught me this way. In L. J. Morrow \& M. J. Kenney (Eds.), The teaching and learning of algorithms in school mathematics: 1998 yearbook (pp. 115-119). National Council of Teachers of Mathematics.

## Websites:

North Carolina Department of Public Instruction. (1999). North Carolina standard course of study: Mathematics, grade 3. http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

# The Centroid 

## Editorial Board

## Editors

Deborah Crocker
Appalachian State University
Holly Hirst
Appalachian State University

Board Members
Betty Long
Appalachian State University Jill Thomley
Appalachian State University Solomon Willis
Cleveland Community College

About the Cover
The image on the cover depicts the centroid of a triangle.

Copyright
Educators are granted general permission to photo-copy material from The Centroid for noncommercial instructional and scholarly use.
Contact the author(s) concerning other copying.

Contact Information
The Centroid
c/o Dr. Deborah Crocker, Editor
Department of Mathematical
Sciences
Appalachian State University
Boone, NC 28608
or send email to
CrockerDA@appstate.edu
Please include a return email address with all correspondence.

An advertisement in The Centroid does not constitute endorsement by NCCTM, and the opinions expressed or implied in this publication are not official positions of NCCTM unless explicitly noted.

The Journal of the North Carolina Council of Teachers of Mathematics


Volume 47, Issue 1 - Fall 2021

## Table of Contents

President's Message ..... 2
Using Leslie Models Built from Scientific Data to Illustrate Matrix Arithmetic ..... 3
Zooming Through Number Talks: Considerations for Virtual Instruction. ..... 7
Problems to Ponder ..... 12
Games A-‘Round’ the Classroom: Supporting Rounding Strategies in Third Grade ..... 13

## Fall 2021 Leadership Seminar Is going Virtual

It is with great regret that the Fall 2021 State Math Conference had to be cancelled to ensure that NCCTM doesn't contribute to the spread of COVID.

We are still holding the Fall Leadership Seminar on November 10, but now it will be through virtual attendance! Please join us for up to date information and professional development on that day!

Watch https://www.ncctm.org/conferences/leadership-seminar/ and Facebook for more information coming soon!

# President's Message 

Stefanie Buckner Hill<br>State President<br>stefanie.hill@bcsemail.org

It is with a heavy heart that we have cancelled the in-person November 2021 State Mathematics Conference. We were hopeful for a face-to-face conference this November. We had a full conference experience planned and were looking forward to being back together again. In addition to planning, the conference committee has kept a dutiful eye on statewide Covid-19 metrics and the operating status of school districts across North Carolina. We also recognize that professional release time for classroom teachers has become increasingly difficult for many schools and school districts. NCCTM's commitment is to NC's classroom mathematics teachers, and it is important that when we have a conference, all roles in our math education family are able to attend. After much deliberation and evaluation of these data points, we feel like meeting face to face in a large group setting is not in the best interest of our membership and organization.

The Conference Committee has pivoted and is working on plans for two virtual experiences. Details are limited at this time but will be shared as they evolve!

First - We are planning a virtual Leadership Meeting to be held on Wednesday, November 10th. This will be an engaging virtual experience with keynote speakers and break out rooms by grade bands. Details will be released by the end of September!

Second - and perhaps most exciting - is a "re-envisioned" virtual conference. This event will be fully online in the early spring of 2022. Sessions will be held across multiple days at various times. We are working on a sliding registration fee schedule according to the number of sessions you are interested in attending. CEU credits will be awarded accordingly.

## Follow NCCTM and NCTM On Social Media

NCCTM and NCTM are on Facebook Instagram, and Twitter! Please like and follow us for announcements about the Council and other interesting information!


NCCTM: https://www.facebook.com/NC.Math.Teachers
NCTM: https://www.facebook.com/TeachersofMathematics/


NCCTM: https://www.instagram.com/ncctm1
NCTM: https://www.instagram.com/nctm.math


NCCTM: https://twitter.com/ncctm1?lang=en
NCTM: https://twitter.com/nctm?lang=en

# Using Leslie Models Built from Scientific Data To lllustrate Matrix Arithmetic 

Holly Hirst, Appalachian State University, Boone, NC

> The author provides an introduction to Leslie matrix population models, with suggestions on where to find realistic population data and matrix arithmetic calculation technology to use with students.

In Fall 2020, teachers implemented the three newly revised "fourth" courses in the North Carolina mathematics curriculum. The standards for each course now include vector and matrix arithmetic, as shown in the unpacking documents (NCDPI, 2020 a, b, c). All of the courses include an introduction to matrix multiplication, with the new version of discrete mathematics including applications to Leslie population models and Markov chains (Table 1).

Table 1. Excerpts from the unpacking documents for the revised courses

| Math 4 | Students will understand that the structure (rows and columns) of a matrix <br> determines how matrices can be combined through addition, subtraction, <br> and moltiplication (including scalar multippication) and use that structure to <br> perform the aforementioned operations with matrices (NCDPI, 2020b, <br> p.4-5). |
| :--- | :--- |
| Precalculus | Students should recognize that in order to multiply two matrices the <br> number of columns in the first matrix must be equal to the number of rows <br> in the second matrix. They should reason about the dimensions of <br> matrices to determine the dimension of the product (NCDPI, 2020c, p. 6). |
| Discrete <br> Mathematics <br> for <br> Computer <br> Science | [SJudents are expected to add, subtract, and multiply matrices in a <br> context. This context can include, but is not limited to, real world <br> problems, the transformation of points on a coordinate plane, Leslie <br> Models and Markov Chains....For both Leslie Models and Markov Chains, <br> students should be able to organize the data into the appropriate matrices <br> (NCDPI, 2020a, p. 4). |

Leslie population models can be used to provide students with a context for matrix multiplication, or more particularly matrix-vector multiplication. While precalculus students are "not expected to...solve problems involving contexts" (NCDPI, 2020c, p. 5), Leslie models could still serve as a motivation for why matrix multiplication is useful in real applications of mathematics. In addition, there is a wealth of data available from actual population studies that can be used to create the Leslie matrix, further demonstrating the utility of matrices in science. In this paper, we provide examples of Leslie models created from actual studies.

## What is a Leslie Model? A Simple (Real) Example

A number of texts on linear algebra, discrete mathematics, and math modeling (e.g., Lay et al., 2016) use data on spotted owls (Lamberson et al., 1992) as a motivating example when introducing Leslie models. In the Lamberson et al. paper, the authors divide the owl population into three age classes: juvenile (up to 1 year old), sub-adult (1 to 2 years old), and adult (over 2 years old). The population is examined at yearly intervals. Since it is assumed that the number of male and female owls is equal, only female owls are counted in the analysis. If there are $j$ juvenile females, $s$ sub-adult females, and a adult females now, then the population of owls can be modelled over time with the following assumptions:

- juveniles and sub-adults do not reproduce
- adults produce 0.33 juveniles per female on average each year
- $18 \%$ of juveniles survive to sub-adulthood
- $71 \%$ of sub-adults survive to adulthood and $94 \%$ of adults continue to survive

This leads to the following formulas for the number in each age group next year based on the numbers from this year:

| New Juveniles: | $.33 a=0 j+0 s+.33 a$ |
| :--- | :--- |
| New Sub-adults: | $.18 j$ |
| New Adults: | $.71 s+.94 a=0 j+0 s+.71 s+.94 a$ |

This set of formulas can be written equivalently as a matrix multiplication, where the coefficients are included in the "Leslie" matrix which is then multiplied by a one column "initial population" matrix. The result will be a one column matrix with the next year's estimated population in each class.

$$
\left[\begin{array}{ccc}
0 & 0 & .33 \\
.18 & 0 & 0 \\
0 & .71 & .94
\end{array}\right] \cdot\left[\begin{array}{l}
j \\
s \\
a
\end{array}\right]
$$

This matrix model is called the stage-matrix or Leslie model for a population. The entries in the first row describe the birthrate (aka fecundity) of the population. The other entries in the matrix describe the survival rates. If we know the number of spotted owls in each age group for one year, we can predict the number for the next year using this multiplication.

Suppose in the region under study that there are approximately 80 juvenile, 30 sub-adult, and 100 adult owls. What does the model predict for the population one year from now? Two years from now? Three? We can see the approximate solutions (rounded to whole numbers of owls) below.

$$
\begin{gathered}
{\left[\begin{array}{ccc}
0 & 0 & .33 \\
.18 & 0 & 0 \\
0 & .71 & .94
\end{array}\right] \cdot\left[\begin{array}{c}
80 \\
30 \\
100
\end{array}\right] \approx\left[\begin{array}{c}
33 \\
14 \\
115
\end{array}\right]} \\
{\left[\begin{array}{ccc}
0 & 0 & .33 \\
.18 & 0 & 0 \\
0 & .71 & .94
\end{array}\right] \cdot\left[\begin{array}{cc}
33 \\
14 \\
115
\end{array}\right]=\left[\begin{array}{ccc}
0 & 0 & .33 \\
.18 & 0 & 0 \\
0 & .71 & .94
\end{array}\right]^{2} \cdot\left[\begin{array}{c}
80 \\
30 \\
100
\end{array}\right] \approx\left[\begin{array}{c}
38 \\
6 \\
119
\end{array}\right]} \\
{\left[\begin{array}{ccc}
0 & 0 & .33 \\
.18 & 0 & 0 \\
0 & .71 & .94
\end{array}\right] \cdot\left[\begin{array}{c}
38 \\
6 \\
119
\end{array}\right]=\left[\begin{array}{ccc}
0 & 0 & .33 \\
.18 & 0 & 0 \\
0 & .71 & .94
\end{array}\right]^{3} \cdot\left[\begin{array}{c}
80 \\
30 \\
100
\end{array}\right] \approx\left[\begin{array}{c}
39 \\
7 \\
116
\end{array}\right]}
\end{gathered}
$$

If we want to know the population in many years, say 30 , we can use the $30^{\text {th }}$ power of the matrix rather than repeatedly multiplying by the matrix 30 times. Checking 30 years into the future by raising the matrix to the $30^{\text {th }}$ power and multiplying by the initial population matrix yields approximately 25 juveniles, 5 sub-adults, and 74 adults, indicating that the population appears to be shrinking over the long term.

Is the population going to continue to shrink in the long term? Consider the percent change of the total population each decade, given in Table 2. The rate at which the population is changing is starting to look about the same; this is actually a property of most Leslie matrices! The long-term decay rate is about $16 \%$ per decade or $1.6 \%$ per year. For more examples of this type of analysis, see Crisler et al. (2000).

Table 2. The change in the owl population over time

| Year | Total | Population | \% Change |
| :---: | :---: | :---: | :---: |
| 0 | $80+30+100$ | $=210$ | -- |
| 10 | $35+6+103$ | $=144$ | $-31 \%$ |
| 20 | $29+5+87$ | $=121$ | $-16 \%$ |
| 30 | $25+5+74$ | $=104$ | $-14 \%$ |
| 40 | $21+4+63$ | $=88$ | $-15 \%$ |
| 50 | $18+3+53$ | $=74$ | $-16 \%$ |

## More on Determining Long-term Behavior: Eigenvalues

Linear algebra theory can explain why this behavior (long-term steady-state percent change) occurs. We need a few facts from a standard linear algebra course (e.g., Kuttler, 2021) to understand what happens.

- A square matrix $A$ with $n$ rows and columns has $n$ eigenvalues usually denoted as $\lambda_{i}$, each with an associated non-zero eigenvector $x_{i}$, where the relationship $A x_{i}=\lambda_{i} x_{i}$ holds.
- If the eigenvalues are distinct, then the eigenvectors are independent and form a basis. If the eigenvalues are not distinct, there may still be "enough" independent eigenvectors to form a basis.
- Any one-column matrix can be thought of as a vector.
- Any vector can be decomposed into a linear combination of basis vectors.
- Matrices with the structure found in Leslie matrices (i.e., the positions of the survival and birth rate parameters and the fact that they are non-negative values) have a unique largest positive eigenvalue.

How does this help us to understand the long-term behavior? Here is a step-by-step sketch of the idea:

1. Rewrite the initial population vector, $v$, as a linear combination of the eigenvectors:

$$
v=a_{1} x_{1}+a_{2} x_{2}+\cdots+a_{n} x_{n}
$$

where the $a_{i}$ are constants chosen to ensure this sum is equal to the initial population vector.
2. Multiply the initial population vector by the Leslie matrix, and reorganize using matrix arithmetic properties:

$$
A v=A\left(a_{1} x_{1}+a_{2} x_{2}+\cdots+a_{n} x_{n}\right)=a_{1} A x_{1}+a_{2} A x_{2}+\cdots+a_{n} A x_{n}
$$

3. Employ the eigenvalue relationship in each term on the right:

$$
A v=a_{1} \lambda_{1} x_{1}+a_{2} \lambda_{2} x_{2}+\cdots+a_{n} \lambda_{n} x_{n}
$$

4. Repeat this process many times, say 1000 :

$$
A^{1000} v=a_{1} \lambda_{1}^{1000} x_{1}+a_{2} \lambda_{2}^{1000} x_{2}+\cdots+a_{n} \lambda_{n}^{1000} x_{n}
$$

5. Suppose that we listed the eigenvalues so that $\lambda_{1}$ was the largest, then eventually for a really high power that first term will swamp all the other terms in the sum, i.e.,

$$
A^{1000} v \approx a_{1} \lambda_{1}^{1000} x_{1} .
$$

Now take one more step into the future:

$$
\begin{gathered}
A^{1001} v=A\left(A^{1000} v\right) \approx A\left(a_{1} \lambda_{1}^{1000} x_{1}\right)=a_{1} \lambda_{1}^{1000}\left(A x_{1}\right) \\
=a_{1} \lambda_{1}^{1000}\left(\lambda_{1} x_{1}\right)=\lambda_{1}\left(a_{1} \lambda_{1}^{1000} x_{1}\right)=\lambda_{1}\left(A^{1000} v\right)
\end{gathered}
$$

What do the first and last terms in this string of equalities indicate in words?
Eventually, the population at one time step into the future will be approximately equal to the current population multiplied by the largest eigenvalue.

Thus, the largest eigenvalue will determine the long-term change in the population.
In our owl example, the largest eigenvalue for the Leslie matrix is approximately 0.9836 . Therefore, eventually the population next year will be 0.9836 times the population now, which indicates a long-term annual rate of decrease of approximately

$$
1-.9836=.0164=1.64 \%
$$

For a more thorough review of eigenvalues and the role they play in long term behavior of Leslie models and other dynamical systems, see any of the many freely available introductory linear algebra texts (e.g., Kuttler, 2021).

## Technology Solutions

To explore these applications with students, we can meet the guidelines in the unpacking document by having the students create Leslie models and preform matrix multiplication to determine population numbers at several time steps into the future. To go further, it is advantageous to employ technology for the actual arithmetic calculations. Table 3 lists several technology solutions for matrix arithmetic in order of increasing complexity.

Table 3. Technology solutions for matrix arithmetic

| Desmos | https://www.desmos.com/matrix | Allows for up to a 6 by 6 matrix and does standard matrix <br> arithmetic as well as reduced echelon form, transpose, inverse, <br> trace, and determinant. Very user friendly. |
| :--- | :--- | :--- |
| Tl calculators | https://education.ti.com/en/guidebook/search | Matrix menu includes a wide range of options for matrices of <br> larger sizes, with the nspire CX CAS having eigenvalue <br> functionality. |
| Octave-online | https://octave-online.net/ | Sophisticated matrix software; an open-source version of the <br> industry standard software, MatLab. Command driven, so there <br> is a syntax learrning curve; but the structure of the commands is <br> quite straightforward. |
| RStudio | https://www.rstudio.com/products/rstudio/ | RStudio provides an interface to the R programming language, <br> which like octave is command driven with a syntax learning <br> curve. |

## Other Sources of Population Data

A search on "<insert species here> fecundity and survival" or some combination of these words on google or google scholar can yield quite a few hits, some of which - while from technical journals in scientific fields - lend themselves to creating new examples with a species of interest to students. Here are a few examples:

- DelGuidice et al. (2007) provide rates for white-tailed deer in Chippewa National Forest.
- Huenneke and Marks (1987) apply Leslie modeling techniques to plant growth, using sprout production and stem growth as a way to divide the plants into "age" classes. The Leslie matrix is provided in the paper.
- Kilgo et al. (2017) provide fecundity rates for coyotes sampled in the US Department of Energy's Savanah River site; age-based survival rates are not provided, but the number and age distribution of trapped animals is provided, which could be used to estimate survival.
- Powell et al. (1996) provide survival and fecundity values for black bears in the Piscah National Forest.


## References

Crisler, N., Fisher, P., \& Froelich, G. (2000). Discrete mathematics through applications, $2^{\text {nd }}$ edition. COMAP.
DelGiudice, G. D., Lenarz, M. S., \& Carstensen Powell, M. (2007). Age-specific fertility and fecundity in northern freeranging white-tailed deer: Evidence for reproductive senescence? Journal of Mammalogy 88(2), 427-435.
Huenneke, L. F., \& Marks, P. L. (1987). Stem dynamics of the shrub alnus incana ssp. Rugosa: Transition matrix models. Ecology 68(5), 1234-1242.
Kilgo, J. C., Shaw, C. E., Vukovich, M., Conroy, M. J., \& Ruth, C. (2017). Reproductive characteristics of a coyote population before and during exploitation. The Journal of Wildlife Management 81(8), 1386-1393.
Kuttler, K. (2021). A first course in linear algebra. Lyryx Learning Inc. https://lyryx.com/first-course-linear-algebra/
Lamberson, R. H., McKelvey, R., Noon, B. R., \& Voss, C. (1992). A dynamic analysis of the northern spotted owl viability in a fragmented forest landscape. Conservation Biology 6(4), 505-512.
Lay, D. C., Lay, S. R., \& McDonald, J. J. (2016). Linear algebra and its applications, 6th ed. Pearson.
NCDPI. (2020a, June 19). 2019 discrete mathematics for computer science unpacking. Mathematics. https://www.dpi.nc.gov/documents/files/2019-discrete-math-computer-science-unpacking
NCDPI. (2020b, June 19). 2019 NC math 4 unpacking. Mathematics. https://www.dpi.nc.gov/documents/files/2019-nc-math-4-unpacking
NCDPI. (2020c, June 19). 2019 precalculus unpacking. Mathematics. https://www.dpi.nc.gov/documents/files/2019-precalculus-unpacking
Powell, R. A., Zimmerman, J. W., Seaman, D. E., \& Gilliam, J. F. (1996). Demographic analyses of a hunted black bear population with access to a refuge. Conservation Biology 10(1), 224-234.

# Zooming Through Number Talks: Considerations for Virtual Instruction 

Savannah Phillips and Monica Gonzalez, East Carolina University, Greenville, NC

> The authors presents a "number talk" strategy for teaching multiplication to build fluency, and report on implementing this interactive approach online.

As we approached 2020, this was supposed to be a riveting year like the 1920's with celebrations and roaring activities. No one expected to have a turn of events that meant a worldwide pandemic. When COVID-19 made its way into the world, it left a lot of panic and confusion with everyone, especially teachers. Teachers were left more confused than ever on how to adjust their interactive, hands-on teaching styles to a virtual environment through a computer screen.

During this time, I was a preservice teacher who used number talks with third-grade students as a tool for developing multiplication fluency. I recognized that memorizing facts is not the best way to build fluency with multiplication facts, so I used number talks as a way to help students build their repertoire of strategies through collaborative learning with their peers. Once instruction moved online, I wanted to continue to learn and practice this teaching strategy in a virtual format. I continued to work with third grade students conducting number talks and analyzing how virtual adjustment affected their learning ability, and my ability, to provide a positive learning environment.

## Developing Multiplication Fluency

There are four different classes of multiplicative structures: equal groups, comparison, area, and combinations (Van de Walle et al., 2019). Third grade students mainly focus on the equal groups structure. This structure involves three quantities: the number of groups (sets or parts of equal size), the size of each group (set or part), and the total of all the groups (whole or part). The parts and wholes terminology helps students make the connection to repeated addition since the equal group is added over and over again. This is important for third grade students to make the connection from repeated addition (additive thinking) to (equal groups) multiplicative thinking since it will produce the same results for positive whole numbers (Van de Walle et al., 2019).

Third grade students in North Carolina are expected to become fluent with their multiplication facts within 10 (North Carolina Department of Instruction, 2018). To be fluent means more than just solving multiplication problems quickly. It is having the skill to find products flexibly, accurately, efficiently, and with appropriate strategies (Bay-Williams \& Kling, 2019). Baroody (2006) suggests that students develop fluency as they progress through three developmental phases:

- Phase 1: students use direct modeling and/or counting to find the answer
- Phase 2: students derive answers using reasoning strategies based on known facts
- Phase 3: students demonstrate mastery through efficiently producing answers

Phase 2 is centered around the development and use of reasoning strategies that allow students to derive facts based on known facts. Derived fact strategies for multiplication include: 1) adding or subtracting a group, 2) doubling and halving, 3) using a square product, and 4) decomposing a factor. Skipping phase 2 does not work for many students as they do not retain the facts they memorized. Memorization of facts discourages students from looking for patterns and relationships and also does not allow students to flexibly apply strategies to find facts, which means they are unlikely to develop fluency with multiplication (Kling \& Bay-Williams, 2015).

## Benefits of Number Talks

Number talks are tools teachers can incorporate into their classroom instruction to enhance students' reasoning strategies and build their mental math skills (Parrish, 2011). Number talks are usually five to fifteen minute conversations around a purposefully crafted computation problem in which students share and discuss computation strategies (Parrish, 2010). The role of the teacher is to facilitate the conversation and make the students' thinking visible to the class to aid in discussion. During these number talks, students have the opportunity to clarify their thinking, investigate and apply mathematical relationships, build a repertoire of strategies, select efficient strategies for specific problems, and then consider and test strategies to check for reasonableness (Parrish, 2011). Number talks allow students to collaboratively explore different students' strategies and develop their fluency with multiplication. As a preservice teacher, I felt this teaching strategy was essential for me to learn to enact and incorporate into future classroom lessons.

## Planning to Teach Online

With the abrupt shift to online instruction, platforms such as Zoom became popular environments for teaching. This video-based communication tool allows for groups to gather synchronously, at the same time, in virtual environments, while maintaining a safe distance. When teachers made the shift to online teaching, they had to consider the functionality of the online platform they chose to use, the technology students had available to them, and how to create a classroom community in this setting. As a preservice teacher, I did not have the experience of teaching online and was suddenly challenged to enact a new teaching practice in an online environment.

As I planned number talks, I decided my goal was for the students to begin with foundational facts and move toward derived facts. Table 1 shows the multiplication problems I planned to pose as well as the strategies I anticipated students might use to solve the problems.

Table 1: Number Talk Planning

| Session | Multiplication Problems | Anticipated Student Strategies |
| :--- | :--- | :--- |
| Session 1 | $6 \times 3,8 \times 4,8 \times 12,4 \times 15$ | Decomposing |
| Session 2 | $7 \times 13,4 \times 5$ | Decomposing, "groups of" |
| Session 3 | $6 \times 9,4 \times 11$ | Benchmarking 10 |
| Session 4 | $8 \times 9,6 \times 12$ | Benchmarking 10, doubling and halving |
| Session 5 | $6 \times 12,5 \times 9,3 \times 4$ | Decomposing, benchmarking 10, doubling and halving, recall |

I decided to work with three boys who were at the end of their third-grade year. I did not know these boys ahead of time, nor did they know each other. These boys' parents agreed to let me meet with each one individually first so we could practice with the technology and so I could assess their current thinking about multiplication using a pre-assessment. I took notes during the pre-assessment on how each boy solved each multiplication problem. We then met as a small group for five different number talk sessions over the span of four weeks. After each session, I would reflect on how the strategies the boys used and any tensions that would arise. Once we completed all five number talk sessions, I then met with each boy individually, again, to assess their multiplication fluency using the same questions I asked in the pre-assessment. I found before the number talk sessions, the boys used a repeated addition strategy and could answer about four out of six multiplication problems. The boys continued to use the repeated addition strategy on the post assessment, but this time they were able to answer all the multiplication problems correctly.

Though the experience of conducting virtual number talks seemed to be beneficial to the boys, what I learned the most was from my reflections after each meeting. I noticed patterns of tensions that arose when I read through
my reflections. These tensions are where I felt other teachers could learn from my experiences to improve their own virtual instruction.

## Tensions When Conducting Virtual Number Talks

Tensions arose throughout my implementation of virtual number talks. These tensions included how to cultivate a classroom community, students not using the anticipated strategies, difficulties with recording students' thinking, and dealing with technical difficulties. Each of these tensions are described below and how I worked through each of these tensions as the sessions progressed.

Tension 1: How do I create a classroom community online? With the mindset that I nor any of those boys had met in a face-to-face classroom setting, I attempted to create a classroom community virtually. During the individual pre-assessments, I spent time with each of the boys, getting to know them, and letting them get to know me. The time spent individually supported the boys getting to know one another during the first synchronous session. I spent time allowing everyone to introduce themselves, share fun facts, and interact. I explained how each session would flow and gave the boys time to play around with the Zoom program features. This served to be very beneficial as it resulted in the boys being willing to share different strategies with one another, listen and encourage one another in the chat boxes, and create an overall positive rapport with each other and myself.

Tension 2: What happens when students do not use the anticipated strategies? I intentionally planned the number talks to bring out specific strategies students might use to solve multiplication problems. During this intentional planning, I considered the problems I chose and questions I would ask the students so they could make connections among strategies and evaluate their efficiency. It is ideal for students to use strategies, such as decomposing or double and halving, and explain their mathematical thinking to their peers during a number talk. However, I did not anticipate that students would continuously choose to use the repeated addition strategy. When the students did not use the strategies I anticipated, I decided to be the one to introduce the new strategies. They would try out the newly introduced strategies, like decomposing or doubling and halving, during the same session, but then they would revert back to using repeated addition in subsequent sessions.

I am curious if they would have been more inclined to adopt new strategies for themselves had they been introduced to new strategies by one of their peers. In the future, I will think of questions to push their thinking beyond using repeated addition such as, "Is there a faster way to do this?" I could also assign multiplication problems where the repeated addition strategy would be inefficient and hopefully lead to choosing other strategies. Looking over the problems I posed for the students, in the future, I would spend more time on the intentionality of the anticipated strategies I want to elicit and modify some of the number selections in the problems to achieve the learning goals.

Tension 3: How do I record student thinking in a way that makes it visible to all students? The biggest obstacle I came across was being able to record students' thinking in a way where they could view it as I recorded it. I tried a variety of ways to represent students' thinking as seen in Figure 1.

Figure 1: Progression of representing students' thinking


First, I wrote on a paper and then held it up. Next, I tried holding up a whiteboard the entire time while I wrote the students' strategies. Finally, I tried posting a sheet of poster board paper behind me to try to create the illusion of a
whiteboard. While the whiteboard illusion was most ideal because I could write and have the students view at the same time, I ran into the problem of the students not being able to see my writing through the computer screen. During the majority of the sessions, I would hold up a whiteboard, write on it, and then hold it closer to the camera for students to see it better. The students could see the whiteboard well, but it was a bit harder for me to hold it up the entire time in a good position for them to view it as I talked. Whiteboards also only allowed me to record one or two of the student's work without having to erase or have smudges.

After going through these number talk sessions, I learned about virtual whiteboards in Zoom where I can write straight onto the board through the computer. This allows students to view what I write on the Zoom screen, which is a lot more effective than the solutions I previously came up with. Sometimes it is helpful for students to be able to represent their thinking themselves during a number talk. In order for students to do this so that everyone in the class can see, they could use another whiteboard such as Whiteboard.fi. By having either the teacher represent student thinking or the students represent their own thinking, the visual representation of the mental math strategies allows students to make connections among different strategies and potentially move forward in their strategy selection.

Tension 4: What should I do when technical difficulties arise? Technical difficulties are inevitable when teaching online. A speaker might not work, the internet connection may be unsteady, the video is lagging, and so on. Over the span of five sessions, we had our fair share of technical difficulties. Since I was only working with three boys, I wanted to help each of them so I could get the most out of these sessions; however, I cannot be there to see what technical difficulties arise on their end, and I could only do my best to walk them through some of the ways to troubleshoot. For instance, one of the boys had a speaker issue where he could hear me but we could not hear him. We solved this problem through utilizing the chat box and the emojis in the Zoom program. I still wanted the student to participate in the number talk, and we were able to find a solution for him. After the session ended, I communicated with the family and they were able to resolve the problem by our next session. Technology can be very unpredictable, so teachers need to be resilient and flexible when using technology to teach. It is also helpful for teachers to have good communication with students' families so that they can work with the teacher to troubleshoot technical issues when they arise.

## Discussion

Overall, this experience was very beneficial to me as I continued to develop my teaching practices in a virtual environment. I was still very new to enacting number talks with students when instruction moved online, and I found it challenging to practice a new teaching skill while simultaneously learning a new platform for instruction. This experience taught me that technological resources (virtual communications programs and virtual whiteboards) are key in being able to enact synchronous online teaching effectively. This experience also seemed to be beneficial to the students I worked with. During the post assessment, I asked the students the questions, "How did you feel being at home learning?" and "Was it hard to understand concepts through the computer?" This conversation during the post assessment was particularly interesting to hear how they felt about their learning. Two of the boys said it was easy to learn through virtual sessions, while another struggled with technical difficulties that posed a barrier to his ability to learn. All of the boys felt it was fun and still felt like school, which was important since these sessions occurred within the first month that COVID-19 made an impact and before all instruction moved online. Unfortunately, I am no longer working with these boys since I am finishing up my senior year of my teacher preparation program. I will be curious to reach out and see how they feel about online instruction now that their online learning environment has continued throughout this school year.

If others are interested in conducting virtual number talks, I suggest using online whiteboard tools, such as the whiteboard tool in Zoom or Whiteboard.fi, to help with recording student thinking and eliciting discussions based on students' strategies. I also suggest spending time building an online classroom community that is a supportive and safe environment for students to provide encouragement to one another through the use of the chat box and emojis. This classroom community will allow students to feel safe to share their thinking, which will benefit all students during virtual number talks.

## References

Bay-Williams, J. M., \& Kling, G. (2019). Math fact fluency: 60+ games and assessment tools to support learning and retention. ASCD.
Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. Teaching Children Mathematics, 13, 22-31.
Kling, G., \& Bay-Williams, J. M. (2015). Three steps to mastering multiplication facts. Teaching Children Mathematics, 21(9), 548-559.
North Carolina Department of Instruction. (2018). 3rd grade unpacking [June 2019 revision]. https://www.dpi.nc.gov/documents/curriculum/mathematics/scos/current/2018-unpacked-content-3rd-grade-math-standards
Parrish, S. D. (2010). Number talks: Helping children build mental math and computations strategies. Math Solutions.
Parrish, S. D. (2011). Number talks build numerical reasoning. Teaching Children Mathematics, 18(3), 198-206.
Van de Walle, J. A., Karp, K. S., \& Bay-Williams, J. M. (2019). Elementary and middle school mathematics: teaching developmentally ( $10^{\text {th }}$ ed.). Pearson.

## Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from Emogene Kernodle, nekernodle@yahoo.com.

## Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Ana Floyd, afloyd@randolph.k12.nc.us.

## Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an ongoing benefit from the grant. In recent years, grants averaged just less than $\$ 800$.
The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [imccormick@rock.k12.nc.us] with questions.

# Problems2Ponder 

Holly Hirst, Appalachian State University, Boone, NC
In each issue of The Centroid, Problems2Ponder presents problems similar to those students might encounter during elementary and middle school Olympiad contests. Student solution submissions are welcome as are problem submissions from teachers. Please consider submitting a problem or a solution. Enjoy!

Problem submissions: If you have an idea for a problem, email Holly Hirst (hirsthp@appstate.edu) a typed or neatly written problem statement, along with a solution. Include your name and school so that we can credit you.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst (hirsthp@appstate.edu) a clear image or PDF document of the correct solution, with the name of the school, the grade level of the student(s), the name of the student(s) if permission is given to publish the students' names, and the name of the teacher.

## Fall 2021 P2P Problems

Problem A: Hilda uses the digits $1,2,3,4,5,6$ to make two three-digit numbers, with each digit used once. She then subtracts the two numbers. What is the largest possible difference?

Problem B: How many three-digit multiples of 21 are there?

## Spring 2021 P2P Problem Solutions

Problem A: Let $\boldsymbol{X}$ represent the sum of whole numbers less than $x$ that are not factors of $x$. For example, $6=9$, because 4 and 5 are the only whole numbers less than 6 that are not factors of 6 , and $4+5=9$. What is the value of $12-10$ ?

Solution: Perhaps the most straight forward way is by an exhaustive list!

$$
12-10=(5+7+8+9+10+11)-(3+4+6+7+8+9)=5+11-3=13
$$

Problem B: One hundred students were asked their opinion on three ice cream flavors. Sixty-five said they liked rocky road, 75 said they liked chocolate, and 85 said they liked butter pecan. What is the smallest number of students who could have said they liked all three of these flavors?

Solution: This one is very tricky! Let's start by determining how many liked two flavors. There are three combinations:

1. Rocky road and chocolate: $65+75=140$, so 40 must have liked both out of 100 total students.
2. Rocky road and butter pecan: $65+85=150$, so 50 must have liked both.
3. Chocolate and butter pecan: $75+85=160$, so 60 must have liked both.

Let's look at each of these and consider what would happen with the third flavor:

1. There were 85 students who liked butter pecan, so if no one liked all three flavors then there would have to be $85+40$ $=125$ students. There were only 100 students, so at least 25 had to like all three.
2. There were 75 chocolate lovers, so if no one liked all three flavors there would have to be $75+50=125$ students again 25 minimum liked all three.
3. There were 65 rocky road lovers, so if no one liked all three flavors there would have to be $65+60=125$ students again 25 minimum liked all three.

Here is a challenge for you teachers out there. Can you prove in general that all three of these approaches had to result in the same answer?

# Games A-'Round' the Classroom: Supporting Rounding Strategies in Third Grade 

Ann H. Wallace, James Madison University, Harrisonburg, VA and<br>Maria Zehr, Cub Run Elementary School, Penn Laird, VA

"Five and above, give it a shove. Four and below, let it go."
How can we help students make sense of these statements? When students understand rounding as a procedure but have no real understanding of the concept, they struggle with the meaning of such statements. The common rule often causes confusion for young children regarding why the rule works, as it fails to explain the importance of incorporating place values. As frequently referenced in the Principles to Actions (National Council of Teachers of Mathematics [NCTM], 2014), there is often a lack of connection between the rules or procedures and the concepts we want children to understand (Fuson et al., 2005; Kilpatrick et al., 2001; Martin, 2009).

## What is Rounding?

Rounding means to approximate and assists in estimating solutions when computing with two or more values. It requires finding a compatible number that is easier to compute mentally (Van de Walle et al., 2018); it is also used to solve problems where exact values are not required. When computing with a rounded number the results are less accurate, but the number is easier to operate on, which facilitates estimation. When rounding, we place a number correctly between two other numbers and determine which is the shorter distance between the two numbers. This requires an understanding of where a number falls in the counting sequence on a number line (Foote et al., 2014).

There is both logic and convention involved when numbers are rounded. Students need to understand what is meant by rounding to a particular place value, such as the nearest ten or hundred. It can be confusing for children when addressing numbers having three or more digits because those numbers can be rounded to multiple places. For example, the number 753 can be rounded to the nearest ten (750) or the nearest hundred (800). In the North Carolina Mathematics Standards for Third Grade (NC.3.NBT.2), students are expected to explore rounding through the use of number lines and similar strategies. Rote memorization of rounding rules without conceptual understanding is not the expectation (North Carolina Department of Public Instruction, 2017).

There are lessons that can help make rounding more meaningful and help to develop students' understanding of the practice. When teaching rounding concepts, it is important to clarify why we round either up or down. For example, the distance on the number line is the same from zero to five as it is from five to ten. The numeral 45 is the same distance from 40 and 50 . By mathematical convention we round the numeral up to 50 . This was determined by the mathematics community who established that when rounding to the nearest $10(100,1000$, etc.), the midpoint that is symmetric to
either end $(5,50,500$, etc.) will result in the numeral being rounded up to the nearest decade, century, etc. This is not something children can determine by observation; it is something that needs to be established as a practice that is commonly accepted and used (Schwartz, 2013).

I collaborated with a third-grade teacher whose students struggled to round numbers to the nearest 10 's or 100's place, especially when asked to round to one of those place values in a three- or four-digit number. I worked with them in small groups of six. Over the course of three days, I presented activities to establish meaning for rounding concepts, followed by games to help reinforce those concepts.

## Why Use Math Games?

Math games are naturally interesting, and because they are exciting, intriguing, challenging, and fun, they motivate children to work at a task over and over again. Peggy Kaye (1987) stated it definitively in her book Games for Math: "Games put children in exactly the right frame of mind for learning difficult things. Children relax when they play - and they concentrate. They don't mind repeating certain facts or procedures over and over, if repetition is part of the game. Children throw themselves into playing games the way they never throw themselves into filling out workbook pages. And games can, if you select the right ones, help children learn almost everything they need to master in elementary math" (p. 236).

The math games were chosen purposefully to encourage students to apply strategies introduced during the lessons and to interact with fellow students. Although we played the games together, the intention was for them to be self-sufficient in order to play the games without help from the teacher. When playing together, we addressed the rules and discussed the content and strategies used by the students.

## The Daily Lesson Plan

Day 1. The objective for day 1 was to use ten frame cards to determine a rule from given rounding scenarios. The ten frame cards used in this lesson were specifically designed vertically for the purpose of helping the students make conjectures about the rounding rule. I placed the cards 0 to 9 in front of the students with a space between cards 4 and 5 (Figure 1). This alignment provides a visual for the rule we were trying to determine.

Figure 1. Layout of ten frame cards showing space in the middle


Rather than simply telling the students a rule for rounding, I gave them the answers to several rounding problems and asked them to make conjectures to help them determine the rule. I targeted two cards and asked the group the following question:

## My target cards are 2 and 3 . If I told you they round down to 0 , what is my rule?

Anika responded that they are on that side of the line (pointing to the left). I further asked what they noticed about the other cards on that side of the line. lan responded that none of the cards have a full column of dots. Emily followed that the cards on the right all have a full column of dots. I repeated the question asking the students if they could determine my rule. A little unsure, Emily responded that they must be less than 5 . I asked if there were other numbers that fit the rule. lan responded " 4 " and Anika "1." When asked how they knew their responses fit the rule, they agreed that the numbers do not have a full column of dots so they both round to zero.

Next, I shared that my target cards 7 and 8 round up to 10 and asked what they thought my rule might be. Abe responded that they are all on the right side so they round up to 10 . Anika followed that the cards to the right all have a full column of dots. Before the rounding rule was articulated, we continued the activity so that others had an
opportunity to reach the same conclusion. We determined that the cards to the left were less than 5 (no full column of dots) and rounded down to 0 . The cards to the right were 5 or greater (full column of dots) and rounded up to 10 .

We transitioned to two-digit numbers. I shuffled the ten frame cards and asked a student to randomly choose two cards from the deck. The student chose 5 and 3 . I placed the cards on a place value mat to represent 5 tens and 3 ones (left side of Figure 2). I asked the students, based on the rule they had determined, whether 53 would round up or down. They were immediately confused by which decade numbers the 53 fell between. I related the one's card back to the rule they had established. I asked what they had determined about the 3 in the original problem. The students were able to articulate that it rounded to 0 because it did not have a full column of dots. They still had trouble articulating that the number would round to 50 . I also recognized this difficulty when we were playing the subsequent game for that day.

I reversed the cards creating the numeral 35 (right side of Figure 2). This time, I asked the students which two-decade numbers 35 fell between. They were more easily able to determine that 35 fell between 30 and 40 and that it would round up to 40 because there was a full column of dots. I asked a student to choose two more cards and we repeated the process.

Figure 2. Cards on a place value mat


I introduced the game Round Up (or Down)! (Britt, 2015, p. 77) to help the students practice their rounding skills to the nearest decade number. For this game, two players are each given a strip of numbers in increments of 10 (Figure 3), and a set of ten frame cards with the 0s and 10s removed. Taking turns, each player draws two cards and creates a two-digit number (on the place value mat) that when rounded to the nearest ten can be colored on his or her game board. I provided the additional two cards so that the number could be created two different ways. For example, if a player drew a 2 and a 9, the player could make 29 and color 30 ; or make 92 and color 90 . If a player cannot make a number that can be colored on the board, the player loses the turn. The first player to color all the numbers on the game board wins. We played the game in three pairs of two giving me numerous opportunities to observe their thinking and to ask questions. I gave each student a different color of chips to record their answer on the game board. Figure 4 illustrates the students engaged during play.

Figure 3. Number strips showing increments of 10

| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Figure 4. Students engaged in the "Round Up (or Down)! game


The student with the red chips drew the cards 0 and 9 and had to determine whether he could round the number 09 or 90 to one of the open spaces on his board. Neither 10 nor 90 were available, and he determined that he did not have a move. I asked him what two cards he would need to draw to have a move for one of his open spaces. He hesitated, so l asked what cards he would need to round to the open 30 on his game board. He was able
to determine that 31 would round to 30 . I next asked if he had drawn the 9 first, what card would he need if he still wanted to round to 30 with his second draw. With help from the other student, it was determined that 29 would also round to 30 . These types of questions helped the students think beyond the idea that only two specific cards were needed; there were multiple cards that could be selected that could round to the numbers on their game boards.

When playing the game with the next group of students, I made a few modifications. First, instead of chips I provided dry erase markers. The chips from the previous group could be easily bumped out of place, and the markers provided a more permanent image. Next, I had the students take turns drawing the cards, but at each turn all the students could choose a number to round and color (Figure 5). These changes generated more play time, less wait time and fueled competition.

Figure 5. Students playing the revised game


From my observations of the game play and the questions I asked, I was able to establish that the students had a difficult time determining the two-decade numbers their generated numeral fell between. For example, 34 falls between 30 and 40 . These observations helped to plan the next day's lesson.

Day 2. The objective for the second day was to help the students determine the two-decade numbers a two-digit number fell between. I introduced a modified 100's chart (Figure 6) that included decade numbers at either end and highlighted the middle 5's column, representing a symmetrical divide and the midpoint between the two-decade numbers. This representation does not help to see why we round up when a number ends in 5 (the way the ten frame cards demonstrated) because the column is equidistant from each of the decade and century numbers.
I began the lesson by highlighting the numbers 14 and 57 in yellow and asked the students to place chips on the twodecade numbers the highlighted numbers fell between. The students were able to determine that 14 fell between 10 and 20, and 57 between 50 and 60 . I related the highlighted column of 5 's back to the ten frame cards and the rule we determined regarding the full column of 5 . The students were able to articulate that the 14 was closer to 10 (and less than the 5's column meaning it did not have a full column of dots) and 57 rounded to 60 because it was closer to 60 and larger than 55 (meaning it had a full column of dots). Noah inquired about a number that fell on the red highlighted column because it fell in the middle. I reminded him about the rule we determined for numbers ending in 5 or more and that by convention we round those to the higher decade number.

Figure 6. The modified 100's chart

| $0-100$ Chart |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |  |  |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |  |  |  |  |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |  |  |  |  |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |  |  |  |  |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |  |  |  |  |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |  |  |  |  |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |  |  |  |  |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |  |  |  |  |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |  |  |  |  |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |  |  |  |  |

Next, we revisited the game from the previous day, but this time I allowed them to use the 100's charts
(Figure 7). After representing the chosen cards on the place value mats, I had the students place a chip on the 100's
chart at the target number's location. They were more confident playing the game with the use of the modified 100's chart.

Figure 7. Students playing the game with the 100's charts available


Another representation I introduced was the open number line (i.e., without values indicated). Number lines help students visualize whether a number should round up or down by identifying the distance between the target number and the two decade or century numbers. The student's role was to determine to which endpoint the target number is closest. The 100's chart helped make this connection. Consider the number line in Figure 8. Where should the number 62 be placed?

Figure 8. Number line problem


It is important for students to determine which numbers to choose for endpoints and midpoints prior to placing the target number 62 along the number line. The endpoints and midpoint for the number 62 are 60,70 , and, 65 respectively. The number line gives a clear visual for why a number can be rounded up or down, depending on the place value being asked, and can be quickly sketched. Next, I provided the students with several open number lines, and one by one, asked them to determine the endpoints and midpoint for a series of numbers $(23,38$, and 86$)$ and then to determine where a target number fell. Some of the students felt more comfortable including all of the numbers between the two endpoints (the left image in Figure 9), while others were able to place the target number given only the endpoints and midpoints (the right image in Figure 9). At first I allowed them to use the 100's chart. Then I asked them to try to visualize the 100's chart to help make the correlation.

Figure 9. Student work on estimating where a number falls on the number line


Once the students were able to determine the endpoints (decade numbers), midpoint (5), and where a target number could be approximately placed, I planned to introduce rounding three-digit numbers to a decade number the following day.

Day 3. The objective for the third day was for students to round a three-digit number to the nearest 10 's place value. To begin, I gave each student a different strip of seven numbers ranging from 110-170, 210-270, etc. in increments of 10 's. Instead of using the ten frame cards from the previous days, I rolled two number cubes. The
students were asked to create a two-digit number on their place value mats and to determine which number they wanted to round. For example, if the numerals 3 and 6 were rolled, they could create a 36 or a 63 .

Abe chose 36 rounds to 40 so he put a chip on the 440 on his strip. Bekka chose 63 and put a chip on 260 on her strip (Figure 10). At times, we had to revert back to considering the two-decade numbers that their created number fell between (using the examples of the 100's chart and the number line). We produced several more numbers for the students to round prior to introducing an additional place value (Figure 11).

Figure 10. Abe's and Bekka's work on rounding


Figure 11. Students working with the number cubes


I exchanged the two-digit place value mat for a three-digit place value mat and added an additional number cube. I began the next activity by rolling only two number cubes and having the students once again create a two-digit number of their choice. I had them find the three-digit number on their number strip, but they were not to place a chip on that number. Whether they would get to color their space depended on the third role of the number cube. My purpose was for them to recognize that rounding to the 10's place was not based on the hundreds place value. We continued this routine for several rounds to get the students adept at selecting the number that would be rounded IF the third roll corresponded to the 100's place on their number strips.

After several practice rounds, I introduced the students to the Roll It! Rounding Game (Field, n.d.). This game requires students to consider different place values when making rounding decisions. Each pair was given a game board with numbers already rounded, a place value mat, and three number cubes. The students were to roll the three number cubes, create a three-digit number, and then round their number to the 10 's place value. The objective for the game was for the students to get four in a row (horizontally, vertically, or diagonally; Figure 12). I discovered early on that the students were not able to consider the three-digit number from afar and be able to make a plan for their choice.

Figure 12. Roll it! game showing a diagonal win

| Roll It! <br> Rounding Numbers to the Nearest Ten |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 110 | 120 | 30 | 140 | 150 | 160 | 170 |
| 210 | 220 | 230 | 240 | 250 | 260 | 270 |
| 310 | 320 | 330 | 340 | 350 | 360 | 370 |
| 410 | 420 | 430 | 440 | 450 | 460 | 470 |
| 510 | 520 | 530 | 540 | 550 | 560 | 570 |
| 610 | 620 | 630 | 640 | 650 | 660 | 670 |

It helped to have the students roll one number cube first and to have that roll represent the value in the hundreds place. They would next find the corresponding vertical and horizontal columns and strips prior to rolling the next two number cubes. It also helped to assign a specific color number cube to represent each place value. They played with these guidelines unless there was a space already covered on their game board; then they would flip the two number cubes representing the 10's and 1's place values to create a different number. Initially, I wanted them to roll all three number cubes and to consider the six different numbers that could be created, but it was too confusing for them. The students were thoroughly engaged during the game play and were able to subsequently play on their own at other times during the school day (Figure 13).

Figure 13. Students engaged in the Roll it: Rounding Game


A benefit of this game to teachers is they can choose the rounding board that best fits their students' level. For one group we used only boards with three strips. Boards can be created with any number of digits, but it is recommended that numbers not exceed five digits. Also, depending on the number of digits found on the playing board, the teacher may choose to have students round to a variety of places. For example, a board containing the four-digit number 3,461 can be rounded to the nearest $10(3,460), 100(3,500)$, or $1,000(3,000)$. The rules can also be modified to allow the students to cover as many spaces as they would like in one turn or to cover just one space. The game provides numerous options for students as they are given multiple choices with each roll of the number cubes.

## Conclusion

Rounding is an important life skill beyond the elementary classroom. Teaching rounding as solely a procedure or trick often leaves students with misconceptions. On the other hand, a series of well-planned activities and games that allow students to connect mathematical representations and help them think about rounding in a visual way will provide children with the necessary tools for building a meaningful understanding of rounding. The games allow students to practice building their rounding skills, and they generate more discussion regarding what the students are doing rather than what procedure they are following. It changes the cognitive demand because students stop thinking about "What 58 rounds to" to "What numbers do I need?" These activities and games help make the connection between rules or procedures and concepts, as previously referenced in NCTM's Principles to Action (NCTM, 2014). Additional rounding games can be found in Bonnie Britt's (2015) Mastering Basic Math Skills: Games for Third through Fifth Grade.

## References

Britt, B. A. (2015). Mastering basic math skills: Games for third through fifth grade. National Council of Teachers of Mathematics.
Field, B. (n.d.). Roll it! rounding game. Games for gain. http://games4gains.com/blogs/teaching-ideas/roll-it-roundinggame
Foote, M., Earnest, D., \& Mukhopadhyay, S. (2014). Implementing the common core state standards through mathematical problem solving, grades 3-5. National Council of Teachers of Mathematics.
Fuson, K., Kalchman, M., \& Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan and J. D. Bransford (Eds.). How students learn: History, mathematics, and science in the classroom (pp. 217-56). National Academies Press.

Kaye, P. (1987). Games for math: Playful ways to help your child learn math, from kindergarten to third grade. Pantheon Books.
Kilpatrick, J., Swoffard, J., \& Bradford, F. (Eds.). (2001). Adding it up: Helping children learn mathematics. National Academies Press.
Martin, G. W. (2009). The NCTM high school curriculum project: Why it matters to you. Mathematics Teacher,103 (3), 164-66.
National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. National Council of Teachers of Mathematics.
North Carolina Department of Public Instruction. (2017). North Carolina K-8 math standards. https://www.dpi.nc.gov/teach-nc/curriculum-instruction/standard-course-study/mathematics
Schwartz, S. (2013). Implementing the common core state standards through mathematical problem solving kindergarten-grade 2. National Council of Teachers of Mathematics.
Schwartz, S. (2005). Teaching Young Children Mathematics. Praeger.
Small, M. (2013). Eyes on math: A visual approach to teach math concepts. Teachers College Press.
Van de Walle, J., Lovin, L., Karp, K., Bay-Williams, J. (2018). Teaching student-centered mathematics grades 3-5 (3rd ed.). Pearson.

## Trust Fund Scholarships: Now $\boldsymbol{s} 1000$

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants \& scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson Plumblee, richards@elon.edu.

## Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313

Raleigh, NC 27636

# NCCTM Board <br> contact information can be found at ncctm.org 

Officers

|  | State | Eastern Region | Central Region | Western Region |
| :--- | :---: | :---: | :---: | :---: |
| President | Stefanie Buckner | Kwaku Adu-Gyamfi | Sara Vaughn | Kanika Turner |
| Past President | Kathy Jaqua | Ginger Rhodes | Denise Johnson | Sheila Brookshire |
| Elementary Vice <br> President | Meredith Stanley | Leigh Belford | Sha Mosley | Amanda Thompson- <br> Rice |
| Middle Grades <br> Vice President | Allison Landry | Katie Martin | Anthony Finlen | Charlcy Carpenter |
| Secondary Vice <br> President | Emily Bryant Hare | Heather Davis | Ebony Jason | Jennifer Reed |
| College Vice <br> President | Vincent Snipes | Kayla Chandler | Perry Gillespie | Erica Slate Young |
| Other State <br> Officers | Secretary <br> Katie Mawhinney | Parliamentarian <br> Wendy Rich |  |  |

## Committee Chairs

Conference and Exhibit Services, Kay Swofford
Centroid, Holly Hirst and Debbie Crocker Convention Services, Marilyn Preddy

Financial Chair, Ray Jernigan Handbook, Ray Jernigan Historian, Kathryn Hill Inclusion and Diversity, Denise Johnson Leadership Conference, Kathy Jaqua Management Services, Joette Midgett
Math Celebrations, Anthony Finlen and Pamela Parham Math Contest, James Beuerle and Phillip Rash

Math Counts, Harold Reiter Math Fair, Betty Long
Mini-grants, Joy McCormick
NCDPI Representative, Lisa Ashe NCSSM Representative, Ryan Pietropaolo NCTM Representative, Debbie Crocker NCSM Representative, Debbie Crocker NCMATYC Representative, Luke Walsh Nominations, Julie Kolb
Rankin Award, Emogene Kernodle Special Awards, Ana Floyd
Student Affiliates, Holt Wilson
Trust Fund, Catharina Middleton Webpages, Holly Hirst

## Becoming a Member

Follow the "Membership Information" link on the ncctm.org website, or go directly to: http://www.ncctm.org/members/register.cfm



