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Problems to Ponder

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics-pre-kindergarten through college levels. The Centroid is published each year with issues in Fall and Spring.

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National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Journal articles:

Perry, B. K. (2000). Patterns for giving change and using mental mathematics. Teaching Children Mathematics, 7, 196-199.
Chapters or sections of books:
Ron, P. (1998). My family taught me this way. In L. J. Morrow \& M. J. Kenney (Eds.), The teaching and learning of algorithms in school mathematics: 1998 yearbook (pp. 115-119). National Council of Teachers of Mathematics.
Websites:
North Carolina Department of Public Instruction. (1999). North Carolina standard course of study: Mathematics, grade 3. http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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## Fall 2023 State Math Conference And Leadership Seminar

Leadership Seminar: November 8, 2023
Conference: November 9-10, 2023
Benton Convention Center Winston-Salem

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- Math Game Night with the NC
 Network of Math Teacher Ciricles
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Earlybird registration ends on September 27. Reduced hotel rates end on October 10.
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# President's Message 

Karen McPherson
State President
karen.mcpherson@bcsemail.org
The theme of this year's math conference is "Be Bold: Teaching Practices to Champion Access and Equity for Each Student." Many of our schools and districts in North Carolina are actively working towards creating, supporting, and sustaining a culture of access and equity. As an organization, NCCTM is fully committed to aiding teachers and leaders in this critical work within the realm of mathematics education. In this context, let's delve into two fundamental components vital for ensuring that every student has an opportunity to succeed in mathematics.

First and foremost, teachers' beliefs about their students' abilities in math significantly shape their expectations and drive their instructional choices. A teacher who firmly believes that all students can learn and excel in math is more inclined to establish high expectations, offer support and differentiation, and cultivate positive math identities among their students. To assess your own beliefs, ask yourself questions like, "Do I believe that all students are capable of tackling complex math tasks? Do I have confidence that every student can actively engage in mathematical discourse? Do I believe that all students can grasp grade-level mathematics?" Your responses to these questions directly impact your teaching approach and the educational experiences you create for your students. Regularly examining and aligning our beliefs with our instructional practices is a crucial step towards providing access and equity for all students.

Secondly, teachers must prioritize grade-level instruction for every student. This involves ensuring that the curriculum, tasks, and resources are of high quality and aligned with grade-level standards, emphasizing the rigor aspects outlined by those standards, such as conceptual understanding, procedural skill and fluency, and application. Grade-level learning is an imperative for each student. Often, in attempts to address unfinished learning, instruction tends to focus on prior grade-level material. Research has shown that out of the 180 classroom hours in a school year, students spent a staggering 133 hours on assignments that were not appropriately aligned with their grade level.** Interestingly, the same study revealed that "students who started the year behind closed the outcomes gap with their peers by more than seven months when they had greater access to grade-appropriate assignments." This underscores the importance of providing students with grade-level instruction while not dismissing the need for just-in-time scaffolding and some remediation. Grade-level instruction must remain the priority, as students greatly benefit from working with grade-level content consistently.

These two fundamental components-believing in every student's ability to learn and demonstrating this belief through expectations and instruction, and prioritizing grade-level instruction through high-quality curriculum and teaching-are not the sole elements required to ensure that every student has an opportunity to succeed in mathematics. However, they serve as foundational pillars in the pursuit of equitable instruction for all students. It is important to acknowledge that understanding and implementing these components can be challenging. Our society bombards us and our students with messages that promote unproductive beliefs about teaching and learning mathematics. Additionally, we operate within systems built upon these unproductive beliefs, leading to limited opportunities for certain student groups and perpetuating educational inequities.

While it may feel overwhelming, we must not lose hope. As a united community, we possess the strength to overcome these challenges collectively. Are we willing to scrutinize and question our beliefs? Are we prepared to challenge our students to reach their full potential? Are we ready to Be Bold?

We wholeheartedly invite you to join us as we explore ways to Champion Access and Equity for Each Student. Come be a part of the NCCTM conference at the Benton Convention Center in Winston-Salem on November 9-10, 2023. We eagerly anticipate your presence and look forward to engaging in these critical conversations together. See you there!
**TNTP. (2018). The Opportunity Myth: What Students Can Show Us About How School Is Letting Them Down - and How to Fix It. Available at https://tntp.org/assets/documents/TNTP The-Opportunity-Myth Web.pdf

# Mirrors And Windows to Analyze Waste Production: A Fifth-Grade Teacher Promotes Equitable Learning With Applying Fractions 

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The authors share the development, enactment, and evaluation of a learning unit designed to promote equity in a fifthgrade bilingual mathematics classroom. Unit lessons were written to encourage student voice and to use multiplication and division of fractions as analytical tools of empowerment. Evidence gathered during three major learning activities indicated growth along dimensions of access, equity, identity, and power through the application of researchbased teaching practices.

Equity-based mathematics teaching requires purposeful planning for student access, achievement, identity, and power as recommended by Gutiérrez $(2009,2012)$. Addressing the dimension of power can be more fully realized as students use tools of mathematics in meaningful ways to mathematize their world (Gutstein, 2006). Learning units that infuse these dimensions through mathematizing students' worlds exemplify meaningful steps towards equity-based mathematics teaching and equitable learning.

Designing learning units to include equity-based mathematics teaching practices is no easy feat and often represents a shift in how teachers perceive and plan for teaching. This shift can be directed when teachers purposefully consider students' needs and experiences along the dimensions of access, achievement, identity, and power. Gutiérrez (2009, 2012) provides examples of access being updated and rigorous texts, appropriate technology, and high-quality teachers; achievement to large numbers of students engaged in mathematics; identity to students using home languages and not parking their identities at the door; and power to giving students voice in the classroom and investigating problems meaningful to them. When teachers consider and address these dimensions in their planning, they can make strides in achieving equity-based mathematics teaching where, "all students can participate meaningfully in mathematics learning and create their own mathematical knowledge" (NCTM, 2014b, p. 1). In this article, we share the story of a fifth-grade teacher, Ms. H, as she developed, delivered, and evaluated a learning unit that addressed dimensions of equity.

Initial planning discussions took place between Ms. H and her professor. To establish content, we used the Common Core State Standards for Mathematics (CCSSM; Council of Chief State School Officers, 2010) for fifth grade mathematics. We planned for students to meet content standards on fractions through the application of the eight mathematics teaching practices (NCTM, 2014a) and considered these practices as having a high capacity to address student access, achievement, identity, and power. For example, practice eight, "elicit and use evidence of student thinking" would assist us with infusing equity dimensions of identity and power as it puts students at the forefront of doing mathematics.

Our planning work did not end with addressing content and practice standards, and we developed a plan to gather evidence to assess how the teacher and students made strides along the dimensions of access, achievement, identity, and power. Under this strategy, we responded to the call by Gutiérrez (2002) for action research with practicing teachers to address equitable learning. In the next section, we honor the voice of Ms. H to describe her process and planning of details in the learning unit. We then describe major activities of the learning unit along with their enactments and highlights of student learning. Evidence of strides made in equity-based mathematics teaching
practices and equitable learning is provided in data gathered while students engaged in learning about the multiplication and division of fractions.

## Planning for Equity-Based Mathematics Teaching Practices

I began planning the learning unit from an agent of change disposition (Sánchez-Robayo \& Torres-Duarte, 2017) where I intended to promote equitable learning for my students. I wanted students to use mathematics as an analytical tool of empowerment with a focus on understanding why procedures worked. I had taught this content previously and noticed that students often struggled with understanding and applying fractions. They needed meaningful experiences that would help them retain what they had learned. I found the dimensions of equity provided by Gutiérrez $(2009,2012)$ and NCTM categories of equity-based teaching practices from their Research Brief helpful to get started. In writing my learning unit, I engaged in the three equity-based teaching practices of reflecting, noticing, and engaging in community (NCTM, 2014b).

Reflecting involved "not just reflecting on your pedagogy and your classroom norms, but also considering how you identify yourself and how others identify you" (NCTM, 2014b, p. 1). This description made me think about reflection in a new way and helped me rearticulate my disposition to teaching. My disposition can be broken into three components: "openness; self-awareness/self-reflectiveness; and commitment to culturally responsive mathematics teaching" (NCTM, 2014b, p. 2). After engaging in both mental and written reflection along these lines, I realized that my desire to enact equity-based mathematics teaching was not always supported with my written teaching plans. I had believed that my advocacy disposition was sufficient to bring about equity-based mathematics teaching in the moment. Upon reflection, I changed the way I approached equity-based teaching towards creating a written record of plans that I could go back to in the future to revise and reenact.

Purposeful planning included a commitment of space and time for noticing. This second equity-based teaching practice involves "attending to students' identities and building upon students' prior knowledge" (NCTM, 2014b, p. 2). I wrote a survey for students to complete at the beginning of the school year that asked them about their perceptions of math and learning math. I felt this information would assist me in building on their prior knowledge and experiences to design a learning unit meaningful to them. I planned to notice and build upon students' identities and knowledge in giving all students a voice in the classroom with strategies that promote discourse (e.g., Batista \& Chapin, 2019). The learning unit would also center around a real-world project that was based on students' interests.

The third component of equity-based teaching was engaging in community. I first communicated with my team of teachers to share curriculum materials that addressed conceptual understanding of fractions. Then we shared discourse promoting strategies that give students voice in the classroom and build mathematics community. Engaging in community caused me to consider and adapt the structure of student work. I wanted students to feel responsible for each other's learning through small group collaboration and sharing thoughts with the whole class. I read about Gutiérrez's use of the terms, mirrors and windows: "a mirror in the sense of offering students a chance to see oneself; a window in the sense of being able to see a different view onto the world" (Gutiérrez, 2012, p. 44). I decided to use this pedagogical idea as a metaphor for how students experience the content. This led me to write the unit theme of seeing ourselves as mathematicians (mirrors) being responsible for protecting the environment (windows).

## Learning Unit and Dimensions of Equity

Reflecting, noticing, and engaging in community resulted in a written learning unit, Mirrors and Windows to Analyze Waste Production, that addressed multiplication and division of fractions in real-world contexts. The learning unit was based on a cluster of content standards within Number and Operations-Fractions (CCSSM; Council of Chief State School Officers, 2010), including, "apply and extend previous understandings of multiplication and division to multiply and divide fractions" (p. 36). Writing a learning unit that aligned to these standards supported the planning of opportunities for students to build conceptual understanding, consider multiple representations, and apply the mathematics they were learning. These opportunities aligned with the eight mathematics teaching practices (Table 1).

The driving theme of the learning unit was waste reduction and the removal of microplastics from bodies of water. Motivation for this theme came from student surveys and solidified when students spontaneously organized
during recess to collect trash and plastics bottles from the park. This learning unit contained activities and word problems that invited students to use mathematics as an analytical tool of empowerment in two contexts. The first was to analyze waste production at the school site along with possible ways to reduce it. The second was in a hypothetical situation in which students were hired as experts who investigated removing microplastics from the ocean. These contexts led students to use multiplication and division of fractions and students' understanding was supported with visual models in the technology GeoGebra.

| Table 1. Eight Mathematics Teaching Practices (NCTM, 2014a, p. 10) |
| :--- |
| Establish Mathematics Goals to Focus Learning <br> Implement Tasks That Promote Reasoning and Problem Solving <br> Use and Connect Mathematical Representations <br> Facilitate Meaningful Mathematical Discourse <br> Pose Purposeful Questions <br> Build Procedural Fluency from Conceptual Understanding <br> Support Productive Struggle in Learning Mathematics <br> Elicit and Use Evidence of Student Thinking |

The unit was planned for ten class sessions with lessons adhering to a launch, explore, discuss format. The fifthgrade class consisted of fifteen bilingual students. The four dimensions of equity were tracked throughout learning unit plans. Table 2 highlights each dimension and pedagogical approaches that were planned to address the dimension.

Table 2. Equity Dimensions and Pedagogical Components

| Dimension | Lesson Component | Dimension | Lesson Component |
| :---: | :---: | :---: | :---: |
| Access - completing <br> rigorous math tasks | Using and explaining <br> multiple representations <br> with area models | Access - using <br> appropriate technology | Geogebra app was the tool used by all <br> small groups to model multiplications and <br> division of fractions |
| Achievement - all students <br> participating in <br> mathematics | Think-pair-share strategy <br> and reflective thinking <br> activity | Achievement-all <br> students' learning <br> needs are addressed | Pre-unit assessment to gauge student <br> readiness for learning unit |
| Identity - students as <br> mathematicians with <br> unique perspectives | Oral and written math <br> explanations done in home <br> language | Identity - students as <br> mathematicians with <br> relevant background <br> knowledge | Students report their impressions about <br> learning math before and during the <br> learning unit |
| Power - investigating <br> problems meaningful to <br> them | Math tasks use data on <br> waste reduction at their <br> own school | Power - recognizing <br> students' voices | Sentence Frames to support student to <br> student discourse and multiple students <br> contribute to whole-group discussions |

Addressing dimensions of equity was not limited to the approaches found in Table 2. These dimensions were coupled with mathematics teaching practices (NCTM, 2014a) as threads throughout the learning unit.

## Enacting Mathematics Practices to Promote Equity

Evidence of equity-based mathematics teaching and equitable learning is organized in describing classroom experiences during three extensive preplanned activities. We describe these activities in the order they appeared in
the unit and highlight how each addressed the mathematics teaching practices (NCTM, 2014a) and dimensions of equity (Gutiérrez, 2009, 2012). For example, students completed tasks with appropriate technology (access) that built procedural fluency from conceptual understanding (practice six).

## Activity One: Conceptually understanding multiplication and division of fractions

Before diving into the real-world context of the learning unit, Ms. H first addressed the underlying mathematics to be applied with her students. She based this decision on results of a pre-unit assessment on multiplication and division of fractions (achievement). To establish conceptual understanding of the multiplication and division of fractions, students worked with mathematical models in activity one. To launch this activity, Ms. H shared two "I can" statements to focus learning with her students (practice one).

- I can multiply a fraction by a fraction using area models,
- I can divide a whole number by a fraction using visual models.

She supported learning by sharing videos on how to use the area model for multiplying fractions and dividing whole numbers by fractions before working through examples as a class.

- Video on Multiplication: https://drive.google.com/file/d/0BwXykaXIkRzvSU1VNjN1U2tCdUE/view?resourcekey=0-zlqGkNlwDmq6EYG4N_veQ
- Video on Division: https://www.showme.com/sh/?h=Oa5n56m

Students used computers in the activity and were reminded that they could reference these videos if they needed help with the models (access). Ms. H continued the activity with preplanned demonstrations of multiplication (Figure 1) and division (Figure 2) of fractions using area models.

Figure 1. Sample area model was used to build conceptual understanding of multiplication.


Figure 2. Sample visual model was used to build conceptual understanding of division.


In this activity, the models were scaffolded with videos and fraction strips as seen in Figures 1 and 2. After the class discussed how to use these models (practice four), Ms. H transitioned to models where the fraction strip scaffolds were removed (Figure 3).

Figure 3. Sample visual model with fraction stripes removed was used to build conceptual understanding of division.


Throughout the work with these area models, Ms. H guided whole group discussions and structured small group work to engage all students (access). To provide all students opportunities to talk (power), she displayed and referenced sentence frames for students to use such as,

I respectfully disagree with $\qquad$ 's idea, and I think $\qquad$ because $\qquad$ .

Students used two applications in GeoGebra to connect mathematical representations (practice three) and develop their procedures for multiplication (Figure 4) and division (Figure 5) (practice six). Student discourse coupled with technology helped prepare them to explain why and how procedures worked for multiplication and division in a whole group discussion (practice six). In this whole group discussion, Ms. H displayed four division problems:

$$
2 \div \frac{1}{5}, 2 \div \frac{1}{4}, 2 \div \frac{1}{3}, 2 \div \frac{1}{2}
$$

She prompted, "let's pay attention to what is changing from one expression to another." Fortino responded, "so the bigger the denominator, the smaller the result."

Figure 4. Sample of GeoGebra app (https://www.geogebra.org/m/AZnX7deX) was used by students to model multiplying fractions.


Figure 5: Sample of GeoGebra app (https://www.geogebra.org/m/bs588zhK) was used by students to model dividing whole number by a fraction.

Division of a Whole Number by a Fraction
Author: mathspace
Topic: Division, Fractions


In the final part of the activity, Ms. H had students work on mathematizing word problems (practice two). For example, she posed the problem, "The Jordan family ordered five small pizzas. A serving is $1 / 2$ of a pizza. How many people could the five pizzas serve?" that required students to translate the problem from words to a division expression and simplify to solve. During class work on these word problems, Ms. H used preplanned questions
(practice five) to guide thinking and sharing. Throughout this activity, Ms. H was able to use formative assessment (practice eight) to address students' understanding of word problems and move them towards mathematizing and explaining why procedures worked. Mathematizing and completing word problems would be essential in the next two learning activities.

## Activity Two: Waste Reduction

At the beginning of this activity, Ms. H engaged students in an ice breaker where students collected trash and recyclables in the park (Figure 6). She then explained that the learning unit was motivated by their concern (power) for keeping the park clean. They talked about the difference between waste and recyclables and briefly discussed how individuals could reduce waste and recyclables.

Figure 6. Ms. H's fifth grade class removes waste and recyclables from park.


After this activity, Ms. H had students read the article, "Students surprised how much food waste, plastic trays reach landfill." The article was found on https://newsela.com, which provides articles written for different reading levels. Students analyzed the reading with a thinking track form to support comprehension. The ice breaker and reading served as a window to view real-world problems.

Ms. H continued the activity with moving from the window to the mirror metaphor. She showed the students a bar graph (Figure 7) of actual average waste production over a three-week period at their school. Due to remote learning, students were present for approximately half of these days.

Figure 7: The graph displays average waste production at school each weekday.


To help students make meaning from the graph they responded to preplanned fill in the blank statements such as, When students enter school, garbage production increases approximately $\qquad$ times.

Ms. H and the class discussed two main ways students can help: (1) reduce their waste; and (2) be thoughtful about what you buy and choose a sustainable option whenever possible.

The class transitioned to mathematizing the problem of waste production. Ms. H recognized students' input (identity) with each student first choosing fractions to represent how much waste they think they produce each day in three categories. For example, Amaya chose $1 / 3$ paper, $1 / 3$ food, and $1 / 3$ cans and bottles to represent her individual production. Individual students then chose a fraction to represent how much they believe they could reduce in each category. The class discussed how multiplying the two fractions together for each category would let them know how much each student could reduce their individual waste in each category. Students used GeoGebra (access) and the turn-and-talk discourse strategy (achievement) to share and assess each other's math work (practice four). The activity ended with students using their home language (identity) to explain why they choose their fraction values and how they solved their multiplication applications. Garcia, summarized his turn-and-talk work, "we said $1 / 2$ times $1 / 2$ equals $1 / 4,1 / 3$ times $1 / 2$ equals $1 / 6$, and $1 / 3$ times $1 / 2$ equals $1 / 6$. I learned how much people waste food, paper, and bottles in my school."

## Activity Three: Removing Microplastics

In this final major activity, Ms. H began with engaging the students in learning about a significant environment issue: the removal of microplastics from bodies of water. She displayed and read a description of microplastics and the challenges of removing them from bodies of water. In this way she offered students a window to understanding a real-world problem and a mirror on what may be done to address the problem. This reading was enhanced with images of microplastics and exploring the National Ocean Service website, https://oceanservice.noaa.gov/facts/microplastics.html.

After setting this problem-solving context, students worked in prearranged small groups. First, small groups chose whether they wanted to analyze microplastic from toothpaste or microplastic from cleaners (identity). Students used GeoGebra to model the division situations in their groups and collaborated in small groups in their home language (identity) to make posters of their choices, work, and results. See Figure 8 for an example.

Figure 8: Sample student modeling of division in GeoGebra helped display their understanding.


This student, William, shared the groups' conclusion: "The most difficult microplastic to remove is the one with the fraction of $1 / 5$. And the easiest microplastic to remove is the one with the fraction of $1 / 2$." However, some students struggled with using the visual models to justify their answers (practice seven) but were able to grasp the connections (practice three) with the help of peers and the teacher.

Luis: We said that the most difficult to remove is 3 divided by $1 / 2$ which equals to 6 and the easiest to remove is 3 divided by $1 / 5$ which equals to 15 .
Ms. H: So, is there any person or group that would like to make a comment about that?
Roxy [her group had chosen a different microplastic scenario with different values]: We said that 2 divided by $1 / 2$ would be the easiest to remove but 2 divided by $1 / 5$ would be the hardest to remove.
Ms. H : Tell me more about what that means.
Roxy: We learned the difference between how hard it is to get the smallest pieces out of the ocean than the biggest. Even though the models [Luis' group's models] are correct, they do not realize that they need to check the size of the pieces.

Luis and his group members nodded to express their understanding. Ms. H continued the whole group discussion (practice eight) on thinking with the models and encouraged students to visualize division to make two important conclusions: (1) that larger denominators represent smaller microplastics; and (2) that smaller microplastics resulted in more microplastic pieces to remove. Ms. H guided students to consider how math was used to understand the environmental impact of microplastics (power). The activity concluded with students reading and discussing a news story about a teenage scientist working on removing microplastics from water
(https://www.motheriones.com/environment/2019/08/a-teen-scientist-figured-out-how-to-suck-microplastics-from-the-ocean-there-may-be-hope-for-humanity/).

## Summary in Context of Equity Goals

Designing and enacting a learning unit to infuse equity-based mathematics teaching practices and promote equitable learning represents challenges for teachers that can be met through reflecting, noticing, and engaging in community (NCTM, 2014b). For Ms. H these challenges involved reconsidering how adequately she planned for equitable learning and the degree to which her students were empowered with mathematics. Many pedagogical approaches are available to teachers such as creating discourse communities (e.g., Batista \& Chapin, 2019), which can work to empower students to use mathematics. Real-world problems may serve as mirrors for students to see themselves in the curriculum and mathematics as a tool to problem solve. Problem solving can be made meaningful when access, achievement, identity, and power (Gutiérrez, 2009, 2012) are accounted for in leaning opportunities. When teachers incorporate research-based mathematics teaching practices (NCTM, 2014a) in the design of these learning opportunities, they can more fully realize equitable learning and empowerment for all students.

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# The Dream Car Project: An Example of a Personalized, Real-World Mathematics Task 

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The authors present a project with real world connections suitable for a capstone to a unit on exponential growth and decay. The actual project text and assessment rubric are provided after discussion of the rationale and context.


#### Abstract

A common question many high school mathematics teachers hear from students is, "When am I ever going to use this?" At the heart of this question, students are begging for teachers to help them see the relevance of the mathematics being taught to their everyday lives. Speaking from experience, it can sometimes be challenging to find engaging tasks that are situated in a meaningful context for students. However, providing an authentic, real-world connection to the mathematics they are learning can be very impactful for students (NCTM, 2000, 2014).


According to Gainsburg (2008), real world connections include simple analogies, word problems, analysis of real data, discussion of mathematics in society, hands-on representations of mathematics concepts, and mathematical modeling of real phenomena (p. 200). Using real world connections in a mathematics classroom encourages students to activate their reasoning, problem solving, and critical thinking skills (NCTM, 2014). In return, we often see increases in student motivation and interest, a deeper understanding of the mathematics, and positive impacts on students' attitudes towards mathematics (e.g., Gainsburg, 2008; Karakoç \& Alacaci, 2015). For these tasks to be effective, they must draw upon students' prior knowledge and be connected to students' actual experiences (Muijs \& Reynolds, 2011).

Additionally, prior research suggests personalization also benefits students' learning (Cordova \& Lepper, 1996; Walkington, 2013). Personalization occurs when instructional activities are differentiated to account for students' varied interests, preferences, and experiences (US Department of Education, 2010). To facilitate personalization, instructional activities may be grounded in a context, or real-world situation, that draws upon students' prior knowledge and interests.
Research has shown that students who engaged in personalized mathematics activities outperformed students who solved similar problems presented in a traditional (non-personalized) format (Walkington, 2013). The students who experienced personalized learning opportunities also continued to outperform their counterparts in subsequent units. While others are still gathering more empirical evidence to measure why such effects are observed (e.g., Bernacki \& Walkington, 2018), we cannot ignore that personalizing instructional activities does positively impact students' learning. Thus, the Dream Car Project was created to engage students in considering a personalized, realworld problem they will likely one day encounter: the decision of whether to really buy that dream car.

## The Context and Personalization of the Dream Car Project

This project was given to fourth-year high school mathematics students as a performance assessment at the end of their unit on exponential growth and decay rather than giving them a traditional summative test; however, the project could also be used in other ways, such as for instruction rather than assessment or within other grade levels for which exponential growth and decay are taught. The context for the project (purchasing a new car) is very relevant for high school students, and at this school, many of the students completing the project had not yet experienced the
car buying process for themselves. So, the project immediately captured their interest as they were curious about buying a car. The start of the project asked them to explore all new models of cars and determine which car they would want to purchase. The freedom to choose any car (foreign or domestic) with any desired features meant that students had the opportunity to determine what was important to them. Once this decision was made, the students determined the price for the car using a dealer website and used this information for the remainder of the project. Other components within the project allowed for additional personalization, such as determining the down payment or interest rate based on the starting letter or number of letters in the student's first and last name.

## The Mathematics of the Dream Car Project

After identifying the price of their dream car, students must calculate the sales tax of the vehicle for their state. Next, they must determine the value of an investment that was started for them by a family member using compound interest and the number of letters in their first and last name to determine the interest rate. This investment will serve as a down payment for the car, so the next step is for the student to decide the amount needed for a loan. After determining the loan amount, students calculate their monthly payment using provided loan information that is again dependent on their name (i.e., first letter of first name). They also consider how much they will have paid in total and in interest by the end of the loan period. Students then consider how the value of their car will depreciate over the course of six years and determine which regression model is most appropriate for their data. While students should select an exponential regression, not all students select this model, but the project does require them to justify their decision and discuss their model within the context of the situation. Using their models, students explain what happens to the value of their car at the five-year mark and whether they expect their car's value to ever reach $\$ 0$. Throughout the project, students are not provided with guidance on how to perform their calculations but must instead rely on their prior knowledge from calculating simple interest, compound interest, amortization, and modeling data to recognize the most appropriate model for the situation.

## Evaluating the Dream Car Projects

Students completed these projects individually and outside of class time. There was time allocated at various points over the course of the two weeks students were given to work on the project for students to ask questions and discuss with peers how things were progressing. Once students turned in their projects, the teacher (the second author) evaluated them using the evaluation criteria for the project. It is important to note that students were given the evaluation criteria when the project was assigned so they could reference it as needed while they completed the project. By using this evaluation criteria, the teacher was able to provide students with feedback related to the accuracy of their mathematical calculations and their explanations throughout all parts of the project.

## Reflections on Implementing the Dream Car Project

Implementing this project was a refreshing experience as the students were genuinely excited to complete it. Many students shared they appreciated that the project was relevant to their own lives. The students also found value in the personalization within the project as this gave them the opportunity to still be able to work together and ask questions if confused but not have the "answer" given away since the numbers within the project were unique for everyone. Overall, using this project proved to be a positive and engaging experience that benefited students and allowed them to demonstrate their understanding of mathematics topics in a different way. We encourage others to use the project in their own classrooms and modify it as needed to fit the needs of their students. This could include updating the personalization pieces or providing the project to students in sections rather than all at once. Most importantly, let the students explore and see how they really will use math in their everyday lives!

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## Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an ongoing benefit from the grant. In recent years, grants averaged just less than $\$ 800$. The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [imccormick@rock.k12.nc.us] with questions.

## Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from Emogene Kernodle, nekernodle@yahoo.com.

## Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Ana Floyd, afloyd@randolph.k12.nc.us.

## Dream Car Project

Name: $\qquad$

Project Overview: You will select your favorite car to purchase. You will research the car, look at loan rates, and go through the process of making your dreams of buying your dream car come true! There is so much more to consider when buying a new car than just the purchase price. For instance, taxes, interest rates, depreciation, and inflation are all factors. So, what is the real cost of your new car? You are going to figure that out and then decide if you would still want to purchase the car.

## Finding the Car

You are going to purchase a 2022 brand new car! And... you have no budget! You can choose any car, as long as it is brand new and from an official dealer. Happy shopping!
I. Use dealer websites to research new cars and pick one that you want to purchase. Use the table to help organize your choices.


What car do you want to purchase and why?

How much is the car you want to buy?
II. What is the sales tax for vehicles in our state? Use this to calculate the purchase price of your new car. (Note: We are not including title and registration fees in our pricing; we are assuming that you can afford those costs out of pocket.)

## Getting the Money

Do you have that much money laying around? Can you pay for the vehicle with the cash in your pocket? If you're like me, you probably don't and can't. So, you will need to go to the bank and get a loan.
III. The day you were born, your Great Uncle Jerry Atrick went to the bank, opened an account in your name, and deposited $\$ 100$. You have decided that you want to use this money to make a down payment on your new car. The rate of growth on the account was determined by the number of letters in your first name and your last name (for example, Tom (3) Holland (7)'s account would have a $3.7 \%$ interest rate).

Assuming that the account was compounded monthly, how much money should be in the account from Great Uncle Jerry Atrick today? (Use your current age!) Explain your thinking.
IV. The money in your account from Great Uncle Jerry Atrick is what you will use as your down payment. Subtracting the down payment amount from your total cost of the car, for what amount will you still need a loan?
V. Mrs. Robyn Debanks, the loan officer at your local BB\&T, tells you that you are eligible for a low interest loan. Use the chart below to find out the details of your loan (for example, Iom Holland's loan will be for 84 months at $5.82 \%$ ).

| First Name Starts With | Loan Information |
| :---: | :---: |
| A - I | 60 months at $5.48 \%$ |
| J - R | 72 months at $5.57 \%$ |
| S - Z | 84 months at $5.82 \%$ |

What loan is available to you?
VI. What will be your monthly payments? (Hint: Think about loan amortization since you will be paying down your loan in equal installments.)
VII. At the end of the term of the loan, how much will you have paid in total? How much will you have paid in interest? Explain your thinking.

## Car Value

Congratulations! After getting a loan from the bank to pay for your car, you are now the proud owner of a 2022 vehicle! How exciting! But you overheard your friend, Rex Kars, say that vehicles lose a lot of value over time. That can't be true, right?
VIII. Fun fact! On average, within the first five years of owning a new car, the value of a domestic car drops by $37.3 \%$, and the value of an imported car drops by $68.8 \%$. Below, we have used this information to create our own depreciation charts. Using the original value of your car (MSRP, without taxes), complete the
depreciation chart that applies to you. Show your work and feel free to use a separate page to answer the questions for this part of the project.

Domestic Cars

| Time since <br> purchase | Depreciated value | Percent value of <br> the car |
| :---: | :---: | :---: |
| 0 minutes |  | $100 \%$ |
| 1 minute |  | $91 \%$ |
| 1 year |  | $84 \%$ |
| 2 years |  | $77 \%$ |
| 3 years |  | $62 \%$ |
| 5 years |  | $56 \%$ |
| 6 years |  |  |

Imported Cars

| Time since <br> purchase | Depreciated <br> value | Percent value of <br> the car |
| :---: | :---: | :---: |
| 0 minutes |  | $100 \%$ |
| 1 minute |  | $91 \%$ |
| 1 year |  | $78 \%$ |
| 2 years |  | $62 \%$ |
| 3 years |  | $31 \%$ |
| 5 years |  | $23 \%$ |
| 6 years |  |  |

IX. Find an appropriate regression model for your data and graph it and your data points below. (You may use a handheld calculator or Desmos to help you find your regression.) Identify the different parts of the graph. What is the equation for your regression? Does your regression model make sense given the context of the situation?
X. What do you notice? Explain what happens to the graph once you reach the five-year mark. What does this mean within the context of the situation?
XI. Explain whether your car's value will ever reach \$0. Use your equation from part IX to help you answer this question. Be sure to explain your reasoning using examples.


Reflect on what you've learned from going through this process. After learning about the mathematical side of purchasing a new car, would you still choose to do it? Explain your reasoning.

| Part | Points | No Credit | Partial Credit | Full Credit |
| :---: | :---: | :---: | :---: | :---: |
| I | 2 | The student did not provide a vehicle and price within the given parameters. | 1 Point - The student provided a vehicle within the given parameters. OR <br> 1 Point - The student provided a price within the given parameters. | The student provided a vehicle and price within the given parameters. |
| II | 2 | The student did not correctly identify the sales tax for our state and did not use it to correctly calculate the purchase price of the vehicle. | 1 Point - The student correctly identified the sales tax for our state. OR <br> 1 Point - The student used the sales tax to correctly calculate the purchase price of the vehicle. | The student correctly identified the sales tax for our state and used it to correctly calculate the purchase price of the vehicle. |
| III | 5 | The student did not use the correct interest formula OR did not use any correct components of the formula. | Students will receive 1 point for each correct component of the compound interest formula. | The student used the correct interest formula and used the correct components of the formula. |
| IV | 1 | The student did not correctly calculate the amount left needed for a Ioan after using Great Uncle Jerry Atrick's gifted money. | N/A | The student correctly calculated the amount left needed for a loan after using Great Uncle Jerry Atrick's gifted money. |
| V | 1 | The student did not correctly identify the loan that applied to them. | N/A | The student correctly identified the loan that applied to them. |
| VI | 2 | The student did not calculate their monthly loan payment. | 1 Point - The student did not correctly calculate their monthly loan payment. | The student correctly calculated their monthly loan payment. |
| VII | 4 | The student did not calculate how much money was paid at the end of the term of the loan and how much money was paid in interest. | 2 Points - The student calculated how much money was paid at the end of the term of the loan. OR 2 Points - The student calculated how much money was paid in interest. | The student calculated how much money was paid at the end of the term of the loan and how much money was paid in interest. |
| VIII | 8 | The student applicable depreciation chart was not completed correctly. | 1 Point (per) - Students will receive 1 point for each correct row of the table. <br> AND/OR <br> 1 Point - The student completed the applicable depreciation chart. <br> AND/OR <br> 4 Points - The student correctly completed the applicable depreciation chart but did not show their work. | The student correctly completed the applicable depreciation chart and showed their work. |
| IX | 3 | The student did not identify an appropriate regression model, did not correctly graph their regression model, and did not correctly identify the different parts of the graph within the given context of the problem. | 1 Point - The student calculated an appropriate regression model and recorded the equation. <br> AND/OR <br> 1 Point - The student correctly identified the parts of the graphs within the given context. <br> AND/OR <br> 1 Point - The student correctly graphed their regression model. | The student did identify an appropriate regression model, correctly graphed their regression model, and correctly identified the different parts of the graph within the given context of the problem. |
| X | 5 | The student did not explain their reasoning behind what they observed at the five-year mark of the graph. | 3 Points - The student's explanation behind what they observed at the fiveyear mark of the graph made minor connections to the context of the problem. | The student explained their reasoning behind what they observed at the fiveyear mark of the graph in a way that was relevant to the context of the problem. |


| XI | 5 | The student did not explain whether <br> their vehicle's value will ever reach <br> $\$ 0$. | 2 Points - The student explained <br> whether their vehicle's value will ever <br> reach \$0, but they made no <br> connection between the vehicle's <br> value and the regression model from <br> part IX and did not use realistic <br> examples. <br> OR <br> 4 Points - The student explained <br> whether their vehicle's value will ever <br> reach \$0 using their equation from <br> part IX. The student's explanation <br> included either sound mathematical <br> reasoning OR provided realistic <br> examples. | The student explained whether their <br> vehicle's value will ever reach \$0 <br> using sound mathematical reasoning, <br> realistic examples, and their equation <br> from part IX. |
| :--- | :--- | :--- | :--- | :--- |
| XII | 5 | The student did not explain why or <br> why not they would still choose to <br> purchase a new vehicle. | The student explained why <br> or why not they would still choose to <br> purchase a new vehicle, but <br> mathematical reasoning is not evident <br> in their response. | The student explained why or why not <br> they would still choose to purchase a <br> new vehicle using mathematical <br> reasoning. |

## Trust Fund Scholarships: $\boldsymbol{\$} 1000$

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants \& scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson Plumblee, richards@elon.edu.

## Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett<br>North Carolina Council of Teachers of Mathematics<br>P. O. Box 33313<br>Raleigh, NC 27636

# Problems2Ponder 

Holly Hirst, Appalachian State University, Boone, NC
In each issue of The Centroid, Problems2Ponder presents problems similar to those students might encounter during elementary and middle school Olympiad contests. Student solution submissions are welcome as are problem submissions from teachers. Please consider submitting a problem or a solution. Enjoy!

Problem submissions: If you have an idea for a problem, email Holly Hirst (hirsthp@appstate.edu) a typed or neatly written problem statement, along with a solution. Include your name and school so that we can credit you.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst (hirsthp@appstate.edu) a clear image or PDF document of the correct solution, with the name of the school, the grade level of the student(s), the name of the student(s) if permission is given to publish the students' names, and the name of the teacher.

Deadline for publication of problems or solutions in the Spring 2024 Centroid: January 10, 2023.

## Fall 2023 P2P Problems

Problem A: A circle, triangle, and square are drawn on a piece of paper. No side of the rectangle is all or part of the triangle. What is the greatest number of points of intersection?

Problem B: Each number from 1 to 16 is written, one to a box in the grid shown. The numbers fall on a path with consecutive numbers in adjacent boxes horizontally and vertically (not diagonally). Given the four numbers shown, find the sum of the numbers in the starred boxes.

## Spring 2023 P2P Problem Solutions



Problem A: The digits 1, 3, 6, and 8 are arranged to form two 2-digit numbers. Each digit is used exactly once. The two 2-digit numbers are multiplied. What are the smallest and greatest products that can be obtained?

Solution: To find the largest: A good start would be to put the largest digits in the tens position. This gives two possible multiplications, the second of which is larger.

|  | 83 <br> $\times 61$ <br> Similarly, to find the smallest: | 81 <br> $\times 63$ <br> 5063 |
| :--- | ---: | ---: |
|  | 38 | 36 |
|  | $\times 16$ | $\times 18$ |
| 608 | 648 |  |

Problem B: In an "up-and-down" counting number, the digits increase to a maximum digit and then decrease; the maximum digit must be internal to the number, i.e., the maximum is not the first or last digit. (For example, 256950 is an up-and-down number that increases to 9 and then decreases to 0 , but 255690 is not, because 2-5-5-6-9 does not strictly increase with each digit.) How many different five-digit up-and-down numbers are there with 6 as the maximum number?

Solution: This problem is harder than intended, due to a typo - I meant to ask how many different four-digit up-anddown numbers there are. The problem is more complicated for five-digit numbers; there are 380 total up-and-down five-digit numbers with largest number 6. To see how to answer this question, let's look at each possible location for the maximum digit 6 . The digit 6 cannot be in the first or last position by definition, so there are three cases.

Case 1: 6 is in the second position. Then

- In the first position, the numbers could be $1,2,3,4$, or 5 (five choices) since they are all less than 6 .
- Then in the third and fourth positions, we could have the $5-4$ or $5-3$ or $5-2$ or $5-1$ or $4-3$ or $4-2$ or 4-1 or 3-2 or 3-1 or 2-1 as shown below, giving 95 possible numbers

| 1, 2, 3, 4, 5 | 6 | 5 | 4 | 3, 2, 1, 0 | $5 \times 4=20$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1, 2, 3, 4, 5 | 6 | 5 | 3 | 2, 1, 0 | $5 \times 3=15$ |
| 1, 2, 3, 4, 5 | 6 | 5 | 2 | 1, 0 | $5 \times 2=10$ |
| 1, 2, 3, 4, 5 | 6 | 5 | 1 | 0 | $5 \times 1=5$ |
| 1, 2, 3, 4, 5 | 6 | 4 | 3 | 2, 1, 0 | $5 \times 3=15$ |
| 1, 2, 3, 4, 5 | 6 | 4 | 2 | 1, 0 | $5 \times 2=10$ |
| 1, 2, 3, 4, 5 | 6 | 4 | 1 | 0 | $5 \times 1=5$ |
| 1, 2, 3, 4, 5 | 6 | 3 | 2 | 1, 0 | $5 \mathrm{x} 2=10$ |
| 1, 2, 3, 4, 5 | 6 | 2 | 1 | 0 | $5 \times 1=5$ |

Case 2: 6 is in the fourth position. Then

- In the fifth position, the numbers could be $0,1,2,3,4$, or 5 (six choices) since they are all less than 6 .
- In the second and third positions, we could have 4-5 or 3-5 or 2-5, or 3-4 or 2-4, or 2-3 as shown below giving 60 numbers

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :--- | :--- |
| $1,2,3$ | 4 | 5 | 6 | $5,4,3,2,1,0$ | $3 \times 6=18$ |
| 1,2 | 3 | 5 | 6 | $5,4,3,2,1,0$ | $2 \times 6=12$ |
| 1 | 2 | 5 | 6 | $5,4,3,2,1,0$ | $1 \times 6=6$ |
| 1,2 | 3 | 4 | 6 | $5,4,3,2,1,0$ | $2 \times 6=12$ |
| 1 | 2 | 4 | 6 | $5,4,3,2,1,0$ | $1 \times 6=6$ |
| 1 | 2 | 3 | 6 | $5,4,3,2,1,0$ | $1 \times 6=6$ |
| total: 60 |  |  |  |  |  |

Case 3: 6 is in the middle (third) position. Let's assume for the moment that 5 is in the second position and look at that situation. Note that the total of 60 comes from 4 choices for the first position times the sum of $5+4+3+2+1$ for the last position.

| $1,2,3,4$ | 5 | 6 | 5 | $4,3,2,1,0$ | $4 \times 5=20$ |
| :---: | :---: | :---: | :---: | :---: | :--- |
| $1,2,3,4$ | 5 | 6 | 4 | $3,2,1,0$ | $4 \times 4=16$ |
| $1,2,3,4$ | 5 | 6 | 3 | $2,1,0$ | $4 \times 3=12$ |
| $1,2,3,4$ | 5 | 6 | 2 | 1,0 | $4 \times 2=8$ |
| $1,2,3,4$ | 5 | 6 | 1 | 0 | $4 \times 1=4$ |
|  |  |  |  |  | total $=60$ (or $4 \times 15$ ) |

How are all the other possible second position numbers $(2,3,4)$ related to this situation? There will be one less choice for values in the first position if 4 is in the second position (i.e., $1,2,3$ ); two less choices if 3 is in the second position, etc.; which gives at total of 225 for all the possible case 3 numbers:

$$
4 \times 15+3 \times 15+2 \times 15+1 \times 15=225
$$

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Follow the "Membership Information" link on the ncctm.org website, or go directly to:
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