The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

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Problem Solving Using a Balance Scale

NCCTM
NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

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The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

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Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

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Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures, and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:
  Bruner, J. S. (1977). *The process of education* (2nd ed.). Harvard University Press.
  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*.
  Journal articles:
  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.
  Chapters or sections of books:
  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). National Council of Teachers of Mathematics.
  Websites:
  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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Fall 2022 State Math Conference And Leadership Seminar

We are thrilled to meet again in person!

November 9: Leadership Seminar
November 10-11: Conference Sessions
Benton Convention Center in Winston-Salem

The theme for this year's conference focuses on celebrating 50+ years of NCCTM as well as the fact that all of us are learners of mathematics and support ALL students' learning of mathematics.

Registration for the meeting is now open, and **hotel reservations** can be made now at the Winston-Salem Marriott. Reduced rate: $159 per night available through October 11.

Register and book now at https://ncctm.org

President's Message

Stefanie Buckner Hill
State President
stefanie.hill@bcsemail.org

Hi Math Friends!

I never know how to start these columns, I end up writing a large number of phrases and construct some really awkward sentences as a direct result and use the delete button more than I should. Pun of a really poor sentence structure fully intended! But without further awkwardness or purposeful grammatical errors… here we go!

In that last issue of the Centroid I wrote about how I have really started to reflect on the value of teaching problem solving in mathematics. This has created a paradigm shift for me. I no longer spend hours trying to find problems that are related to every career pathway or interest; rather, I focus my time on getting to know my students as individuals and work to find ways to value and incorporate their perspectives and approaches. Valuing their original thoughts celebrates and validates their thinking leading to more robust opportunities for mathematics. If we can teach our students to think, to reason about problems, to decontextualize a problem to apply mathematics and then recontextualize to describe a viable solution, to reason about unknowns and persevere I would claim we are "doing right" for our students. We are providing a deeper, richer, more transferable experience that prepares them for jobs we do even know exist. Please don't misread or misunderstand me, standards are critical and, yes, there is some mathematics that must occur. I am simply saying our "why" and "how" of teaching mathematics has to be as great as the "what" of mathematics.

Also in my last issue, I wrote briefly about the networks across North Carolina ready to help in these endeavors of teaching problem solvers. Teaching mathematics for a deeper, more transferable experience is not an easy task and working alone is daunting. As a North Carolina native, I will always be biased and say we are the best. But truly, I know of no other state that has the network opportunities for math teachers that we have here in NC. This past spring and summer I was fortunate enough to participate in outstanding opportunities for NC math teachers. While reflecting at NCCAT Cullowhee as a Math Camp participant, it occurred to me that I had experienced, first-hand, some of the math networks across NC. I needed to share.

Let's start by backing up to February. We had amazing participation in our February Frenzy - a two day virtual conference experience organized and hosted by NCCTM. We (the NCCTM Board of Directors and Conference Planning Committees) heard that you want options with flexibility to your schedule. You want networking opportunities in a variety of formats across a variety of days. Your participation in these events told us this! And while we are moving back to a face to face conference this fall (yay!), we have embarked on a new wave of supporting you and will continue to offer virtual options in the coming years as a way to better support a variety of needs across our membership of NCCTM.

Fast forward a little bit to this summer. I had the opportunity to attend the NC²ML gathering to work on Developing a High Quality Equitable Vision of Mathematics statewide this past June at High Point University. If you don't know NC²ML is the "North Carolina Collaborative for Mathematics Learning" and is a formal partner with NCCTM. NC²ML also recently received a NSF grant to develop a statewide vision of high quality equitable mathematics instruction K-12. It was so exciting to sit alongside my K-12 peers, NCDPI, and IHE (institutes of higher education) partners and discuss what the vision for NC math instruction is and what is needed to make this happen. It all works together - how do we prepare our K-12 students for a problem solving rich, unknowns abounding post secondary world in cooperation with each other. If you want to know more about NC²ML, visit their website, https://www.nc2ml.org/, but also be on the lookout for some of their sessions at our fall conference.

Most recently, I attended Math Camp. Yes, that's right I went "camping" overnight to do math problems. Side note, texting you "can't make the 6 mile run tomorrow morning [Saturday] because you're literally at math camp" is a conversation starter like none other to your running friends, and spouse for that matter! But for real, this is a thing. And it's a wonderful thing. I spent two days at NCCAT in Cullowhee doing what I love - math. This experience is brought to the teachers of NC by another NCCTM partner, the North Carolina Network of Math Teacher Circles (NCNMTC). (More information about this network, statewide circles and math camps is at their website, https://sites.google.com/site/ncnmtc/). As I was persevering in solving problems, working collaboratively with peers from across North Carolina, trying my best to explain my reasoning to complex problems it once again hit me - this was what I talked about in my January column! Teaching a student to think, to reason with unknowns, to explain and justify their reasoning can't happen until WE, their teachers and math educators, give ourselves that same experience. And math camp did just that. It provided me with the personal opportunity to preserve in problem solving. (It also provided an opportunity for aqua Zoomba. I'm just going to leave that there, but also as a promise to you - if the Marriott in Downtown Winston Salem has an indoor pool I will work to add this session to our fall conference.)

What a summer; what a fun way to explore mathematics pedagogy and instruction and rich problems in collaborative environments. Developing mathematical thinkers - celebrating that we are all math people - is best done in collaboration and I encourage you to take advantage of the opportunities to collaborate that North Carolina offers! Explore those websites, reach out with questions, become involved with NCCTM and our partners!

Looking forward to this fall, I would be remiss to leave out the opportunity to JOIN US LIVE, FACE TO FACE, not on a flat screen, and all completely unmuted at our 52nd Annual Conference (technically it's our 50th conference but should be our 52nd) on November 10 and 11, 2022 at the Benton Convention Center in Winston-Salem. Registration is open and our hotel block of reduced rate is filling quickly - don't delay, register today! All of this information is on our website, www.ncctm.org. Hope to network and collaborate with you in November! And, happy new school year!

Stefanie

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800. The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [jmccormick@rock.k12.nc.us] with questions.

Fun with Measurements

Solomon Willis, Cleveland Community College, Shelby, NC

The author presents a series of activities designed to help students understand the formulas for area and volume of cylinders and prisms.

Effective teachers at all grade levels have spent lots of time for many years brainstorming and creating fun ways to make learning more interesting to their students. Time after time, studies have shown that hands-on activities make learning more meaningful to students and help them to better retain the knowledge that is at hand (Chang et al., 2016; Jones & Tiller, 2017). Teaching measurement within a geometry course or any course that includes geometry opens a great opportunity to conduct some engaging and meaningful lessons with students. When it comes to working with perimeter, area, surface area, and volume, many ideas can be explored kinesthetically. Students learn to recognize and classify shapes very early on, and by middle school, they are familiar with many of their properties (Dossey et al., 2002). It is not long after they develop a basic recognition of various shapes until they start learning how to measure such objects. Having a few manipulatives on hand can certainly help with hands-on lessons. When students visualize math, they are often more successful with learning math (Kang & Liu, 2018).

For many years, math classes, including geometry, have been taught by the teacher showing the students how to work a problem, and then having them work a large number of similar problems. However, developing knowledge by increasing the amount of interaction between the student and the concept will increase student understanding (Malloy, 1999). By the high school years, students learn to analyze two- and three-dimensional figures. By studying and understanding the relationship between these objects, students take an important step toward understanding objects that make up our world (Dossey et al., 2002). Showing real-world applications of math concepts is always an asset in the classroom.

Concerning geometry, students also learn to use tools that help them with reasoning skills. Knowing how to use measurement tools such as a ruler or a meter stick, understanding slope and distance, and working on a coordinate plane are all integral parts of middle-school, high-school, and college math curriculums (Dossey et al., 2002). When students are taught geometry by only using various formulas, it often confuses them and their temporary knowledge of the concept is not retained (Malloy, 1999).

When distinguishing figures or components of figures, students often rely on numbers to make comparisons. Using numbers is generally easy for students, because they are easy to work with. However, to fully understand the conceptual ideas behind measurement, students must also understand which attributes can be measured and which units are appropriate for measuring various figures. Applications and physical models of figures can help students understand the various measurements that are associated with two-dimensional and three-dimensional objects (Dossey et al., 2002). Many hands-on tools can be used to help understand measurement and geometric problem solving. Algebra tiles can be used for two-dimensional figures and pattern blocks can be used for three-dimensional figures. When students can visualize an operation or concept, they can retain the information more easily (Kang & Liu, 2018). Later, knowledge of such geometric concepts will also aid students in more advanced classes such as precalculus and calculus (Dossey et al., 2002).

Lesson: Determining Surface Area & Volume of Prisms, Pyramids, and Cylinders
Materials Needed: Color cubes, copies of the pyramid template (Figure 2), scissors, tape, rulers, empty soup cans (with lids safely removed), pre-scored measuring cups, water, calculator (optional).

Student Learning Objectives**:**

- Discover properties of finding the surface area of prisms, pyramids, and cylinders.
- Discover properties of finding the volume of prisms, pyramids, and cylinders.
- Connecting hands-on discoveries and properties to the actual formulas for finding similar computations numerically.

This lesson addresses the following NCTM Principles and Standards for School Mathematics for Measurement (2000):

Grades 6–8 Expectations:

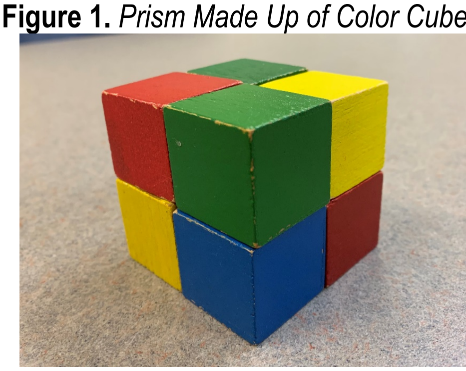
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.
- Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes.
- Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders.

Grades 9 – 12 Expectations:

- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.
- Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.

Surface Area and Volume of Prisms
One way to introduce students to the idea of surface area and volume is to have a rectangular prism made up entirely of smaller, square-shaped cubes. The cubes can be made up of manipulatives such as color blocks or dice. Students would then be asked to devise a way to find the surface area and volume of the shape, without counting every cube individually that makes up the object (Mangum, 2018). Figure 1 shows an example of a "modeled" prism.

Figure 1. *Prism Made Up of Color Cubes.*

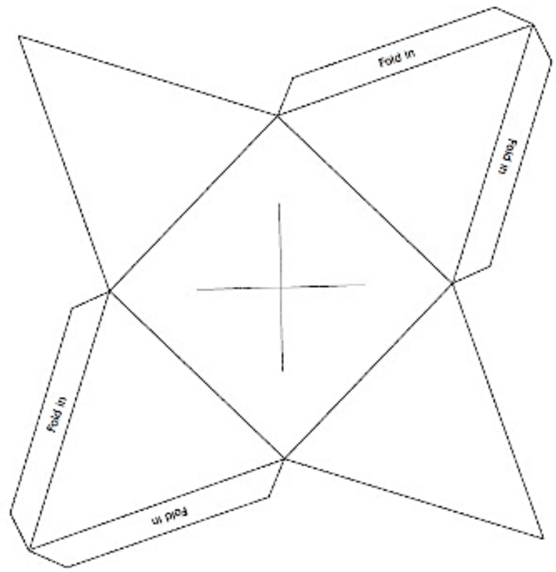


Their goal is to find how many square units make up the surface area of the object and how many cubic units make up the volume of the object. Students should work in groups and then each group will share their final answers with the class as well as what methods they used to find their answers. As a follow-up, student groups can make their own rectangular prisms, and then ask another group to find the surface area and volume of the newly created prism.

Surface Area and Volume of Pyramids

To start a discussion on the volume of a pyramid, student groups will be provided with three copies of the pyramid template shown in Figure 2.

Figure 2. *Pyramid Template* (First Palette, 2018).



Using scissors and tape, students will cut-out and form three pyramids and then fit them together to show why the volume of a pyramid is one-third of the volume of a corresponding rectangular prism. By using the paper pyramids, students can draw conclusions about what is happening and how the two formulas used to find these volumes are similar (Mangum, 2018).

With one of their paper pyramids, students will use a ruler to find the height of one side of a triangle, in centimeters, and then find the length of its base. Using the areas of all five sides that make up the pyramid, students will find the surface area of the pyramid by adding the areas of these sides together, arriving at the conclusion that the area of two of the triangles combined would equal the area of a corresponding quadrilateral (Mangum, 2018).

Surface Area and Volume of Cylinders

To investigate the volume of a cylinder, if plastic cylinders are available, then they can be used with pre-scored measuring cups and water to determine the volume. Empty soda cans or empty soup cans, with the lid safely removed by using a can opener that does not create a sharp edge, can also be used to help students "discover" how much volume various cylinders can hold (Mangum, 2017). In addition, this is also a great time to tie-in a discussion of π and the relationship between the circumference of a circle and its diameter.

To investigate the surface area of a cylinder, students will carefully remove the label from a soup can, lay it flat, and find the area of it as if it were a rectangle. They will then find the area of the base, multiply this measurement by two (to account for the top and the bottom), and then add all sides together.

Conclusion

After students have spent some time "discovering" the surface areas and volumes of the different shapes involved, the teacher will then present the formulas for surface area and volume that are used for each computation, as given in Table 1. Hopefully, a conceptual understanding of these formulas will be gained by the students. In the real world, everything that is mass produced must be designed, from a box of cereal sitting on a store shelf to a new car. The formulas have many uses for helping design objects in the real-world and are the basis for many engineering concepts.

After reviewing each formula, students will discuss how these formulas are related to the exploratory methods that they previously used to determine the surface areas and volumes of the various objects with which they worked. The hands-on activities that were done at the beginning of the lesson will help the students understand how the formulas work and hopefully help them retain this knowledge more so than just providing the formulas up front and working every problem using only pencil and paper. As a follow-up, the teacher may want to provide additional problems for practice that use the formulas.

Table 1. *Surface Area and Volume of Select Prisms, Pyramids, and Cylinders*
Adapted from Hoffer et al., 1998, p. 606, 611, 616, 627, 632, & 636.

Measurement:	Formula:
Surface Area of a Right Prism	$SA = LA + 2B$ or $SA = p \cdot h + 2B$ *LA* is the Lateral Area and is given by the product of the perimeter of the base and the height of the prism. *B* is the area of the base.
Volume of Right Prism	$V = L \cdot W \cdot H$
Surface Area of Pyramid (with square base)	$SA = LA + B$ or $SA = \frac{1}{2} p \cdot s + B$ *LA* is the Lateral Area and is given by one-half the product of the perimeter of the base and the slant height of the pyramid. *B* is the area of the base.
Volume of Pyramid (with square base)	$V = \frac{1}{3} B \cdot H$ where *B* is the area of the base, or one side squared.
Surface Area of Cylinder	$SA = LA + 2B$ or $SA = 2\pi rh + 2\pi r^2$ *LA* is the Lateral Area and is given by the product of the circumference of its base and the height of the cylinder.
Volume of Cylinder	$V = B \cdot h$ or $V = \pi r^2 h$ *B* is the area of the base.

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Playing With Blocks

Gail Kaplan, Towson University, Towson, MD

The author presents an activity using physical manipulatives to visualize the algebraic formula for the difference of two cubes.

Block City*

What are you able to build with your blocks?
Cubes and solids, models of math.
Ideas may be challenging and others give up
But you can persist and take the time
To build understanding from bottom to top.

**Poem Modeled after Block City in*
A Child's Garden of Verses by Robert Louis Stevenson (1946)

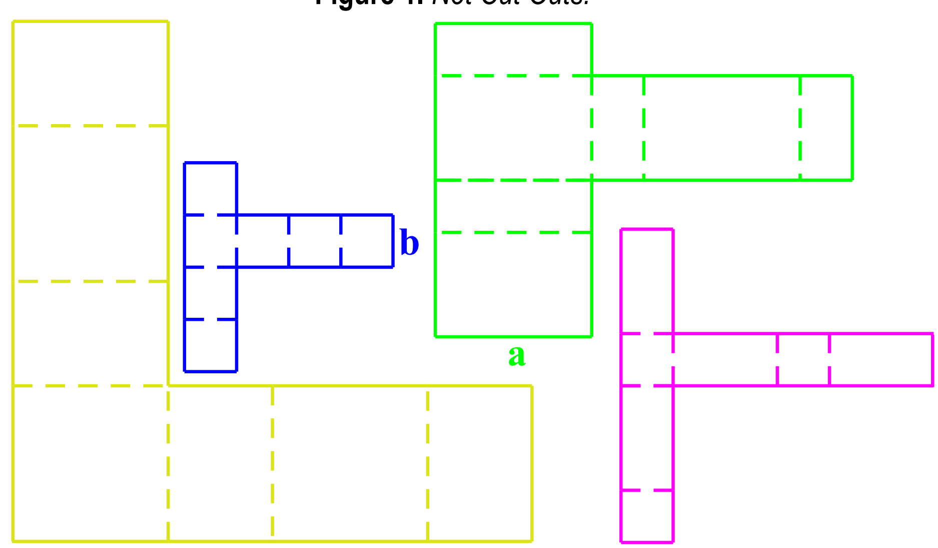
An in depth understanding of mathematical ideas is often challenging for students. In this article we explore a hands-on approach to discovering how to factor the difference of two cubes. Students work in groups as they cut out nets, create 3-dimensional rectangular blocks from the nets, and use these blocks to create a large cube. By removing a small cube from the corner of the created cube, students gain a visual understanding of the formula for factoring the difference of two cubes. Playing with the rectangular blocks provides the opportunity for students to gain insight on why the formula for factoring the difference of two cubes is valid. This activity supports the North Carolina Algebra Standards from the Standard Course of Study (NCSCOS) NC.M1.A-SSE.1 and NC.M3.A-SSE.1, which state "Interpret expressions that represent a quantity in terms of its context" (NCDPI, 2019a, 2019b).

To begin students are given the four nets shown in Figure 1, and directed to do the following.

1. Cut along every solid line.
2. Fold along every dashed line.
3. Use tape to create a rectangular block from each of the 4 nets.

Note that on each net the measure of one side is given. Using all of the pieces it is possible to find the dimensions of each rectangular solid created.

Figure 1. *Net Cut Outs.*



Once formed, students should have constructed the four blocks shown in Figure 2. Note that the only rectangular block that is a cube is the small blue block. Using all of these blocks, students create the multicolored cube (Figure 3) with the yellow rectangular block on the bottom and the blue cube at the top right corner. The longest sides of the yellow block are a units long; therefore, the volume of the cube created from all of the rectangular blocks is a^3 cubic units.

Figure 2. *Rectangular Solid Blocks*

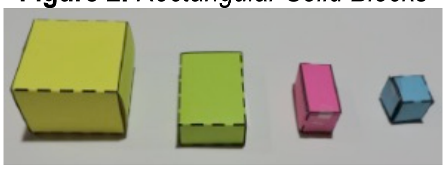
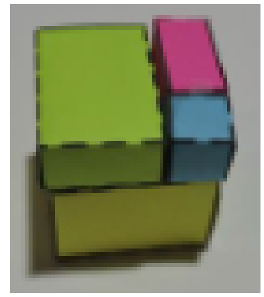


Figure 3. *Cube Created from Rectangular Blocks.*



Each side of the blue cube is b units long, so the dimensions of the blue cube are $b \times b \times b$. Students now complete the row for the blue solid in the Dimensions and Volume Chart (Table 1).

Notice that the height of the multicolored cube formed using all of the rectangular blocks created from the four nets is the sum of the height h of the yellow rectangular solid plus the height of the blue cube, $h + b$. Since the multicolored solid is a cube, this means that $h + b = a$ and $h = a - b$. This tells us that the dimensions of the yellow rectangular solid are $a \times a \times (a - b)$, so the volume of the yellow rectangular solid is equal to:

$$(a)(a)(a-b) = a^2(a-b) = a^3 - a^2b$$

Using this type of reasoning, students work in groups to complete the Dimensions and Volume Chart using only the letters a and b.

Table 1. *Dimensions and Volume Chart (completed)*

Solid	Dimensions of the Solid	Volume of the Solid
Yellow Solid	$a \times a \times (a-b)$	$a^2(a-b)$
Blue Solid	$b \times b \times b$	b^3
Green Solid	$a \times b \times (a-b)$	$ab(a-b)$
Pink Solid	$b \times b \times (a-b)$	$b^2(a-b)$
Cube Created from All Above Solids	$a \times a \times a$	a^3

This activity supports the North Carolina standards for Math 3, “Seeing Structure in Expressions” (NCDPI, 2019b):

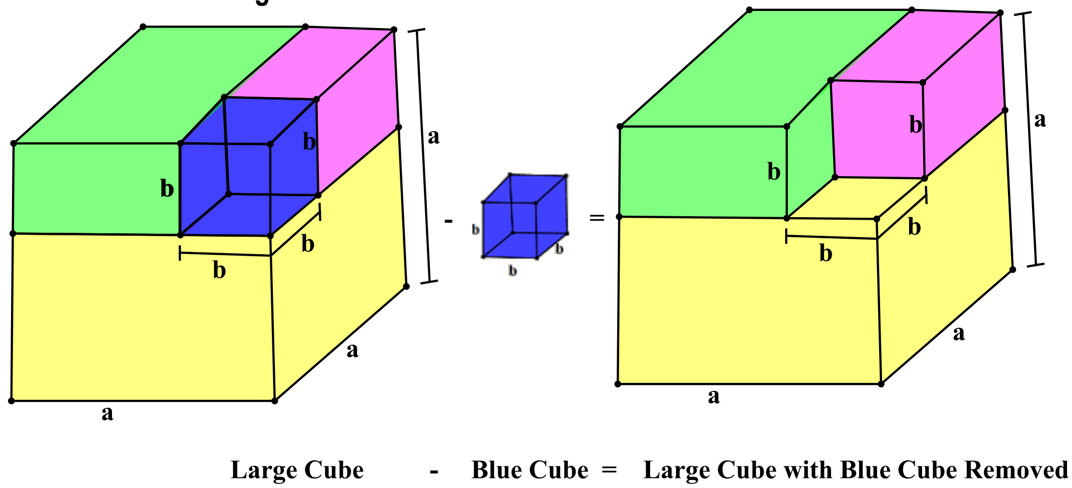
- NC.M3.A- SSE.1: Interpret expressions that represent a quantity in terms of its context. a. Identify and interpret parts of a piecewise, absolute value, polynomial, exponential, and rational expressions including terms, factors, coefficients, and exponents; b. Interpret expressions composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.
- NC.M3.A- SSE.2: Use the structure of an expression to identify ways to write equivalent expressions.

Using the volumes from the completed chart, we see that the volume of the multicolored cube, minus the volume of the small blue cube is $a^3 - b^3$. Notice that the large multicolored cube with the blue cube removed consists of the green, pink, and yellow rectangular solids (Figure 4). This leads to the following equations.

Volume of Large Cube – Volume of Blue Cube =
Volume of yellow solid + Volume of green solid + Volume of pink solid

$$a^3 - b^3 = a^2(a-b) + ab(a-b) + b^2(a-b) = (a-b)(a^2 + ab + b^2)$$

Figure 4. The Volume with the Blue Cube Removed.



This exploration also supports the following North Carolina Standards for Math 1, "Seeing Structure in Expressions Interpret the structure of expressions" (NCDPI, 2019a).

- NC.M1.A-SSE.1: Interpret expressions that represent a quantity in terms of its context.
- NC.M1.A-SSE.1a: Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.
- NC.M1.A-SSE.1b: Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

The student handout that follows this article enables students to discover the formula to factor the difference of two cubes by building the multicolored cube and then applying the formula to other more typical examples as shown in the following student handout. It is a great "AHA!" experience and supports the *Standards for Mathematical Practice* (CCSSI, 2022):

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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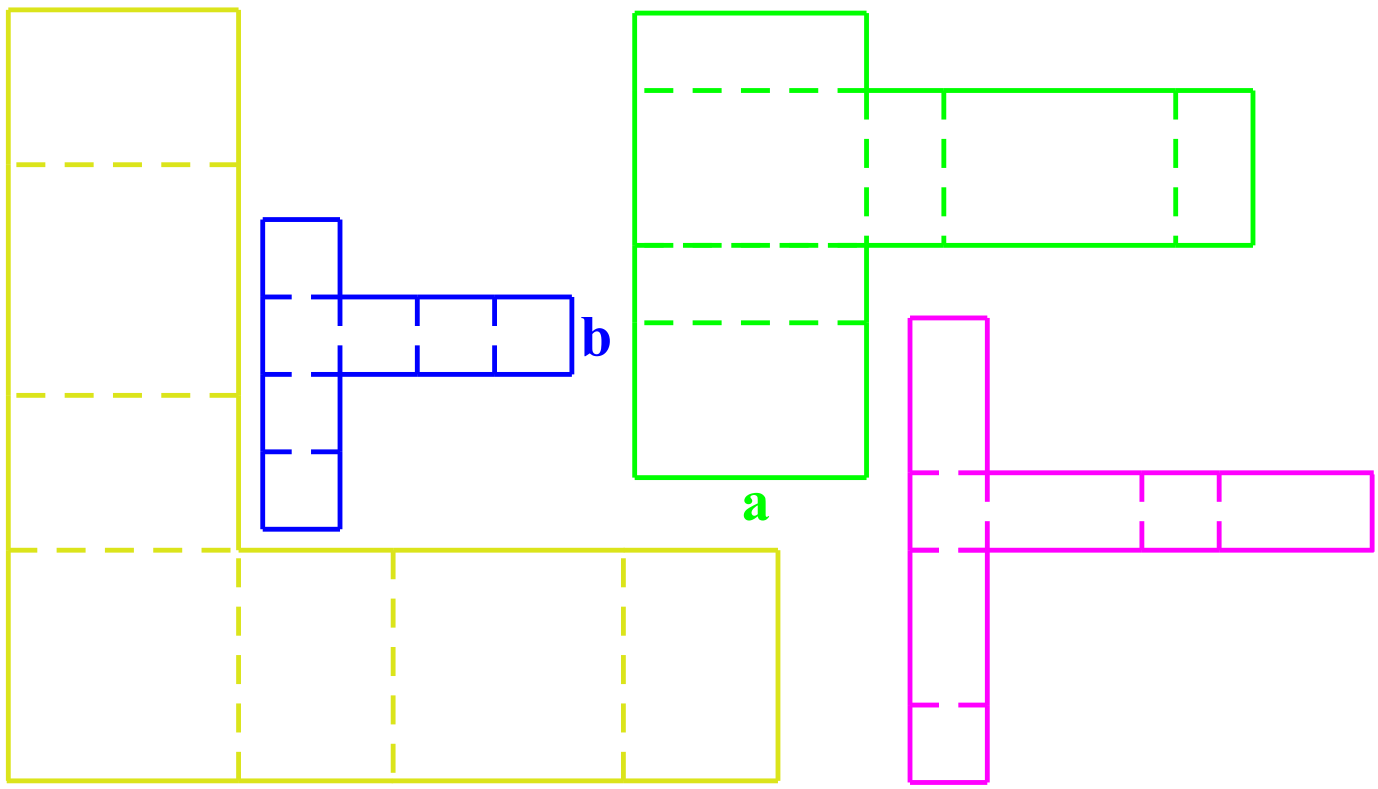
Student Handout: Playing with Rectangular Prisms to Discover the Difference of Two Cubes

I. For each of the nets below,
   a. Cut along every solid line and fold along every dashed line
   b. Use tape to create a rectangular block from each of the 4 nets.

   Which of these rectangular blocks is a cube? Why?

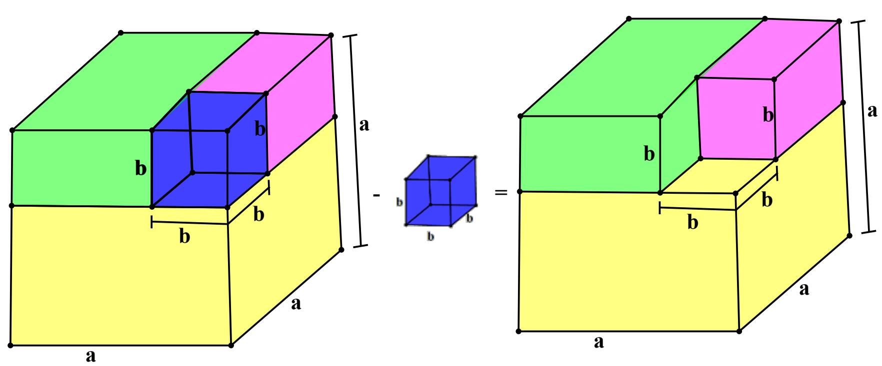


II. Use all of the rectangular solids created from section I to form a single cube with the yellow rectangular solid on the bottom and blue rectangular solid in the front right corner.

   a. The volume of this cube is ______________ cubic units.

   Remove the small blue cube with side length b from the front right corner of the larger cube.

   b. The volume of the Large Cube with the Blue Cube removed is __________________ cubic units.



Large Cube - Blue Cube = Large Cube with Blue Cube Removed

III. Find the dimensions of each rectangular solid to complete the chart below. The first row is done for you.

Rectangular Solid	Dimensions of the Solid	Volume of the Solid	Volume of the Solid In Factored Form
Yellow	$a \times a \times (a-b)$	$a^3 - a^2b$	$a^3 - a^2b = a^2(a-b)$
Blue			
Green			
Pink			
Multi Colored Cube			

The volume of the solid obtained with the corner cube removed is $a^3 - b^3$. Using the volumes from the chart above, we see that the volume of the large multicolored cube minus the volume of the small blue cube, $a^3 - b^3$, is the sum of the volumes of the green, pink, and yellow rectangular solids. Complete the equation below.

$a^3 - b^3$ = Volume of the yellow solid +Volume of the green solid + Volume of the pink solid

$a^3 - b^3$ = ______________________+______________________+______________________

Factor out the common factor.

$a^3 - b^3$ = (______________________) (______________________)

You have now found the formula for factoring the difference of two cubes! Carefully read the following examples showing how to use this formula.

Example #1
$x^3 - 8 = x^3 - 2^3$
so $a = x$ and $b = 2$.
$x^3 - 8 = x^3 - 2^3 = (x-2)(x^2 + 2x + 2^2)$

Example #2
$x^3 - y^3$
so $a = x$ and $b = y$.
$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Example #3
$8m^3 - 27n^3 = (2m)^3 - (3n)^3$
so $a = 2m$ and $b = 3n$.
$8m^3 - 27n^3 = (2m-3n)[(2m)^2 + (2m)(3n) + (3n)^2]$
$= (2m-3n)(4m^2 + 6mn + 9n^2)$

Follow the pattern in the above examples to factor each expression below.

1. $64k^3 - 27s^3$ =
2. $125t^3 - 343r^3$ =
3. $(x-7)^3 - (y+1)^3$ =
4. Create another expression that is a difference of two cubes and factor that expression. Show all details.

Problem Solving Using a Balance Scale

Gregory K. Harrell, Valdosta State University, Valdosta, GA

The author illustrates how a two-pan balance scale provides a real-world context for problem-solving that helps students understand how to solve linear equations.

Algebra is a major part of the middle-grades mathematics curriculum. The 6th, 7th, and 8th grades North Carolina Standard Course of Study (NCSCOS) includes approximately 80 standards, objectives, and elements within objectives related to algebra and algebraic thinking (NCDPI, 2022c, d, e). As we teach these standards, we find that algebraic ideas can be very challenging for some students. Each classroom is filled with learners who differ in their learning styles and mathematical cognition. While some middle school students may be able to generalize abstractly, many struggle with this level of algebraic thought. Many students need more concrete representations, such as hands-on activities, to build algebraic understanding. Exposing students to algebraic ideas before they are ready can lead to frustration and negative attitudes toward mathematical learning (Fennell, 2008; Gojak, 2013). As mathematics educators, we can help students build their understanding of algebraic ideas and facilitate their ability to manipulate algebraic symbols if we provide them with extensive real-world context activities (NCTM, 2000). One such real-world context is the measurement of weight. The 3rd and 4th grade NCSCOS standard "Solve problems involving measurement" (NCDPI, 2022a, b) provides a platform for algebraic learning. Using a balance scale representation of algebraic ideas allows students to explore, through hands-on manipulation, the symbols of algebra in a real-world context; in turn, this allows students to build new, practical algebraic knowledge.

This article demonstrates how the balance scale is a useful tool for developing students' conceptual understanding of linear equations and building their problem-solving abilities related to linear equations. It also shows how the balance scale is a useful tool for differentiating instruction as it provides an important link between lower-level strategies—such as guess, check, and revise—and symbolic strategies—such as the ability to set up and solve linear equations.

Problem Development and Classroom Resources

The problems used for balance scale exploration are designed so that each group of students has a two-pan balance with a weight set containing one-gram, five-gram, and ten-gram weights, such as a 54-piece stackable Hexagram Weights set. The hex weights serve as the objects of known weight for the students, and one set of weights can be shared by two groups of students. There are many varieties of two-pan balances available, but the two-piece stacking balances are inexpensive and can easily be stacked in a small amount of space when not in use. Prior to working with the students, I chose household and classroom items, which are named in each of the five balance scale problems, to serve as the objects of unknown weight for the students. This not only allows students to become familiar with their tools for learning, but it also allows students to see these tools being practically used with every day, recognizable items.

Since these problems are introductory hands-on problems, I chose objects that weigh close to whole-numbered grams so that students can find the unknown weight using hands-on manipulations without requiring "breaking apart" the one-gram weights, which is not possible using the given weight set. The U.S. Mint provides specifications, including the weight, for U.S. coins. The nickel weighs 5.000 grams and the Native American Sacagawea $1 coin weighs 8.1 grams (United States Mint, 2022). The two-pan balance scale is not accurate enough to detect a difference between 8.0g and 8.1g, but be aware that placing numerous $1 coins on one side of the scale will add up, so ten $1 coins will be 81g and will not balance with 80g from the weight set.

Weigh-Too™ cubes, which were available in my particular classroom, were used for the unknown weights in Problem 3. They weigh precisely 8g each, so three of these cubes balanced 24 one-gram weights on the scale. However, they are no longer in production, so the Sacagawea $1 coin may be used instead. Three of these coins weigh 3 x 8.1 = 24.3g, which will tilt the pan very little, if any, when placed on the scale in the pan opposite the 24 one-gram weights

Balance Scale Rule

While it is important for 3rd and 4th grade students to understand measurement of weight and use of the balance scale to measure, it is also important for 6th through 8th grade students to know that measurement and algebraic thinking are not synonymous. When measuring on the balance scale, we start with each side (pan) of the scale empty. We place the object of unknown weight on one pan, which causes the pan to tilt. We then place known weights on the other pan until the scale balances. Placing or removing weights until the scale balances is measurement. When solving linear equations, however, we start with a scale that is balanced with objects of unknown and known weights on the pans. When we start with a scale that is initially balanced (=), we must maintain that balance (=). Before removing objects from the scale, we must first decide what objects to remove from the scale pans in order to maintain balance. Following this "Balance Scale Rule" requires algebraic thinking and fosters algebraic thinking through a visual, hands-on approach.

Classroom Use of the Problems

The balance scale problems are designed to be used by mathematically diverse groups of students in a problem-solving classroom as an introduction to solving linear equations. By allowing each student to problem-solve using a strategy of his or her own choosing, each individual student can build new knowledge, thus providing access to learners who have various levels of knowledge. The problems can also serve as a formative assessment to determine each student's current level of knowledge, so they have been used in one-on-one help sessions as well. These problems fit well in classrooms using differentiated instruction. For example, if tiered lessons are used, teachers can use the balance scale as a tier to help students transition to solving linear equations using the symbols of algebra. If parallel tasks are used, problems 1-3 serve as parallel tasks (Small, 2009). Each problem is a challenge, but a different sort of challenge. Specifically, problem 1 requires the subtraction property of equality on the balance scale; problem 2 requires both the subtraction and division properties of equality; and problem 3 requires the division property of equality. No matter which parallel task a student chooses to solve, he or she will still be able to contribute to the whole-class discussion. Similarly, problems 4 and 5 serve as parallel tasks. Problem 4 represents the "infinite solutions" case when solving linear equations, while problem 5 represents the "no solutions" case when solving linear equations. Many students who find problem 4 daunting can more readily determine that problem 5 describes an impossible balance scale.

Sample Student Work

Rising 6th through 9th graders who were in the same classroom during summer school were given the problems, and the teacher explained the Balance Scale Rule before starting. The students were then allowed to choose how to work the problems. For these activities, they worked in heterogenous groups. All students in each group had their own balance scale, weights, and objects so they could explore individually as well as participate in group discussion.

<u>Problem 1</u>. *Two nickels and two one-gram weights weigh the same as one nickel, one five-gram weight, and two one-gram weights. How much does one nickel weigh in grams?*

After placing two nickels and two one-gram weights in the left pan and one nickel, one five-gram weight, and two one-gram weights in the right pan, Antonio, a seventh grader, said that he didn't know how to do this problem, but he wanted to try using the balance scale. The teacher reminded him of the Balance Scale Rule, and Antonio worked on the problem using the balance scale. He later shared, "I took off the same pieces on each side. They were a nickel and two one-gram weights. I knew that the scale had to be balanced after I took off the pieces because I took the same pieces off each side. I then knew the nickel weighed five grams because the scale was even with a nickel on the left side and a five-gram weight on the right."

Lacretia, a seventh grader, also used the balance scale, but she did not have a solid understanding that the balance scale is used to measure the quantity of grams, not the quantity of objects. After placing the objects on the scale, she said, "I can take one nickel from the left side and one five-gram weight from the right side. The scale still balances." When asked, "How did you know that the scale would still balance before removing the nickel and five-gram weight?" Lacretia replied, "Since they are the same number. There is one of each of them, so I removed one from both sides."

Andy, an eighth grader, set up an algebraic equation and wrote,

$$
\begin{aligned}
&2n+1g+1g=1n+5g+1g+1g\\
&-1n \qquad\qquad\quad -1n\\
&1n+1g+1g=5g+1g+1g\\
&\qquad -2g \qquad\qquad -2g\\
&1n=5g\\
&1\ nickel\ \ weighs\ 5\ grams.
\end{aligned}
$$

Problem 2. *Three dice and two one-gram weights weigh the same as one die, two five-gram weights, and two one-gram weights. How much does one die weigh in grams?*

Katie, a sixth grader, struggled to find a solution using the guess, check, and revise strategy. She knew the strategy and wanted to use it, but she was not sure how to start the strategy in this context. The teacher suggested, "Here, hold this die. What do you think that it weighs in grams?" After some thought, Katie responded, "Maybe ten grams?" The teacher then said, "Okay, now check ten grams in your story problem. Does it work?" Katie shared, "If I was right, the three dice and two one-gram weights would be 32. The one die, two five-gram weights and two one-gram weights would be 22, not 32."

While Katie needed some help determining her first guess, she was very skilled at continuing the strategy. Her second guess was five grams. When asked why she chose five grams, she said, "Well 22 and 32 have a difference of ten. Guessing nine grams would only change it by three, so I knew not to stay close to ten grams. Three dice and two one-gram weights is 17. One die, two five-gram weights and two one-gram weights is 17. So the answer is five grams."

Antonio (7th grade) shared, "First, I took two one-gram weights off each side. And there were three dice on one side and one die and two five-gram pieces on the other side. Then I took one die off each side and that left two dice on the left and two five-gram weights on the right so one die is five grams."

David, a seventh grader, used the balance scale in the same way as Antonio; however, when he had two dice on the left and two five-gram weights on the right, he turned one of the two dice so that six dots showed on top instead of five. When asked why he did this, he said that he wanted to see if the scale stayed balanced. From further questioning, David understood that the balance scale measures weight, not the number of dots on the dice; however, David's years of experience playing dice-based board games with his family at home had left a strong imprint that "different dots mean different values." He was not surprised by the results when he turned the one die, but he needed to resolve the conflicting ideas that he had in his mind.

Andy (8th grade) again set up an algebraic equation. He wrote,

$$
\begin{aligned}
&3d + 1g + 1g = 1d + 5g + 5g + 1g + 1g \\
&-1d \qquad\qquad -1d \\
&2d + 1g + 1g = 5g + 5g + 1g + 1g \\
&\qquad -2g \qquad\qquad\qquad -2g \\
&2d = 10g \\
&\div 2 \quad \div 2 \\
&1d = 5g \\
&1 \textit{ dice weighs } 5 \textit{ grams.}
\end{aligned}
$$

Problem 3. *Three Weigh-Too™ Cubes weigh the same as 24 one-gram weights. How much does one Weigh-Too™ Cube weigh?*

Katie (6th grade) shared, "Three cubes is 24 grams. So I need 1.3 of 24 and that is eight. One cube is eight grams." Andy recorded, "$24g \div 3c = 8$. Weigh-Too™ cubes weigh eight grams each."

Antonio (7th grade) said, "One cube weighs eight grams." When asked why, Antonio said, "I just know. Three of eight is 24." When asked if he could also use the Balance Scale Rule to remove equal weights from both sides of the scale, however, he struggled. He said, "I can remove one cube from one side and eight grams from the other side because I know that one cube weighs eight grams." When asked if he could remove a fraction of the weight from both sides, Antonio thought for a while. He later removed 1/3 of the weight from both sides, leaving two cubes on one side and 16 one-gram weights on the other side. He then removed 1/2 of the weight from both sides, leaving one cube on one side and eight one-gram weights on the other side.

Problem 4. *Two jumbo paper clips and three one-gram weights weigh the same as two jumbo paper clips and three one-gram weights. How much does one jumbo paper clip weigh in grams?*

Katie (6th grade) held the paper clip in her hand as she had previously done with the die. She later wrote in her journal, "I guessed the paper clip weighed two grams. And then I realized if it was one gram that would be right. And two grams and three and four. So, it could be any amount."

Andy had exhibited strong skills in problems one through three as he set up equations and solved them symbolically step-by-step. However, he struggled with problem four. Andy wrote,

$$
\begin{aligned}
&2c + 1g + 1g + 1g = 2c + 1g + 1g + 1g \\
&\quad -1c \qquad\qquad\qquad -1c \\
&1c + 3g = 1c + 3g
\end{aligned}
$$

He then asked the teacher for help. He did not know what algebraic steps to take next. He wanted $1c$ to stay in the equation, but he didn't want it to stay in the equation on both sides. Andy said, "No matter what I do I'm still going to end up with the same things on both sides." The teacher then had the following conversation with Andy.

Teacher:	*Could the clip weigh five grams?*
Andy:	*I don't think so.*
Teacher:	*Why not?*
Andy:	*Well, I think it weighs two grams.*
Teacher:	*Why?*
Andy:	*Because it's small.*
Teacher:	*Based only on the information in the story, what could it weigh?*
Andy:	*I don't know.*

Teacher: *Try checking five grams in the story, then another weight, say two grams.*
Andy: *Five grams gives 13 grams equals 13 grams. Two grams gives 7 grams equals 7 grams.*
Andy: *Oh… It could weigh anything.*

Andy showed strong measurement sense as he recorded in his journal, "No matter what I do I'm still going to end up with the same objects on each side. With this information, a jumbo paper clip could weigh anything, but I think it weighs about 2 grams." While Andy and Katie were in different grades, both had important understandings that contributed to group discussion.

Problem 5. *Three pennies and two one-gram weights weigh the same as three pennies and five one-gram weights. How much does one penny weigh in grams?*

Katie wrote in her journal, "Three pennies and two one-gram weights can't weigh the same as three pennies and five one-gram weights no matter what one penny weighs." Andy wrote, "$3p + 2g = 3p + 5g$. That's impossible because $5g$ is more than $2g$. There is no such p."

While Katie relied more on her numeric guess, check, and revise understanding and Andy continued to rely on his algebraic equations, Antonio depended more on the balance scale representation. As he pointed at the objects on the scale, he said, "This [three pennies and two one-gram weights] doesn't weigh the same as that [three pennies and five one-gram weights]. The scale isn't even." When asked, "Then how do you answer the problem? How much does one penny weigh, in grams?" Antonio was uncertain how to respond. To help Antonio understand there is no such penny weight, p, it may help to ask him, "Do any penny weights exist that will balance the scale as described in the story?"

Conclusion

Providing the balance-scale problems, along with a balance scale, facilitates differentiation of mathematics instruction. This differentiation occurs by accommodating differences in students' learning styles as well as differences in their cognition. In reviewing Katie, Antonio, and Andy's problem-solving strategies, we see the students used three major strategies for determining the unknown weight on the balance scale. Katie tended to use her ability to estimate weight as well as the guess, check, and revise strategy; Antonio used the balance scale; and Andy set up an algebraic equation and solved the equations symbolically on paper. The problem-solving environment allowed them to meet the problems from the perspective of their own knowledge. As they shared their results, each person contributed within the groups and across the groups, which helped all of them learn from each other. The differences in grade level between the students contributed to differences in their prior knowledge, and in result, their problem-solving strategies and answers.

We can see that the balance scale provides a rich context for learning as the students all struggled at various times with their knowledge in relation to measuring weight, using the balance scale to find the unknown weight, and setting up and solving linear equations based on the real-world context of a balance scale. However, by the end of these hands-on activities, all of the students understood these ideas, as well as how the ideas link together, much better.

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Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the “awards” area of the NCCTM Website, www.ncctm.org. More information can be obtained from Emogene Kernodle, nekernodle@yahoo.com.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the “awards” area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Ana Floyd, afloyd@randolph.k12.nc.us

Problems2Ponder

Holly Hirst, Appalachian State University, Boone, NC

In each issue of The Centroid, Problems2Ponder presents problems similar to those students might encounter during elementary and middle school Olympiad contests. Student solution submissions are welcome as are problem submissions from teachers. Please consider submitting a problem or a solution. Enjoy!

Problem submissions: If you have an idea for a problem, email Holly Hirst (hirsthp@appstate.edu) a typed or neatly written problem statement, along with a solution. Include your name and school so that we can credit you.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst (hirsthp@appstate.edu) a clear image or PDF document of the correct solution, with the name of the school, the grade level of the student(s), the name of the student(s) if permission is given to publish the students' names, and the name of the teacher.

Deadline for publication of problems or solutions in the Spring 2023 Centroid: December 30, 2022.

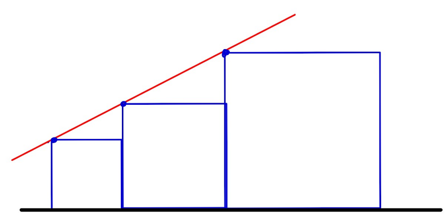
Fall 2022 P2P Problems

Problem A: The arithmetic mean of the numbers denoted by A and B is 18, and the arithmetic mean of the numbers denoted by C, D, and E is 43. What is the average of all five numbers A, B, C, D, and E?

Problem B: The fraction $\frac{5}{7}$ can be approximated by a decimal. What is the digit the lies in the 2022 place in its decimal expansion?

Spring 2022 P2P Problems

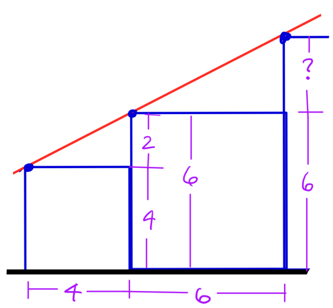
Problem A: Three adjacent squares rest on a horizontal line as pictured. Their upper left corners all lie on the slanted red line. If the left-most square has area 16 and the middle square has area 36, what is the area of the third square?



Solution 1: Use the concept of slope.

$$\frac{rise}{run} = \frac{2}{4} = \frac{?}{6}$$

Thus the unknown height is 3 and the at the
third square is 9 by 9 with area 81.



Solution 2: Use similar triangles.

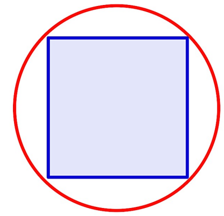
$$\frac{vertical}{horizontal} = \frac{2}{4} = \frac{?}{6}$$

The legs are in proportion, and we arrive

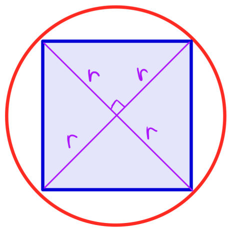
same calculation!

Interesting follow up question: If we continue to draw adjacent squares whose upper left corners fall on the red line, what are the areas of the next three squares?

Problem B: A square with an area of 18 square cm is inscribed in a circle. Find the size of the unshaded area inside the circle that is outside of the square.



Solution: If we draw the diagonals through the square, we have 4 internal congruent right triangles (why)? Since these triangles have equal area, each area is $\frac{18}{4} = 4.5$ square cm.



We also know that each of these right triangles has area $\frac{1}{2}\, bh = \frac{1}{2}\, r^2 = 4.5$, and so $r = 3$. Now we are ready to calculate the difference between the area of the circle (πr^2) and the area of the square:

$$\pi \times 3^2 - 18 = 9\pi - 18 = 9(\pi - 2)$$

If you would be happier with a decimal approximation: 10.27 sq cm.

Trust Fund Scholarships: $1000

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
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- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the “grants & scholarships” link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson Plumblee, richards@elon.edu.

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Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

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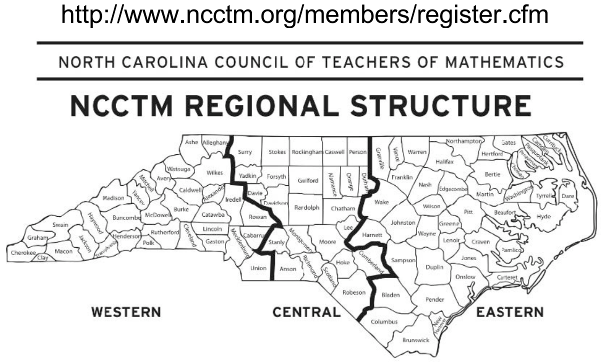

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