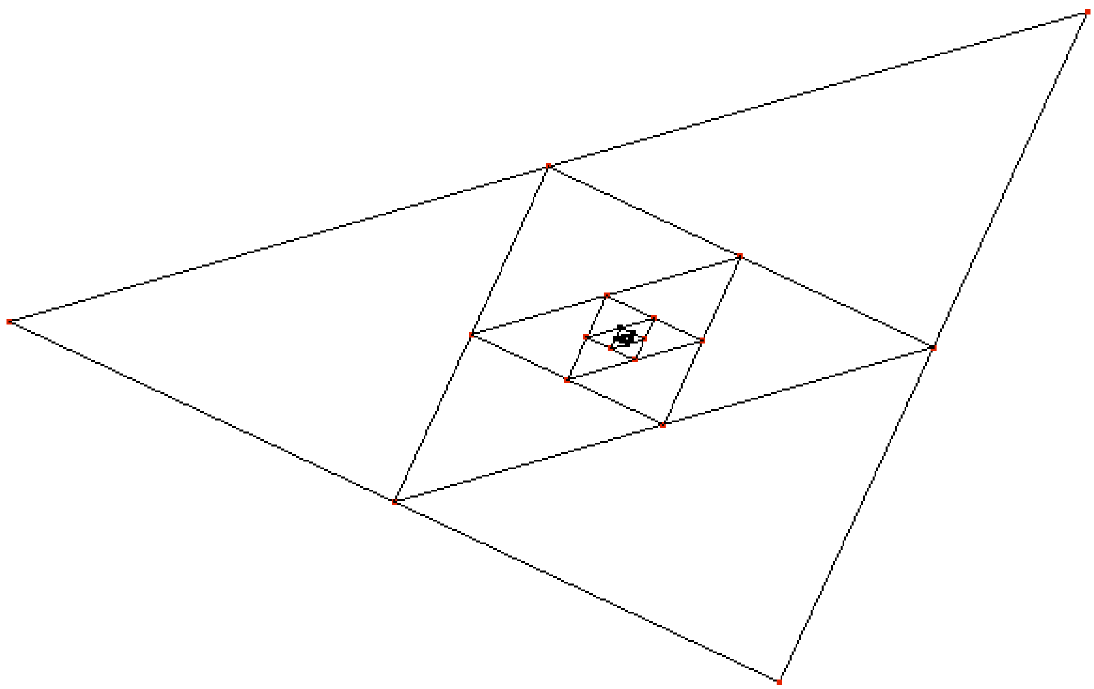
The Centroid



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OFFICIAL JOURNAL OF THE NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS
VOLUME 32 • NUMBER 1 • SPRING 2006

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through teacher education. *The Centroid* is published in January and August. Subscribe by joining NCCTM; see the Membership Form on the last page.

Submission of Manuscripts

We invite the submission of news, announcements, and articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. To be considered for inclusion in an issue, news and announcements must be received by November 1 for the spring issue and by June 1 for the fall issue.

Manuscripts that have not been published before and are not under review elsewhere may be submitted at any time to the address below. Submit one electronic copy via e-mail attachment (preferred) or diskette in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page. Manuscripts should not exceed 10 pages double-spaced with one-inch margins. Figures and other pictures should be included in the document in line with the text (not as floating objects). Scannable photos are acceptable and should be large glossy prints mailed to the editor or minimum 300 dpi tiff files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.

Manuscripts should follow APA style guidelines from the fifth edition of the *Publication Manual of the American Psychological Association* (2001). References should be listed at the end of the article, and should also follow APA style, e.g.,

Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, Grade 3* [On-line]. Available: http://www.ncpublicschools.org/ curriculum/mathematics/grade_3.html

Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.

Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

General articles are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

About the Cover

The Centroid logo is based on the following theorem: The limit of the sequence of midtriangles of a triangle is the centroid of the triangle.

Copyright

Contact Information

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The Centroid

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Articles

News & Information

From the Editor

A Time of Change for NCCTM

Holly Hirst
Appalachian State University
Boone, North Carolina

Happy New Year! I hope your new terms have all started well. NCCTM is undergoing some big changes in 2006: We are adding two new awards: The Horizon Award and the Hallmark of Achievement Award; watch the web for information. We will be working with a new management service to handle financial and membership functions; the new contact information is on the inside back cover.

We were saddened by the death of NCCTM founding member Bill Palmer in November. For the last two years, Brian and I have worked with him and his wife Anne in their capacity as business managers for NCCTM. We had only met him a few times, but it was clear from the first meeting that Bill Palmer was instrumental to the success of NCCTM. In preparing the memorial on the next page of this issue, Bill Paul and I were heartened by the outpouring of sentiment from current and past NCCTM board members in response to our emails. We only printed portions of the glowing tributes, but even those of you who did not know Bill Palmer will be convinced of his dedication to the organization along with his humor, honesty, and kindness.

We hope you will enjoy the articles in this issue. *It's Elementary* documents a practice useful at all grade levels for encouraging students to take ownership in their mathematics. *Math in the Middle* provides insight into teaching the arithmetic of fractions that is appropriate for elementary teachers as well. *A Teacher's Story* describes a prospective teacher's "aha" moment: observing first hand that students learn more by doing. The article also lists some ideas for activities to use in Advanced Functions and Modeling, something many teachers are searching for. The *Women and Minorities in Mathematics* lesson highlights the contributions of North Carolina's own Marjorie Lee Browne, one of the first African American women to receive a doctorate in mathematics.

We have included our very first *Teaching with Technology* submission on the mathematics of tipping points. Tipping points are related to the mathematical concept of an equilibrium in dynamical systems, difference equations, and differential equations – topics that are somewhat advanced for pre-college students – but with technology students can investigate these ideas to see the richness of mathematics while practicing algebra and function skills. We would like to see more submissions in this category.

As always, we encourage you to consider assisting with *The Centroid* by:

- **submitting a manuscript** – general articles are welcome, as are the following special categories of articles: *A Teacher's Story*, *History Corner*, *Teaching with Technology, It's Elementary!*, *Math in the Middle*, and *Algebra for Everyone*.
- **becoming a reviewer** – please send e-mail to me if you are interested in helping in this way.

Contact information. Feel free to contact us at any time with submissions, news items, questions, or concerns.

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http://www.mathsci.appstate.edu/centroid/

In Memoriam
Dr. William Fisher Palmer
1934–2005

Dr. William Fisher "Bill" Palmer, died at his home in Salisbury North Carolina in November 2005. He was instrumental in the growth and development of the North Carolina Council of Teachers of Mathematics, and we are saddened by his passing.

Bill graduated from Catawba College with a bachelor's degree in mathematics and went on to complete a master's degree and a doctorate in mathematics education from the University of North Carolina at Chapel Hill. Bill taught in the High Point schools before moving to Raleigh to work for the North Carolina Department of Public Instruction, and later joining the faculty at Mercer College in Georgia. He returned to North Carolina in 1973 to teach at Catawba College, where he served as a professor in the Education Department for 23 years.

A founding member of NCCTM, Bill's involvement with the organization started over 32 years ago. Bill was instrumental in establishing and publishing the early issues of this Journal. In 1981 he was elected to the position of Treasurer for the organization. In 1982, the By-Laws were changed to create the paid position of Business Manager. Bill was recruited to fill that position, and held it jointly with his wife Anne until the time of his death. In 1986 he was honored with the W.W. Rankin Award for his commitment to the organization.

Such are the facts, but there was much more to this man as can be seen in the following testimonials. As stated so well by our President, Jeane Joyner, "Bill's work with NCCTM as an officer and then with the management service helped to shape the organization we have become today." Bill, NCCTM will miss you sorely.

—Holly Hirst and Bill Paul

His steady, reliable, honest, unselfish approach was always such a comfort. We always knew we were in good hands when Bill was around... He will be greatly missed by all of us, and especially by those of us lucky enough to have the opportunity to work closely with him. —Diane Frost

Over the past few years, each time I would see Bill, there was always that twinkle in his eye whether he was describing the adventures of his family or talking about mathematics... High integrity, Honesty and trustworthiness are words that described Bill Palmer in whatever he did. He will be missed…not only for the things he did but as a valued friend and colleague.

—Jeanette Gann

As the organization grew and management tasks became a burden on NCCTM officers, Bill began taking on those tasks by creating a management services organization with his wife, Anne... Bill's dedication, ideas and attention to detail will be sorely missed.

—Ron Hann

During the two years that I served as President of NCCTM...Bill and Anne agreed to work with no salary increase while we put the budget of NCCTM in order. Most of the members have no idea about the personal and financial sacrifices that Bill and Anne made in order for NCCTM to be the successful organization that it is today. —Robert Joyner

I feel that he often went beyond the call of duty to assist the organization...He will be sorely missed by all who knew him—for his keen abilities, his wit, and his love of mathematics and the NCCTM, the organization to which he devoted much energy and effort for many years. —Bob Jones

My earliest memory of Bill was when I attended my first conference... I had no idea what to do or how to do it. Kindly, Bill took the time to walk me through the steps of on-site registration and pointed me in the right direction...We shall miss his many kindnesses, his knowledge and historical background of the organization, his shy smile, and his huge heart.

—Jan Wessell

I was always impressed with Bill Palmer's energy and professional activity... he was willing to say yes when someone asked for help. Our profession and the NCCTM organization is certainly indebted to Bill's outstanding service for such a long period of time. We will miss him. —Harold Williford

Presidents' Messages

State President
Jeane Joyner
joynerj@meredith.edu

As educators, we often think of the new year beginning when a new group of students arrive at our doors; for others the new year begins each January. For NCCTM, 2006 will be a year of challenges and opportunities.

Let's begin by looking back at 2005.We had successful spring conferences and a fall conference that brought many new ideas to our membership. We hosted a Leadership Seminar for school district leaders and higher education colleagues. We supported math fairs and contests, awarded mini-grants and scholarships, honored outstanding educators, choose a new logo, and provided information through the Centroid. In November we lost a friend and long-time advocate, Bill Palmer. Bill's work with NCCTM as an officer and then with the management service helped to shape the organization we have become today.

Looking forward, we strive to become an even stronger organization by supporting mathematics education through multiple venues. As always, we seek ways to involve more members in active roles. Briefly, here are four challenges that need your attention:

Choosing our leaders: Participation by members in our election process in the past has been abysmal. Choosing NCCTM's leadership is every member's responsibility. You have a ballot–please vote for the leaders who will shape our organization in the next few years!

Using our resources wisely: Communicating with our membership is increasingly expensive, so we are investigating ways to improve our website and take advantage of electronic communications. We plan to create improved links that will give you more up-to-date information about NCCTM activities and provide ways for you to contact appropriate committee chairs. A committee is currently working on our website design. If you have suggestions email them to me, and I will pass them along. Get in the habit of checking the website <http://www.ncctm.org> to learn the latest about the organization.

Honoring outstanding leaders in mathematics education: Part of the work on our website will involve posting information about NCCTM's awards programs. In addition to the W.W. Rankin Award, the Innovator Awards, and the Outstanding Mathematics Education Student and Outstanding Teacher Awards, NCCTM is implementing two additional awards in 2006: the Horizon Award and the Hallmark of Achievement Award. Both awards will honor NCCTM members who make significant contributions to mathematics education in our state. Who will be named by NCCTM depends upon your identifying those who deserve to be honored. As soon as details about all of the awards and nomination forms are posted this spring, please send in your nominations.

Improving our annual fall conference: What suggestions do you have for improving our fall conference? What role are you willing to take on? I have had a number of communications from members who were pleased with this past year's conference and from others who stated concerns and made suggestions for changes. Both kinds of communications are helpful. The conference and program committees are already at work on the fall conference (October 5 and 6 in Greensboro) even as others are preparing for our spring meetings February 18 in Boone and February 25 in Raleigh. Bill Scott, 2006 Program Chair for the fall conference, has provided a speaker proposal form in this issue of the Centroid for you to fill out and send in.

I hope that you will accept the challenges and see them as opportunities to become involved in your professional organization. If you are interested in serving on a committee or running for an office, contact me or other officers. Many hands will certainly make lighter the work of NCCTM!

Eastern Region President
Julie Kolb

jkolb@wcpss.net

Greetings to (and from) the members of the Eastern Region. Wow–the school year is flying by. I hope that you are enjoying your classes and that you have tried some new activities to keep yourself and your students engaged.

I was deeply saddened to hear that Bill Palmer passed away in November. An integral part of our organization for many years, Bill generously contributed many hours to managing our funds and running our conferences. As members of NCCTM from its beginning, he and Anne understood that the main goal of our organization is helping mathematics teachers help their students grow and develop mathematical fluency. His guidance and commitment to the success of our organization will be missed.

The fall conference was a great time to renew friendships and glean some wonderful ideas for use in my classroom. There was a wide choice of sessions at every level. The conference is an opportunity to engage in energizing professional development appropriate to what you teach in a setting that is relaxed and enjoyable. If you did not take the opportunity to attend the Plenary Session, you missed a very lively session including a fashion show of math tee shirts and door prizes. It was like no other business meeting so be sure that you put it on your agenda for next fall's meeting. The NCCTM fall conference will be in Greensboro on October 5 and 6. Make plans now to attend and please consider being a speaker. This would be a wonderful opportunity for you to share classroom strategies, activities, and resources with your colleagues. The speaker form is available in this issue of the Centroid. Remember that it is your active participation in our organization and its conferences that make NCCTM such a wonderful network of professional educators.

As of this writing, plans are underway for a fantastic spring regional conference. The Eastern and Central regions will once again host a joint conference at Meredith College on Saturday, February 25. The theme of the conference is "Success for All in Algebra" with several additional sessions planned that are related to grant writing and the implementation of grants received. Dr. Ron Preston from East Carolina University is the featured speaker for the general session that will focus on Algebra K-12.

Thank you once again for allowing me to serve as your president. If there is any way that I can assist you or if you would like to know more about how you can contribute your time to maintaining the quality of NCCTM, please contact me.

Central Region President
Emogene Kernodle

nkernodle@yahoo.com

Greetings from the Central Region. We hope the beginning of this school year has been rewarding and successful. As 2006 begins, it is our wish for each of you to experience the guidance and support you need to provide quality mathematics education in North Carolina.

The Eastern Region has allowed the Central Region to join it again for the Spring Regional Conference. It will be held Saturday, February 25, 2006 at Meredith College in Raleigh. Check the NCCTM website for more information and the registration form.

You will also find on the website information about other spring activities that may be of interest to you and your students. The Math Logo Contest is for all grade levels. The deadline for entries is March 1. Information and downloadable registration forms for the regional Math Fairs and sites for upcoming Math Contests can also be found on the website. If you are pursuing graduate studies in mathematics education, you may want to apply for the NCCTM Graduate Scholarship. Applications will be reviewed and granted March 1.

Do consider sharing your expertise by speaking at the 2006 State Mathematics Conference to be held at the Koury Convention Center in Greensboro on October 5 and 6, 2006. Bill Scott would like to have your speaker form by April 16.

Please see the Centroid and the NCCTM website <http://www.ncctm.org> for information about NCCTM activities.

Your Central Region officers want to serve you. Please let us hear from you to let us know your needs and interests.

Western Region President
Carmen Wilson

cwilson@ashe.k12.nc.us

Happy New Year from the Western Region! I hope that many of you are planning to come to the 2006 Western Region Mathematics Conference on the campus of Appalachian State University on February 18th. Once again we will be hosting a "conference within a conference." The theme for our conference for Pre-Service teachers will be "Get Mountains of Help for Your First Year of Teaching." Preservice teachers will attend sessions for grade bands K-2, 3-5 or 6-8 led by outstanding educators in our field. Our conference for inservice teachers has been titled "Climbing to the Top with the New Curriculum" and will feature sessions for teachers in K-2, 3-5, 6-8 and 9-12 grade spans. Session leaders include some of our state's best classroom teachers as well as the new math folks from the Department of Public Instruction. Check-in will be at 8:30 AM in the lobby of Walker Hall. Sessions will begin at 9:00 AM and will end at 1:00 PM. Registration forms will be mailed to all Western Region NCCTM members and curriculum coordinators in each school system in early January, so watch your mail! If you need more information you may contact me via email or phone <336-246-2400>.

The 2006 Western Region Math Fair will be held on Saturday, April 1, at Appalachian State University. Students attending school in North Carolina are eligible to enter the Math Fair. Projects may be entered in the following categories: Grades K-2 (individual or class projects), Grades 3-5 (individual or class projects), Grades 6-8 (individual projects), or Grades 9-12 (individual projects). Projects must be pre-registered to be accepted for competition, and there is a limit of nine projects per category per school. On the day of the Math Fair, each project must be represented by one or two students whose names appear on the project, and the projects must be checked in at the registration desk between 9:00 AM and 9:30 AM. For more information and a registration form, contact Dr. Betty B. Long <longbb@appstate.edu, 828-262-2372>.

NCCTM Trust Fund Scholarships

Thinking about going to graduate school in mathematics education? Already in graduate school? NCCTM offers scholarships to any NCCTM member who is currently employed as a K-12 teacher in North Carolina. The funds can be used to pay for coursework in mathematics or mathematics education. Recently took a class? It is not too late. The funds can be requested by members who have completed a course within the previous four months of the application deadline (March 1 and November 1).

Interested? The application is on the last page of this issue!

It's Elementary

Ownership of Students' Ideas

Donna Godley[1]
Dana Elementary
Dana, North Carolina

"I tried Shelby's strategy, and it worked!" said 11-year-old Kurt in math class today. He had just won a round of *Close to 1000*, a math game in our Investigations book. The day before, Cassie had learned to successfully complete multiplication of fractions problems using Sara's strategy.

Giving students a sense of ownership of their mathematical discoveries has added a whole new dimension to mathematics in my classroom. During the past few years I have taught second, third, and now fifth grades. Although the "look" of that ownership has been a little different at various grade levels, the results have been the same. The students are excited and proud of their mathematical discoveries. Their success in being able to solve problems has increased along with their self confidence.

This technique of publicly acknowledging students' ownership of ideas is one that evolved in my second grade classroom as a result of my view that a teacher should be a facilitator of communication and learning, as opposed to a lecturer or demonstrator. It all began when I noticed that Mattie was repeatedly successful at solving difficult word problems. I asked her to come to the overhead and show the class exactly how she had solved a problem. She nonchalantly said, "It's easy. All you have to do is draw a picture." She proceeded to tell the class the same things they had heard from me, but this time it was different. Drawing a picture was seen as Mattie's personal revelation. With her telling the class how she figured out what to draw, the students were interested and amazed that she could figure the problem out by herself. They even gave her suggestions to make the drawing easier to understand. Playing off that interest and hoping to encourage the students to use the strategy more often, I gave Mattie a piece of construction paper and asked her to copy her drawing. I then labeled it, "Mattie's Strategy-Draw a Picture," and taped it on the wall. Afterwards I heard the students saying, "I am going to use Mattie's strategy," and "Mattie's strategy works!"

Later that week Luke showed the class how he solved a problem by putting counters into groups of ten. He was very eager to draw a representation of his strategy and have it added to our wall. We titled it "Luke's Strategy-Group by Tens." From that point our wall grew to include "Ike's Strategy-Use Tally Marks," "Carrie's Strategy-Think of Money," "Mike's Strategy-Counting On," etc. One vital key to this whole process was that the students were talking and explaining the math to each other. They were proud of their techniques and eager to teach them to their classmates. We often discussed whose strategies helped us solve a particular problem and if any new strategies had been discovered. I worked very hard to make sure each child had at least one problem solving strategy on the wall. I wanted every child to be an "expert" who could be consulted by their classmates.

[1] *Mrs. Godley has taught in grades 2 through 6 for 13 years in Henderson County. She is currently a 5th grade teacher at Dana Elementary and is a member of the TEAM II project.*

I had one child in class who only used one strategy to solve all problems. If that strategy wouldn't work for a particular problem, he would just give up. I had several other students who were stuck on using only two or three strategies. One Monday I told my class that I was giving them a great opportunity that week. We were going to try to get everyone in the class to use every single strategy on the wall at least once during that week. If they could work together and help each other accomplish this task, we would celebrate with an ice cream party on Friday afternoon! Each child had a record sheet that I stamped in a different color each day when they used each strategy. At the end of every day I asked which strategies had been used during the day. We put a star on the papers on the wall if that strategy was used during the day. I could plan the problems for the next day based on which strategies had few or no stars. During that week I found children solving a problem using one strategy, then doing it again using another strategy, and sometimes even using a third strategy. They were determined to get all their stars. During that week all students tried all the strategies. My "Tally Mark King" actually learned two new strategies that he used consistently the rest of the year.

The routine in our class had now become working each problem using more than one strategy. Underneath the first solution, most students would draw a line and show a second way to find the answer. I noticed that having a large selection of strategies in full view of the students all year long gave them a ready recollection of things they could try. It also gave them confidence in themselves as problem solvers.

Now that I am teaching fifth grade, I use the same concept but have modified it somewhat for the more advanced students. There are usually three or four strategies up on the wall at a time that students have discovered. They usually all deal with the same type of problems, such as fractions.

Recently, for example, our class was working together to make a t-chart of the sum of the measures of the angles of different polygons. After studying the pattern a while, Janelle commented, "I think that if you count the number of sides and subtract two, you can multiply that times 180 to get the answer." We discussed how we could represent Janelle's idea and arrived at

$$(s-2)\times 180 = \text{the sum of the measure of the angles in a polygon.}$$

I put her idea on a chart paper and titled it "Janelle's Conjecture." (I do this with any suppositions or assumptions students make aloud.) Classmates are encouraged to add to the chart by agreeing or disagreeing with the conjectures. They might show their work for several problems that support a conjecture. The conjecture stays up on the chart until someone finds a problem for which it will not hold true. If the student's conjecture is an incorrect assumption and no one in the class challenges it, I will write a problem on the chart that would not support the conjecture and write, "What about this problem?" Conjectures that hold up are posted on the wall with the author's name.

After we play a new math game I make a poster with strategies that were used in winning the game. This poster is generated through our class discussion, and students' names are posted beside the strategy they figured out. The next time we play the game, we bring out the poster. I encourage students who were not very successful the last time we played to choose a strategy on the chart that they understand and try it. If someone is still struggling, I will have the student choose a person to be a partner and help in using a particular strategy. I am still trying diligently to make sure every student is recognized for a strategy he or she has used or a conjecture he or she has made. I think if I can influence my students to perceive themselves as problem solvers I have won half of the battle.

I have noticed that now the students do not give up on a problem quickly. They can work through the information in a problem when they are uncertain of how to get the answer, because they have a repertoire of strategies to try. They will usually stick with a problem until they can't think of anymore strategies to try. With many strategies at their ready recollection, they can usually find a way to arrive at an answer.

A child that I had nick named "Fraction King" recently said to me, "I always thought I was bad at math, but I found out I'm pretty smart."

NCCTM Fall Conference

36th Annual Conference of the
North Carolina Teachers of Mathematics

October 5 and 6, 2006

Joseph S. Koury Convention Center
Sheraton Greensboro Hotel at Four Seasons
Greensboro, North Carolina

Registration Fees: $55 ($45 for members, $5 for students)

Registration: Forms will be available on line <http://www.ncctm.org> or at the conference Wednesday evening, or Thursday and Friday all day.

Speakers Needed!

The 2006 State Mathematics Conference is a wonderful opportunity to share research, classroom strategies, activities, and resources with your colleagues that make mathematics come alive for your students. Take the time to fill out a Speaker form and be a part of this annual professional development opportunity. Encourage colleagues to present as well.

The form appears on the next page, or it can be downloaded from the NCCTM website <http://www.ncctm.org> .

Hope to see you there!

Speaker Proposal Form

NC Council of Teachers of Mathematics
Greensboro, NC October 5-6, 2006
(Print/type your name and school as they should appear on the program)

____
First Name MI Last Name

Are you currently a member of NCCTM? ☐ Yes ☐ No

School/LEA/College/University/Organization ______________________

Preferred Mailing Address ______________________

City, State, Zip Code ______________________

City, State, Zip Code ______________________

E-mail: ______________________

Work Phone: ______________________

Home Phone: ______________________

Fax: ______________________

Title of Presentation: ______________________

Description of Presentation (25 words or less):

____

Please check the grade level for which your presentation would be most appropriate (Check ONLY one)

___PreK-2 ___3-5 ___6-8 ___9-12 ___College/Univ ___General ___Pre-service

Type of Presentation: __45 min session __90 min session __90 min workshop

Would you like to repeat your session? ☐ Yes ☐ No

Which day would you prefer? ☐ Thursday, October 5 ☐ Friday, October 6 ☐ Either

One overhead projector will be available in each room. A VCR/monitor or internet connection may be requested. Wireless internet will be available and speakers will be reimbursed for any charges related to their presentation. Other specialized equipment will be the responsibility of the speaker.

Check if you require a VCR/monitor. ☐

Additional Presenters:
Name: ______________________

School/Organization: ______________________

Name: ______________________

School/Organization: ______________________

Return completed form by April 16, 2006:

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A (Prospective) Teacher's Story

Ideas on Advanced Functions and Modeling

Natasha L. Mabe [1]
Fort Chiswell High School
Wytheville, Virginia

As part of my student teaching last Spring, I taught the advanced functions and modeling (AFM) class that has been recently introduced into the North Carolina mathematics curriculum. Student teaching in itself has been the most eye-opening experience thus far in my postsecondary education, especially after teaching college algebra for three semesters at Appalachian as a graduate teaching assistant. Since coming from a small-town high school in southwestern Virginia five years ago, I have found out just how different high school structures and curricula can be from college and from one high school to the next.

After various observations at area high schools, I requested and was placed for student teaching at Ashe County High School in West Jefferson, North Carolina, with Ms. Dana Long. Her teaching load, which became mine soon enough, included AFM, which was being taught for only the second time.

The Course and the Students

AFM has been designed for those students who are not prepared for calculus or precalculus, but need to take a fourth mathematics course in order to be admitted into college. The students will most likely not end up on a math-related track for a degree, and so the class is designed to teach students to solve problems and to interpret the solution in the context of a real-world situation. These students have had algebra and geometry courses in the past, so ideally they would be bringing that prior knowledge to the class.

However, I observed while teaching this class that students do not have this knowledge to bring to the AFM class, so the review of concepts needed for the activities is prolonged in order to prepare students to understand the math carried out in the first place. I encountered students who did not recall algebra skills that were introduced in Algebra 1 and then reinforced in Algebra 2. Is the old cliché "if you don't use it, you lose it," correct? As an enthusiastic and eager new high school teacher, I believe that these students have not had enough time in between their math classes at this point to lose the information. I believe that the students are simply not *learning* the concepts initially. They memorize the information long enough to be tested, and then it leaves their minds.

While teaching this class, I came to the conclusion that constructing their own knowledge is what the weaker mathematics students seem to need the most. I found that participating in hands-on and simulation activities allows the students to be more involved in figuring out and hence understanding processes. They are able to investigate relationships and discover patterns or ideas that already exist in mathematics. The amount of retention is greater when this is accomplished. I would like to share the list of the topics that we covered during the course and also a few of the activities that were used for those topics—activities

[1] *Natasha finished her MA in Mathematics Education at Appalachian State in 2005 and is in her first year teaching high school. She is currently teaching Geometry and dual-credit Precalculus and Calculus.*

centered around hands-on exploration both with and without technological simulation.

Topics

Since AFM is not an end-of-course tested class in the mathematics curriculum, teachers have some flexibility in the topics they cover. The textbooks we used include *Discovering Advanced Algebra* by Murdock, Kamischke, and Kamischke (2004) and *Algebra and Trigonometry* by Stewart, Redlin, and Watson (2001). We covered the following topics, listed in the order in which they were presented in class, along with the corresponding objectives from the newest North Carolina Mathematics Standard Course of Study for Grades 9-12 in Advanced Functions and Modeling (NC DPI, 2005):

Topics

- Geometric and arithmetic sequences
- Exponential and logarithmic functions
- Quadratic functions
- Data collection and analysis
- Probability and statistics
- Trigonometry

Objectives

1.01 Create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions of bivariate data to solve problems.

1.02 Summarize and analyze univariate data to solve problems.

1.03 Use theoretical and experimental probability to model and solve problems.

2.01 Use logarithmic (common, natural) functions to model and solve problems; justify results.

2.04 Use trigonometric (sine, cosine) functions to model and solve problems; justify results.

2.05 Use recursively-defined functions to model and solve problems.

Sequences and Recursion Activity

With the first topic, we tried to instill in students that with recursion, any particular value depended on the previous one. In one especially successful activity, students simulated the process of the kidneys flushing medicine out of the body and determined how much of the medicine remained after a certain number of hours or days. In groups of three or four, students were given a liter of water and food coloring, and they were to mix the water with 16mL of food coloring (to represent the medicine). The food-colored water would model the amount of medicine and blood in the bloodstream. To simulate one day's kidney function, the students removed 25% (254mL) of the colored water and replaced it with the same amount of fresh, plain water. The students repeated this process for a time equivalent to four days to see the color of the water getting weaker from one day to the next. After students completed the simulation, they constructed the following recursive formula:

$$u_o = 16$$

$$u_n = u_{n-1}(1-.25)$$

The initial amount of medicine was 16mL and 25% of the medicine was removed—or 75% of the original amount remained from one day to the next. The equations were entered into TI-84+ graphing calculators in sequence mode. Students used this recursive formula to calculate how much of the medicine remained at the end of each day for several days. Then they answered several questions about the amount of medicine remaining in the blood stream at specified times and the times at which specified amounts of medicine were present.

Probability and Statistics Activities

One of the most activity-based sections covered involved various ways of describing data. Topics for this area included finding mean, median, mode, and standard deviation of data sets, ranges of values, box plots and histograms, and percentile ranks. Directly after the chapter on data, we discussed probability and statistics (Murdock, Kamischke, & Kamischke, 2004). I learned a lot while teaching this chapter. In order to open up the sections on probability, students were paired up and given a bag of ten dice. Within each bag were different numbers of red, green, and white six-sided dice. Students were to draw a total of 50 times out of the bag, drawing one die at a time and then replacing it before drawing again. As they went along, they tallied up how many of each color they were pulling out of the bag. After the 50 draws and tallies, students counted the number of times out of 50 each color occurred. Some students were frustrated with their selections because some bags only had two colors of dice instead of all three. The students converted the frequencies of each color to a percent to ultimately determine the exact number and color of the ten dice in each of their bags. They had to continue to guess the number and color of the dice until they were exact on the numbers. This activity introduced the concept of experimental probability, after which we discussed theoretical and geometric probability.

Another activity introduced the concept of a random variable and expected value. The activity was taken from *Discovering Advanced Algebra* (2004). Here we had them do the activity first, and then we discussed the definitions so the students could relate back to the activity. Students used TI-84+ graphing calculators and the *Probability Simulation* application. They simulated rolling a six-sided die, recording the number of times it took to roll a 4. Some students got a 4 on two rolls; others took as many as 29 rolls. It was so exciting to watch them do this activity. After they rolled a 4, students recorded how many rolls they did to get that 4 under Trial 1 in Table 1.

Trial	1	2	3	4	5
# of Rolls					

Table 1. Recording rolls until 4

They then repeated rolling to get a 4 for a total of five times. After each had finished this, they came together row by row and found an average of the number of times it took to roll a 4 for their row. We then came together and averaged the rows so that we had a class average. This led us into the discussion of random variables (in our case, rolling a 4 on a die) and expected value (the average number of times it took to roll that 4). This allowed students to attach an example to the definitions they were given at the end instead of merely stating definitions then trying to explain complicated vocabulary the students had never seen before.

One of the last topics we covered in this particular chapter on probability dealt with permutations and combinations—the number of arrangements or groupings when order does and does not matter. When discussing combinations using the Murdock, Kamischke, and Kamischke text (2004), I gave the students an opportunity to play the lottery. I made a sheet full of lottery-like cards (Figure 1), nine on each piece of paper, and for each card the students selected numbers that they would use to play the lottery.

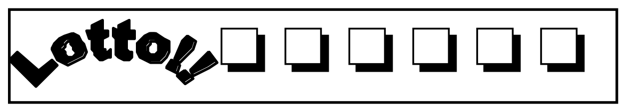

Figure 1. A lottery ticket

We discussed the fact that it did not matter in what order they wrote their six numbers on each ticket; however, the numbers on each ticket could not repeat. I used the TI *Probability Simulation* application again, but this time chose the *Random Numbers* option to generate the “winning ticket.” I set the options so that only one number was chosen at a time to model the way the numbers are drawn.

We played for several rounds. I told the students that I would give some extra credit if anyone had a winning ticket that matched all six numbers. After many rounds, no one had six matches but some were close with three or four. After playing, we discussed what the probability would be of one person having a winning ticket of all six numbers. Using the combination operation on the calculator, ${}_{47}C_6$, we came up with over ten million combinations of six numbers, and the probability was extremely small. Their reactions were “How can anyone ever win?” and “That’s ridiculous!" It was great, and the students were able to apply the math we had talked about to determine that probability.

Conclusion

Overall, teaching AFM has been a wonderful experience. I can see the potential for this class as a wonderful extension of the algebra and geometry concepts that students have learned in previous classes. Students who express a lack of interest in mathematics seem to jump at the chance of participating in an enriching activity, and what is so great about these activities is that students are doing mathematics without *worrying* about doing mathematics. However, I believe more needs to be done in order to prepare students mentally to handle the mathematics that goes on behind the topics covered in order to truly meet the goals of the course.

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Mini-Grants from NCCTM

NCCTM is pleased to be able to offer Mini-Grants to teachers who need some support to implement an innovative idea at their schools. There are no preconceived criteria for projects except that students should benefit from the grant. Possible projects for consideration include math clubs, field days, contests, workshops for parents, math activities, math laboratories, and research topics.

Each of the three NCCTM state regions has $5000 to award to teachers in their area. The average mini-grant is about $600 but some have been awarded for as little as $100 or as much as $2000. Applications will be accepted only from persons who are NCCTM members as of 1 September 2005. Don't let the application process intimidate you! See the sample application on the website and use it as a guide!

Completed applications must be received by 15 September 2006 to be considered. For more information and submission guidelines, see the website <http://www.ncctm.org> or contact Phyllis W. Johnson <pwjohnson210@earthlink.net, 252-752-1796>.

2005 NAEP "Nations Report Card"

The 2005 National Assessment of Educational Progress math scores were released in October. There were definite signs of improvement: Average 4th-grade scores increased by 3 points and average 8th-grade scores increased by 1 point since 2003. Average scores for white, black, and Hispanic students in both grades 4 and 8 were higher in 2005 than in any previous assessment year. Both male and female fourth graders' average scores were higher in 2005 than in any previous assessment year. Average scores for male and female eighth-graders were higher in 2005 than in 2003 or 1990. To learn more about the findings and see a breakout by state, check out the website.

<http://www.nationsreportcard.gov>

Math in the Middle

Adding and Subtracting Fractions: Is It How Much or How Many?

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Consistently, if we ask students, "How much is $5+4$?" or "How much is 3 from 8?", we get the kind of response we seek–addition or subtraction. Similarly, if we ask, "How many apples are in the basket?" we also get the behavior we expect–a count of apples. Although we often ask, "How many are $5+4$?" or "How many is 3 from 8?", we usually do so at the end of a word problem such as

> Rashida has five stamps and gets four more from her Dad. How many does she have in all?

or

> Adam had eight bags of candy to sell. By noon, he had sold three of them. How many bags of candy does he have left to sell?

In either case, the expected behavior, adding or subtracting, can be performed using rote knowledge: $5+4=9$ or $8-3=5$.

Now, consider the addition and subtraction of fractions, such as $1/4+2/3$. The vast majority of teachers ask students, "How much is $1/4$ and $2/3$?" However, in asking, "How much…?", teachers unknowingly ask students to use rote knowledge that doesn't exist. That is, students don't typically recall that $1/4+2/3=11/12$ like they know that $5+4=9$. In fact, when teachers ask students, "How much is $16+8$?", teachers expect students to combine their rote knowledge of $6+8$ and $1+1$ with their understanding of regrouping and place value to compute the sum, 24. Over time, students learn that their rote knowledge of basic facts plays a critical role in whole-number computations that address the question, "How much...?" When they encounter the addition and subtraction of fractions, they reasonably draw from learned behavior and expect the connection between "How much…?" and rote knowledge to continue. However, asking "How much…?" does not use the conceptualization and representation of addition and subtraction that students already have. Specifically, the idea that discrete, separable objects can be combined or compared by counting is not emphasized. In fact, asking "How much…?" encourages students to accommodate a continuous representation of fraction addition and subtraction rather than a discrete one.

Although basic facts such as $1/3+2/3=1$ or $1/4+1/4=1/2$ may help students estimate the answer to "How much is $1/4+2/3$?" such

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basic facts do not facilitate the implementation of the traditional algorithm for computing these sums. In fact, asking "How much…?" causes most students to ask themselves after they realize that they don't know a basic fact to assist them, "What do I do?" Of course, we can help students decide what to do even if we ask, "How much is $1/4+2/3$?" However, the point of this paper is that if we ask, "How many is $1/4+2/3$?", students may be better able to construct and reconstruct for themselves how to get the sum.

Students' Understanding of Fractions

The meaning that children initially give to fractions is related to their understanding of whole numbers (Pitkethly & Hunting, 1996; Kieren, 1995). Studies show that students who invent their own procedures for solving problems involving fractions resort to counting pieces as individual units, rather than seeing them as fractional parts (Gray, 1993; Mack, 1993). Counting like parts of the whole ("How many…?) should be the focus of instruction on addition and subtraction of fractions and must precede the introduction of purely symbolic algorithms ("How much…?") (Kieren, 1992). When asked, "How many…?", students respond by counting. Teachers can exploit the strong connection between "How many…" and counting in their instruction on adding and subtracting fractions. However, middle grades students often need to be reminded about the subtleties of the counting process. Namely, when people count they count like objects. Unfortunately, adding $14+19$ or subtracting $42-37$ in school mathematics has contributed to middle grades students' notion that counting is divorced from the objects being counted. Consequently, it is necessary to remind students that the counting process is used to count objects, and the objects must be alike. As a result, three apples and four oranges are not seven apples or seven oranges.

Furthermore, we must know what the object is. What does the whole (object) look like? Is it an apple, a square, or a red counter? How are the parts of the whole related to the whole and to themselves? Is it essential that all of the parts of the whole be equal-sized pieces (Caldwell, 1995)? An understanding of the role of the unit and its parts in fraction addition and subtraction is essential to developing meaningful algorithms (Witherspoon, 1993; Pitkethly & Hunting, 1996). A concrete procedure for dividing units into parts and parts into subparts is a critical component in the development of addition and subtraction algorithms for fractions. In fact, mastering the division and partitioning of units has been identified as the key to success in understanding fraction concepts and algorithms (Behr, et. al., 1983; Mack, 1990; Pitkethly & Hunting, 1996).

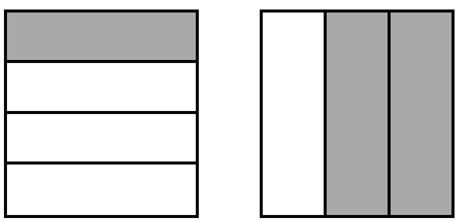
Pointing out obvious elements of counting can help students improve their ability to add and subtract fractions by reminding them of a representation that they already understand. If asked, "How many are $1/4$ and $2/3$?", an algorithm for combining $1/4$ and $2/3$ can be determined using arithmetic reasoning. For example, since $2/3$ represents two objects of some type, are the objects represented by $1/4$ and $1/3$ alike? If so, then we should be able to immediately count all like objects to determine the total or sum. If the objects represented by $1/4$ and $1/3$ are not alike, can we modify the given objects so that they become alike? If so, we count like objects to derive the sum or difference. To illustrate, $5+4=9$, but what are 5 apples and 4 oranges? In order to add these objects together, we must modify our thinking about them, perhaps seeing them as 5 pieces of fruit and 4 pieces of fruit. Then, the total is 9 pieces of fruit.

In an analogous way we can modify unlike fractions so that we can count to determine the sum or difference. Consider the following concrete representations of the addition and subtraction algorithms for fractions.

Adding Fractions

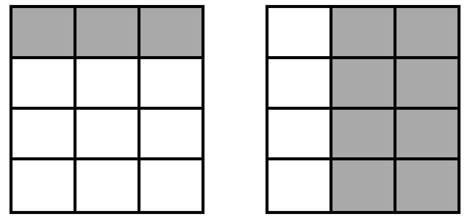
Consider, "How many is $1/4+2/3$?" To solve this problem, we do the following.

1. Start with unit squares drawn on student worksheets. We recommend using 6 cm-by-6 cm squares whose side lengths are not ruled. Any unit square whose dimensions permit students to easily determine halves, thirds, fourths, fifths, etc. is appropriate. It is important that side lengths are not ruled to allow students to subdivide or "cut" the sides during the addition/subtraction process.
2. Represent each fraction by subdividing and shading the unit squares. In our example problem, the fraction $1/4$ is represented by the unit square having horizontal cuts, and $2/3$ the one with vertical cuts.



1/4 2/3

3. Determine whether the cuts of each square are alike in size. Students should be allowed to reason and talk about the similarities and differences of the horizontal and vertical cuts. Observe that we use squares as units because vertical and horizontal cuts of the same size will have the same shape. Obvious questions to stimulate a dialogue are: (a) Are the vertical cuts the same size as the horizontal cuts? (b) How can our physical models be used to demonstrate this? (c) Can we provide logical rationales why the cuts are the same or different?
4. If the cuts are alike, we can add or subtract by using counting techniques or basic addition and subtraction facts. If the cuts are not alike, then we must modify the representation of the fractions to make them alike by doing the following: (a) Make the same vertical cuts in the first square that were made in the second unit square. (b) Make the same horizontal cuts in the second unit square that were made in the first.



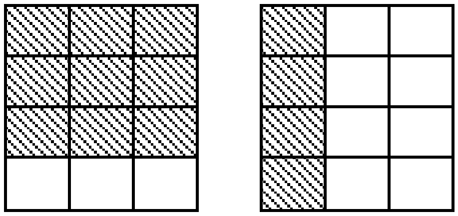
1/4 = 3/12 2/3 = 8/12

   Whenever the process is used, it is important to ask questions about, and discuss the meaning of, the cuts and crosscuts until students demonstrate their understanding of the elements of the process. For example, (a) Does the shaded part of the first unit square still represent 1/4? (b) What equivalent fraction is also represented by the modification to the first unit square? (c) Are the cells created by the crosscuts in the first and second unit squares alike (i.e., of the same size)? Explain. (d) How important is it that the unit squares of each fraction are of the same size?
5. Once the two fractions have been modified so that they are represented by same-size parts of the whole, students may add by counting or by using the relevant basic addition fact. We may represent what we did with the unit squares, cuts, and crosscuts in symbolic terms:
$$1/4+2/3=3/12+8/12=11/12\,.$$

Subtracting Fractions

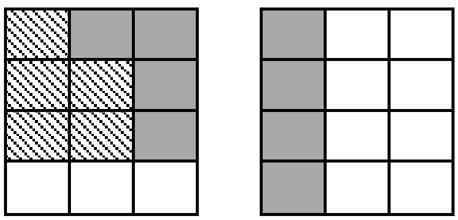
Here is a subtraction example: How many more is $3/4$ than $1/3$? Note that we have posed the question as a comparative subtraction problem. We could ask, "How many is $3/4-1/3$?", but in the beginning we want to emphasize the comparative interpretation of subtraction because we want to use two unit squares rather than one (as would be used in the take-away interpretation of subtraction) to determine the difference. (To read about the interpretations of subtraction, see Troutman & Lichtenberg, 1995.)

Implementing Steps 1 through 4, we get:



9/12 4/12

For step 5 we compare the number of shaded cells of $9/12$ with those of $4/12$.



We observe that there are five more shaded cells in $9/12$ than there are in $4/12$. Hence, the difference is $5/12$. Or,

$$3/4 - 1/3 = 9/12 - 4/12 = 5/12.$$

Increased Understanding

As students use the above representation of fraction addition and subtraction more and more, they are able to describe the addition and subtraction algorithms in great detail and with understanding. They easily address such questions as: (a) How can one tell if two fractions are alike? (b) How can one modify two fractions to make them alike? (c) How is the number of crosscuts in a unit square related to the representation of a fraction? (d) How is the shaded part of the unit square affected by crosscuts of the unit square? (e) How are numerators and denominators represented in the addition and subtraction process? And, as these questions are addressed, students construct referents to the elements of the traditional algorithms for the addition and subtraction of fractions.

This approach to adding and subtracting fractions recognizes that students should be given broad experiences in partitioning objects to provide them experiences with the relationships between the size and number of parts of the whole (Pitkethly & Hunting, 1996). Textbooks seldom reflect the instructional value of partitioning as a starting point for work with fractions. Most textbooks only present children with drawings that are pre-marked into equal sized pieces (Dorgan, 1994). Although this is certainly a valid device, if it is the only context encountered during the development of fraction ideas, students will miss many important underlying concepts and skills, including the development of strategies for handling fractions represented by a variety of different shapes and sizes (Witherspoon, 1993).

When it is time to teach addition and subtraction of fractions, most students have ample relevant knowledge to conceptually understand and represent the process. Unfortunately, they do not see how the symbolic forms for the algorithms relate to the whole-number addition and subtraction algorithms previously learned. In fact, if students have sufficient experiences in subdividing units, partitioning sets, and determining the equivalence of parts of a whole, they should be encouraged to construct meaningful addition and subtraction algorithms for fractions without the use of formal symbols (Mack, 1990; Mack, 1993; Empson, 1995).

Unfortunately, many students' understanding of fraction addition and subtraction is limited to their rote knowledge of the procedures rather than an awareness of the concepts that lead to the algorithms. It is exactly this rote knowledge of procedures that hinders students from successfully building on that part of their prior knowledge that would be useful in developing meaningful fraction algorithms (Hiebert & Wearne, 1988). Studies show that fraction concepts are better developed with students who can construct meaningful algorithms by building upon this prior knowledge (Hiebert & Wearne, 1988; Resnick et. al., 1989; Mack, 1990). The connection we have made between the representation of fraction addition and subtraction using countable objects and the development of an algorithm based on the manipulation of those objects helps students better understand fraction addition and subtraction and better able to perform the associated paper-and-pencil algorithms.

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The Materials Marketplace at the NCCTM Conference

The Marketplace will be back this fall, and the organizers need your help! Please consider donating materials to the marketplace. We are looking for new or gently used supplies such as manipulatives, posters, books, professional development materials—anything that would be useful to new teachers.

Preservice and new inservice teachers will be invited to come and purchase at rock bottom prices all sorts of materials—textbooks, technology, supplies, etc.—to start building their resource base.

Please contact coordinators Kim Aiello and Shana Runge if you have materials to contribute.

<ncctmmarketplace@hotmail.com>

Teaching With Technology

Mathematics of Tipping Points

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In 421 BCE Hippocrates (466-377 BCE) recorded the first description of influenza. We are still thinking about how flu spreads. Consider a very simple model for the spread of the flu. Suppose we know how long people are actively spreading the flu, how many uninfected people they meet each day, and what percentage of the contacts results in a new case of flu. We want to calculate the spread of the flu under three different public health scenarios, shown in Table 1.

	Days Exposing Others	Contacts per Day	Contacts Infected	Notes
1	2 days	50 people	2%	No intervention
2	2 days	50 people	1%	Improved hygiene
3	1 day	50 people	1%	Stay home and use better hygiene

Table 1. Three Flu Scenarios

If we start with 16 cases of the flu, the behavior for the three scenarios differs as shown in Table 2.

	Number of Infected People			
	Day 0	Day 2	Day 4	Day 6
1	16	16(2)(50) (.02) = **32**	32(2)(50) (.02)=**64**	64(2)(50) (.02)=**128**
2	16	16(2)(50) (.01)=**16**	16(2)(50) (.01)=**16**	16(2)(50) (.01)=**16**
3	16	16(1)(50) (.01)=**8**	8(1)(50) (.01)=**4**	4(1)(50) (.01)=**2**

Table 2. Number Infected in the Three Scenarios

This model is extremely simple, but it demonstrates a key principle: By understanding the behavior of a system under different assumptions, we can be more effective at controlling the system to behave as desired. Obviously, scenario 2 represents some kind of boundary between two very different situations. It is a “tipping point” between exponential growth and exponential decay.

Tipping Points

A tipping point is a model component for which small changes have large consequences. The idea became popular with the publication in 2000 of Malcolm Galdwell’s book, *The Tipping Point: How Little Things Can Make a Big Difference*. Now tipping points are popping up all over, in describing the spread of new technologies, the effective implementation

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of organizational change, the beginning of revolutions, and the effects of greenhouse gases.

Mathematically tipping points come in two forms. The first is an unstable value of a control parameter in an equation. The second is a value of the dependent variable at which "things get out of hand."

The control parameter version of the tipping point is illustrated by the function

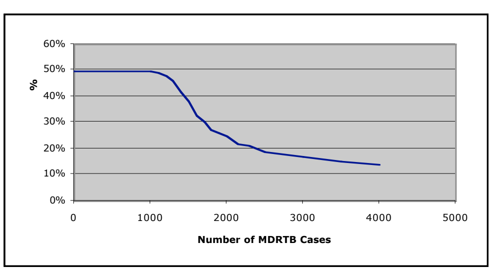
$$y = AB^x$$

or, equivalently, the recursive relationship

$$y_n = By_{n-1} \text{ and } y_0 = A$$

since it will grow exponentially if $B > 1$ and decay if $B < 1$. The three simple flu scenarios fit this model with B being 2, 1, and 1/2 respectively. In practice, finding a value for B for real diseases is hard, but the public health objective is clear: Getting long-term control of the disease requires keeping B small through quarantines, intense treatment or public education.

The mathematics of the tipping point as a value of a dependent variable is less straightforward. For example, in epidemiology there are only so many trained public health workers. Thus, if the number of cases of multi-drug resistant tuberculosis (MDR-TB) is small enough, caseworkers can have personal and intense contact with each infected person. As the number of infections grows, the load on caseworkers becomes larger and the interventions become less effective. Suppose that without intervention, an average person with MDR-TB causes 1.5 new infections if there is no public health intervention and the effectiveness of intervention is variable and is modeled by Graph 1.



Graph 1. Variable level of effectiveness

If the number of cases is below 1000, intervention decreases new infections by 50% according to the graph, so on average a person with MDR-TB infects only $B = (.5)(1.5) = 0.75$ others, and the disease is under control. If there are 2000 cases, the effectiveness has fallen to 25%; each case causes on the average 1.125 new infections, and the epidemic will spread. Somewhere between 1000 and 2000 cases is a tipping point for the control of MDR-TB.

The MDR-TB example illustrates a second key principle: In practice, we may need a computer model to investigate and visualize tipping points and other characteristics of the system we are studying.

Model Components

A first step in understanding and building computer models is to express what is going on in the components of the system as functions. That is, one uses the activities described in the North Carolina Mathematics: Course of Study and Grade Level Competencies for middle school students (NC DPI, 2003).

> Students…use the language of function, identifying patterns and relationships in context and expressing them algebraically. Variables are used to describe the interdependence of quantities and build an understanding of slope as rate of change between quantities. In order to solve problems, ordered pairs of data are generated and used to identify a linear relationship between quantities graphically and algebraically. From tables and graphs students recognize nonlinear relationships and functions.

In helping students to understand functions and see the equivalence of different forms, the visual, dynamic aspects of technology tools can be very useful. For example, the Function Flyer Tool from The Shodor Education Foundation, Inc.'s Project Interactivate website (Shodor, 2005) can be used to demonstrate how the graph of the function

$$y = Ax + Bx^2 + CD^x$$

depends on the parameter values for A, B, C, and D, as well as how the graphs are related for different values and where tipping points delineating different types of behavior occur.

For example, Figure 1 shows variations in the base D.

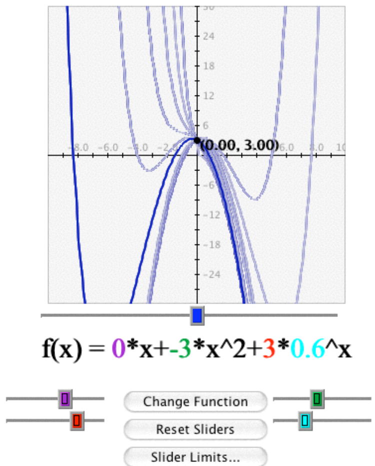


Figure 1. Function flyer for parameter experimentation

More Complex Models

Once students understand the component functions, they can build models with complex interactions. Let's go back to the flu. In the real world, there is not an infinite number of people to be infected with the flu, so epidemics can't grow exponentially forever. Furthermore, two infected people meeting don't add any new infections, and someone who is recovered is generally immune.

A slightly more realistic and commonly-used model is the Susceptible, Infected, and Recovered (SIR) model. The susceptible people haven't had the disease yet. The infected people have the disease and can spread it to others. The recovered people had the disease and are now immune. In each time period, the number of susceptible people who become infected is

(susceptible)(infected)(infection-rate).

If each infected person is sick N days, the number of people who recover in each period is (infected)/N.

Using a systems modeling program such as STELLA™ (as in Figure 2) or Berkeley Madonna, the SIR model can be investigated for many values of the infection rate and starting populations.

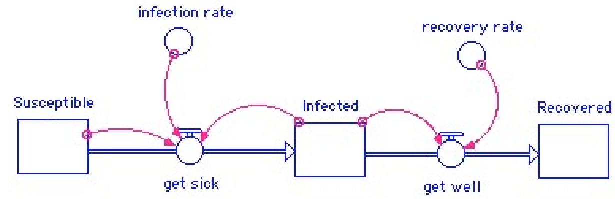


Figure 2. A STELLA™ SIR Model

Running the model with different parameter values for infection rates results in graphs such as those in Figure 3.

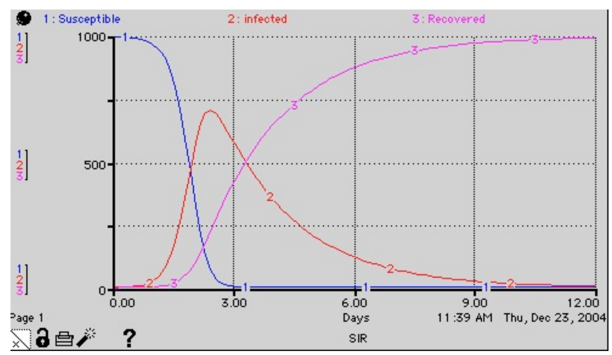


Figure 3. Typical SIR population curves

Infections increase with more contacts between susceptible and infected people. As more people are infected, there are fewer susceptible people to infect so the outbreak reaches a tipping point where infections start to decrease. Moreover, there is a more subtle tipping point in the infection rate. Above the tipping point, everyone is eventually infected, but below it, the graphs flatten out with substantial populations still susceptible as in Figure 4.

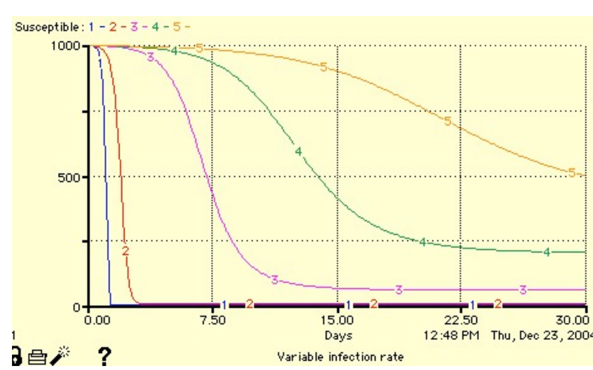


Figure 4. Effect of infection rate on long-term behavior of the susceptible population

A more complete model would also include public health measures such as encouraging people to get vaccinated and to stay home when they are ill. Once someone is vaccinated, they are indistinguishable from someone who is recovered. People who stay home are infected but not contagious. The Shodor Education Foundation, Inc., has developed the STELLA™ model in Figure 5 as part of its SUCCEED program (Shodor, 2002). This model incorporates more of these factors and, along with the basic SIR model, is available on line.

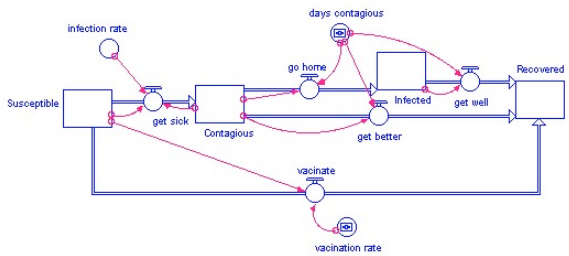


Figure 5. A STELLA™ flu model with more factors

The behavior of this model is more complicated than the SIR model, and students can learn concepts through changing parameters and observing how those changes cause changes in the behavior of the system. This kind of computer model can be useful in the classroom as a way for students to participate actively in their own learning.

Conclusion

The tipping point is just one example of a mathematical idea that is now more accessible because of technology. The idea in this form can be investigated by middle school and high school students, and can give teachers a new way for these students to learn more about the basic mathematics of algebra and functions. Technology tools such as Project Interactivate and STELLA™ allow this to happen in a more active, inquiry-based way.

In addition, scientists are increasingly using modeling and visualization tools in their research into complex systems such as fluid flow and turbulence in physics, environmental impacts, business processes and molecular structures. Because of the number of political decisions that are made based on computer models – where to put roads, what is too much pollution, what are the costs and benefits of a change in tax policy – all students need some appreciation of how modeling tools work.

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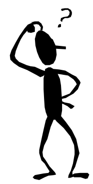
NCTM 2006 National Meeting

The national meeting will be held in St. Louis, Missouri on April 26 through 29, 2006. The theme is ***Asking Questions, Generating Solutions***. Pre-register before March 31 and save! See the NCTM website <http://www.nctm.org/> for more details. Keynote speakers include Neil deGrasse Tyson from the Hayden Planetarium, NCTM President Cathy Seeley, NCTM President-Elect Francis (Skip) Fennell, and 2005 National Teacher of the Year Jason Kamras.

Problems to Ponder 

Spring 2006 Problems

Gregory S. Rhoads
Appalachian State University
Boone, North Carolina

Grades K–2	The Jones family has fifteen children. Each child goes to the local apple orchard, picks 2 apples, and places them all in a basket. They wish to share the apples with the Smith family, which has five children. Each of the Smith children takes one apple from the basket. How many apples are left in the basket for the Jones family?
Grades 3–5	What is the smallest positive integer that has exactly 5 different integer factors (including 1 and itself)?
Grades 6–8	On their initial end of grade tests, John scored 26 out of 40, Pauline scored 21 out of 35, and Mike scored 17 out of 30. They were allowed to take the test again and John scored 27 out of 35, Pauline scored 23 out of 30, and Mike scored 30 out of 40. Which student had the largest increase in their percentage score on the test?
Grades 9–12	Suppose that (x, y) is a solution to the system of equations $$xy = 6 \text{ and } x^2y + xy^2 + x + y = 63$$ The point (x, y) lies on a circle centered at the origin. What is the radius of this circle?

Directions for submitting solutions

1. Neatly print the following at the top of each solution page:
   - Your full name (first and last)
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page.

Students who submit correct solutions will be recognized in the next issue of *The Centroid*. We publish creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission. Keep in mind that proper acknowledgement is contingent on legible information and solutions.

Send solutions by 30 April to:

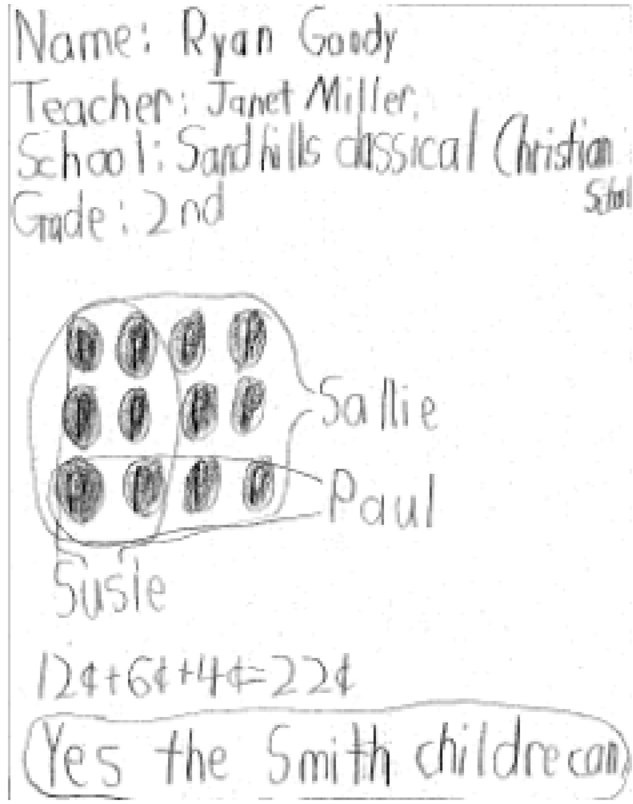
Problems to Ponder
c/o Dr. Greg Rhoads
Dept. of Mathematical Sciences
Appalachian State University
Boone, NC 28608

As these problems are intended to stimulate independent thinking, it is expected that a submitted solution indicates that the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

Solutions for Problems from the Fall 2005 Issue

Grades K-2

The Smith children are counting their pennies to see if they can buy an ice cream cone. Sallie has 12 pennies. Susie has half as many as Sallie and Paul has one third as many as Sallie. If they combine all of their pennies together, do they have enough to buy the ice cream cone that costs 20 cents?



Solution: By Ryan Gandy, 2nd grade, **Sandhills Classical Christian School** (Teacher: Janet Miller)

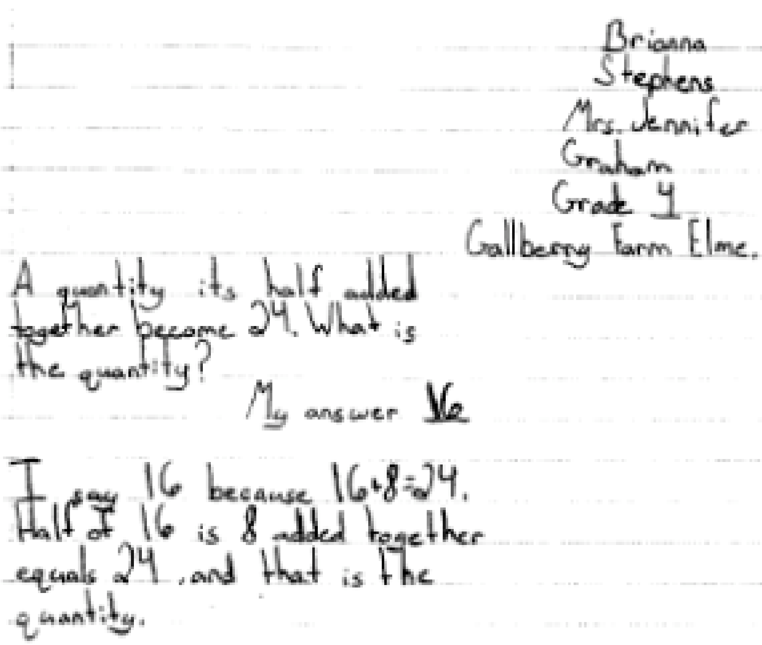
Correct solutions were received from: Mari Joe Sanqui of **Hardin Park Elementary** and Caitlin Burgess, Mary Furby, Ryan Gandy, Thomas Johnson, Kay Lauren Rhea, and Maddi Smith of **Sandhills Classical Christian School.**

Grades 3-5

A quantity and its 1/2 added together become 24. What is the quantity?

Solution: By Brianna Stephens, 4th grade, **Gallberry Farm Elementary** (Teacher: Mrs. Jennifer Graham).

Correct solutions were received from: Casey McGuirt of **David Cox Elementary,** Sarah Chenault, Nick Kalning, Chris Kim, Justin Lee, Carlos Lopez, Will Pearce, Pooja Shah, Shivani Shah, Anita Simha and Vishak Venkataranan of **Davis Drive Elementary,** Scott Kallianos, Matthew Szolnoki, Bill Vogel and Neusha Zadeh of **Easley Elementary,** Hannah Burnette, Lane Lon, and Fisher Reaves III of **Florence Elementary,** Brittany Anderson, Ashley Barefoot, Adam Bremer, Morgan Collins, Cory Efird, Ian Frazier, Sarah Harper, Madison Heath, Leona Heyward, Alyssa Like, Preston Lollis, Amanda Marley, Alexis Martinez, Joseph Martinez, James McBride, Logan McBride, Trevor Pollitt, Andrew Rhiner, Kyle Spegal, Brianna Stephens, Dominique Walker and James Westfall of **Gallberry Farm Elementary**, Daniel Broadnax, Tyler Crosby, Ayla Crum, Olivia Doherty, Megan Hoover, Gary John Hurley, Trae Lane, Wyatt Lemons, Connor McMahan, Steven Miller, Ash-

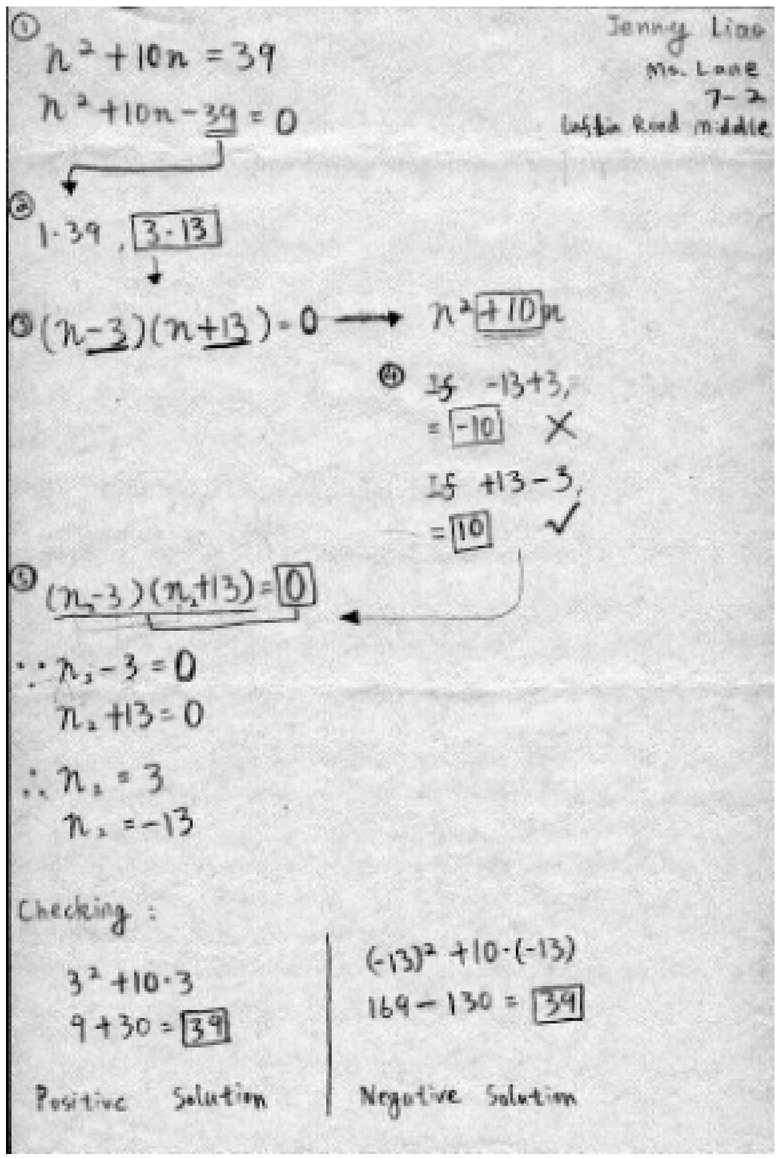


lee Schaffer, Kayla Trader, Coty White and Jeffrey White of **Moyock Elementary,** Luke Athans, Amy Fogleman, Johnny Gandy, Joshua Rhea, Alex Stroud and Dylan Walker of **Sandhills Classical Christian,** Chloe Carroll and Corbin Carroll of **Shining Light Academy,** Caleb Baker, Emily Phillips, Kat Thomson and Jonah Turcotte of **South Mebane Elementary.**

Grades 6-8

"…a square and 10 roots are equal to 39" (in modern terms: the square of a quantity plus 10 times the quantity equals 39), what is the number?

Solution: By Jenny Liao, 7th grade, **Lufkin Road Middle School** (Teacher: Ms. Lane).


Jenny Liao
Ms. Lane
7-2
Lufkin Road middle

① $n^2+10n=39$

$n^2+10n-\underline{39}=0$

② $1\cdot 39$, $\boxed{3\cdot 13}$

③ $(n-\underline{3})(n+\underline{13})=0 \longrightarrow n^2\boxed{+10}n$

④ is $-13+3$, $=\boxed{-10}$ ✗

is $+13-3$, $=\boxed{10}$ ✓

⑤ $(n_1-3)(n_1+13)=\boxed{0}$

$\because n_1-3=0$

$n_2+13=0$

$\therefore n_1=3$

$n_2=-13$

Checking :

$3^2+10\cdot 3$

$9+30=\boxed{39}$

Positive Solution

$(-13)^2+10\cdot(-13)$

$169-130=\boxed{39}$

Negative Solution

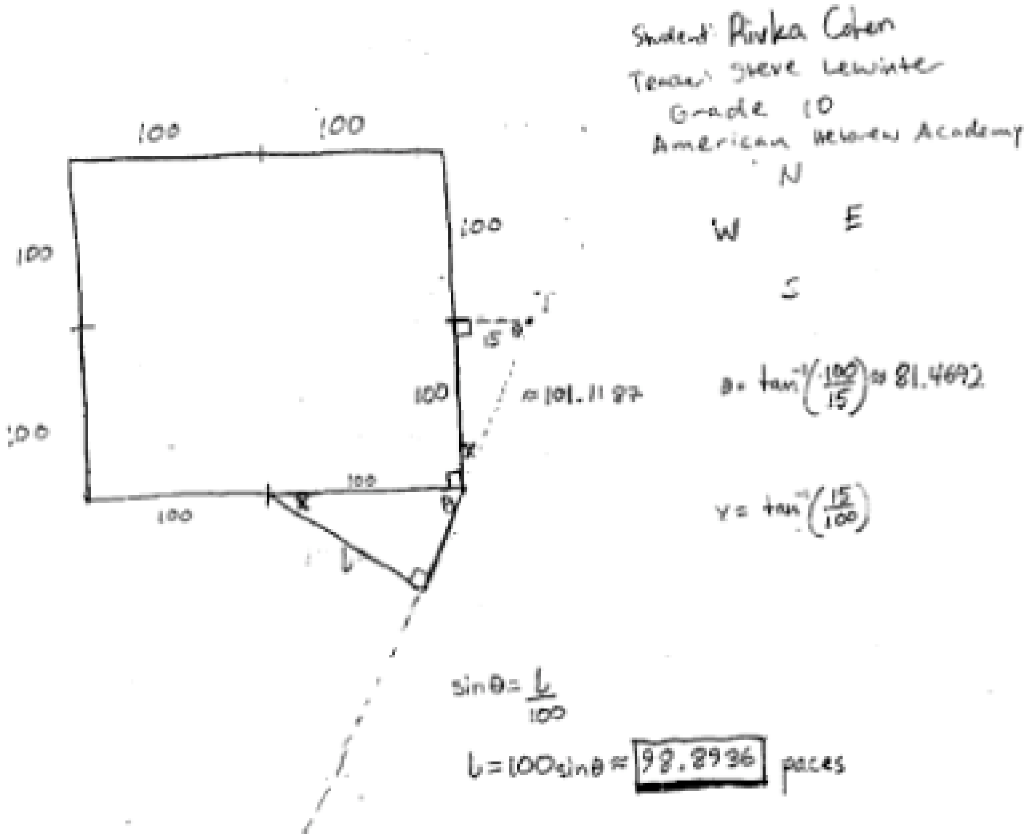
Correct solutions were received from: Jeremy Dammeyer of **Currituck County Middle,** Edward Cain*, Jenny Liao*, Aneesha Sehgal*, Priya Sharma*, Walter Squier, and Justin Trasti* of **Lufkin Middle,** Megan Blalock*, Holly Clayton*, Gibson Gillespie*, Eric Reed*, and Amber Williams* of **Northern Granville Middle,** Adam Fogleman* of **Sandhills Classical Christian,** Savannah Allen, Alissa Cline, Austin Kendrick, Olivia Meeks, and Christopher Rider of **Shining Light Academy,** James Bryan Bailiff, Chance Cockrell*, Emily Hinshaw, Logan Hurley and Corri Marshall of **Southeast Guilford Middle,** Sarah Balance*, Zykiatra Green*, Scott Hardy*, Jonnisha Hardy*, Nikki Harrell* and Nagawasa Thompson* of **Southwestern Middle,** Saangyoung Lee* of **Turrentine Middle,** Daniel Benfield, Chelsea Brown, Jacob Bumgarner, Emily Eckard, David Gilbert, Kristen Howe, Devan Isaac, Dylan Jolly, Nou Lee, Will Marlowe, Graham Marshall, Vanessa Pennell, Nicole Rauscher, Avery Ritchie, Jordyn Setzer, Allison Sigmon, Emily Sinclair, Zach Stafford, Olivia Starnes, Jacob Taylor, Crystal Tibbetts, Alex Vannoy, Ally Walker and Jonathan Williams of **West Alexander Middle,** Danielle Eylers, Ben Holden* and Zachary Staley of **Woods Charity.**

(* indicates the student correctly identified the negative solution also).

Grades 9-12

A square, walled city measures 200 paces on each side. Gates are located at the centers of each side. If there is a tree 15 paces from the east gate, how far must a man travel out of the south gate to see the tree?

Solution: By Rivka Cohen, 10th grade, **American Hebrew Academy** (Teacher: Mr. LeWinter).



Editor's Note: There were different ways of interpreting this problem based on which direction the person would walk out of the south gate. Most of the solvers below indicated that if you walked 100 steps along the wall, you get to the corner of the building and can see the tree. The solutions of Rivka Cohen and Sam Mize found the shortest distance the person could walk to see the tree. Another interpretation is that the person would walk due south out of the south gate. In keeping with the historical context, I used the most common translation from the Chinese in wording the problem.

Correct solutions were received from: Rivka Cohen of **American Hebrew Academy** and Sam Mize of **Turrentine Middle**.

DPI Math Resources on the Web

The website you are accustomed to for DPI mathematics resources is moving. Please check the new site

<http://community.learnnc.org/dpi/math/>.

Marjorie Lee Browne: North Carolina Educator

Sarah J. Greenwald[1]
Vicky Klima[2]
Katie Mawhinney[3]
Appalachian State University
Boone, North Carolina

Marjorie Lee Browne was a pioneering woman of color in mathematics. Completing her PhD at the University of Michigan in 1950, she was one of the first African American women to receive a PhD in mathematics, with Martha Euphemia Lofton (PhD from Catholic University of America in 1943) and Evelyn Boyd Granville (PhD from Yale University in 1949) finishing earlier (Williams, 2004). During her career as a teacher and professor, she became well known as an extremely caring and effective North Carolina educator.

Early Black Women Mathematicians

The earliest black women in mathematics faced many barriers (The Journal of Blacks in Higher Education, 2001):

> Over the years, black women who might be disposed to pursue a career in mathematics faced the "double whammy" of racism and sexism. Like blacks, women were not considered to have the mental skills necessary for advanced mathematical inquiry. For all women, and especially for black women, the field of mathematics was essentially shut tight.

Things began to change as black women obtained PhDs in mathematics, making it easier for others who followed, but there is still much work to be done. Today, less than 1% of all mathematicians are black, and of those, approximately 25% are women (Williams, 2005).

Background and Education

Marjorie was born on September 9, 1914 in Memphis, Tennessee. While it was unusual at the time due to racial barriers, her father had been able to attend two years of college and was known for being a "whiz" in mental arithmetic. He helped Marjorie and her older brother in their mathematical studies, both financially and by passing on his love for mathematics. While her brother majored in mathematics as an undergraduate, he obtained a master's degree in physical education and pursued that as a career.

It is not surprising that Marjorie attended and taught exclusively at historically black colleges and schools, since there were very few opportunities for African Americans at other learning institutions. She attended an excellent private high school in Memphis. Even though college funding was difficult during the depression, with the help of loans, scholarships, and

[1] *Sarah Greenwald is associate professor of mathematics, and spends time on Simpson's math and orbifold geometry.*
[2] *An assistant professor of mathematics, Vicky Klima works in Lie Algebra and has studied one-parameter subgroups.*
[3] *An assistant professor of mathematics, Katie Mawhinney is interested in point set topology and in math education.*

hard work, she graduated with a bachelor's degree from Howard University. Next she taught at a black high school in New Orleans. When a neighbor informed her that the University of Michigan was open to minorities and within her price range, she attended during the summers to obtain her master's degree. She then taught at Wiley College in Texas as she worked on her PhD at Michigan. Her thesis advisor was George Rainich, who had encouraged and advised numerous African American students, including Wade Ellis, Sr.

Marjorie's PhD thesis was on *Studies of One Parameter Subgroups of Certain Topological and Matrix Groups*. Although she applied to research universities, she was politely rejected (Fletcher, 1999):

> Browne was acutely aware of the obstacles which women and minorities faced in pursuing scientific careers... and resolved that her greatest contributions would be directing programs designed to strengthen the mathematical preparation of secondary school teachers and to increase the presence of minorities and females in mathematical science careers.

She accepted a position at North Carolina College, which is now North Carolina Central University.

North Carolina Central University

Marjorie poured her heart into her job (Kenschaft, 1980):

> If I had my life to live again, I wouldn't do anything else. I love mathematics.

As a mathematics professor, Marjorie taught fifteen hours a week, published articles, ran workshops for secondary teachers in the summers, and became the chair of the mathematics department. Browne described herself a "pre-*Sputnik* mathematician," referring to pure mathematics preparation and research done for the sake of intellectual pursuit instead of applied investigations (Fletcher, 1999). When she was asked for a definition of this term, she said that it was (Fletcher, 2005)

> a mathematician who appreciates the beauty, power and eloquence of mathematics as one of the greatest art forms; one who responds to its ability to stir the imagination and who recognizes mathematics as the sole custodian of precision.

She was an outstanding teacher, and she was the advisor for ten master's theses. Dr. Asamoah Nkwanta, who was a student at Central and met her shortly before her retirement, said (Nkwanta, 2005):

> She was a very nice and down-to-Earth type of person. She always encouraged us as students. I was and have always been impressed with her mathematical endeavors. I was honored to have met her.

Dr. Fletcher, who first met her as a student in Marjorie's calculus class in 1954, said (Fletcher, 2005):

> It was Dr. Browne who first showed me and countless other bewildered students that mathematics could be a delightful, creative pursuit; and her forbearance and encouragement made life for us - as beginning mathematics students - challenging and brighter. She brought mathematics within our reach and opened up a world of beauty and opportunities that we did not know existed... Her philanthropic philosophy included the axiom that no good student should go without an education simply because he or she lacked the financial resources to pay for it. Thus, it was not uncommon for her to provide financial assistance for many of her students - for tuition, books, food, clothes, funds to attend scientific meetings.

At the national level, she advocated the integration of mathematics meetings and conferences, and she was an outspoken critic of the discrimination that was prevalent in funding agencies (Fletcher, 1999). She received numerous grants, honors, and awards, including the first W. W. Rankin Memorial Award for excellence in mathematics education from the North Carolina Council of Teachers of Mathematics. She died in 1979.

Activities and NCTM Standards

The following activities relate to her mathematics and address numerous points in the NCTM *Principles and Standards for School Mathematics*. While the inclusion of her mathematical achievements can easily be incorporated into college level modern algebra courses, aspects relating to her work may still be incorporated into courses at other levels.

In grades three through five, students explore how variables are related to each other, and the one-parameter activity below can be adapted for use there or in the higher grades to explore the connections between algebra and geometry. As in the worksheet presented below on matrix groups, which is designed for use in the high school classroom, the number and operations standard specifies that students should develop their understanding of properties and representations of addition and multiplication of matrices. The standards also discuss the fact that "mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement" (NCTM, 2000). The North Carolina Mathematicians activity can help with this second goal.

One-Parameter Activity

Begin with an introduction to Marjorie Lee Browne's life, and explain that her thesis related to one-parameter objects. Have your students search the internet for the definition of a parameter. Ask them to create a function of one variable, to write the (x,y) coordinates of a circle in terms of one variable, and to compare their work with the definition of parameter they found. Ask them to share their findings with the rest of the class and discuss the difference between a variable and a parameter. Search the internet for applications of one-parameter objects within other fields such as statistics and the mathematics of diseases.

North Carolina Mathematicians Activity

Have student groups research the lives and work of Marjorie Lee Browne and mathematicians who taught in North Carolina such as Alfred Brauer (UNC Chapel Hill and Wake Forest University), Ethelbert Chukwu (North Carolina State University), William Cochran (North Carolina Institute of Statistics), Elbert Frank Cox (Shaw University), Gertrude Cox (North Carolina State University and North Carolina Institute of Statistics), Max Dehn (Black Mountain College), William Fletcher (North Carolina Central University), Johnny Houston (Elizabeth City State University), Witold Hurewicz (UNC Chapel Hill), Mary Ellen Rudin (Duke University), Beauregard Stubblefield (Appalachian State University), and William Whyburn (UNC Chapel Hill). These are just a sampling of North Carolina mathematicians. By using a google search ("North Carolina" site: www-groups.dcs.st-and.ac.uk), and other similar site searches (e.g., The Mathematics Genealogy Project; Williams, 2005), many additional names can be found.

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Activity Sheet: Matrix Groups

Marjorie Lee Browne was the third black woman in the United States to receive her PhD in mathematics, and she was known as an extremely caring and effective North Carolina educator, teaching at North Carolina Central University for most of her career. Regarding her career choice, Dr. Browne said: “If I had my life to live again, I wouldn't do anything else. I love mathematics.” In this worksheet we will explore topics related to her thesis work and published paper.

Concept 1: The Square Orthogonal Matrices

A *matrix group* is a set of matrices that satisfy specific algebraic relationships. One of the many groups of matrices studied by Dr. Browne was the group of square orthogonal matrices, represented by the notation $O(n)$, where n represents the dimension of the matrices. In order to understand what it means to be an orthogonal matrix, we must first investigate identities and transposes.

We learn in elementary school that multiplication by one leaves the value of a number unchanged. One is called the multiplicative identity because of this property. Sets of square matrices also have multiplicative identities, called *identity matrices*. An identity matrix $\boldsymbol{I}$ is a matrix with 1’s down the diagonal and 0’s everywhere else. For any $n\times n$ matrix A, the product of A with the $n\times n$ identity matrix $\boldsymbol{I}$ is equal to A.

<u>Question A</u> Check the identity property with $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ and $\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

<u>Question B</u> Is multiplication with $\boldsymbol{I}$ always commutative ($A\boldsymbol{I}=\boldsymbol{I}A$ for any matrix A)?

The *transpose* of a matrix is a new matrix whose rows are the columns of the given matrix. For example, $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ has transpose $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$. A square matrix is *orthogonal* if the product of itself with its transpose is the identity matrix, i.e., $AA^T = \boldsymbol{I}$.

<u>Question C</u> Are $B = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ and $C = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ -2 & 0 \end{bmatrix}$ orthogonal matrices?

$O(2)$ is the set of all 2×2 orthogonal matrices, and in geometry these matrices can be interpreted as representing the symmetries of the circle $x^2 + y^2 = 1$. For example, the orthogonal matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ acts on the points of the circle by matrix multiplication: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$. Points on the circle are sent to the corresponding points with the opposite x-value, but the same y-value, i.e., each point on the circle is reflected across the y-axis. Since this reflection “preserves” the circle (the new set of reflected points is exactly the same circle), it is called a symmetry of the circle. Similarly, the symmetries of the sphere $x^2 + y^2 + z^2 = 1$ can be represented by $O(3)$, and in general $O(n)$ is extremely important in science and engineering.

Question D Show that the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is orthogonal by checking that $AA^T = \boldsymbol{I}$. Next explain why it preserves the circle by describing how it acts on the points of the circle as a rotation of a certain degree.

Concept 2: One-Parameter Subgroups

Marjorie Lee Browne's PhD thesis was on the *Studies of One-Parameter Subgroups of Certain Topological and Matrix Groups*. The matrix $\begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$ is a one-parameter subgroup of $SL(2)$, the matrix group of 2×2 matrices with real entries and determinant equal to one. The parameter here is t, a real number, and for each value of t we generate a matrix in the subgroup.

Question E Recall that the determinant of the 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$. Check that $\begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$ has determinant equal to one.

The matrix $R(t) = \begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a one-parameter subgroup of $O(3)$, the group of orthogonal matrices that act as symmetries of the sphere $x^2 + y^2 + z^2 = 1$. Again the parameter is t, a real number.

Question F To show that $R(t)$ is a symmetry of the sphere, calculate $\begin{bmatrix} \cos(t) & -\sin(t) & 0 \\ \sin(t) & \cos(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and show that the new values of x, y and z satisfy $x^2 + y^2 + z^2 = 1$.

Question G For a given real number t, what geometric transformation does the matrix $R(t)$ represent? First try some examples of specific values of t such as $t = 0^\circ$, $t = 90^\circ$, and $t = 180^\circ$ and examine the action of $R(t)$ on the sphere $x^2 + y^2 + z^2 = 1$.

While her thesis contained completely original work, her published paper represented an attempt to help people understand matrix groups. In Marjorie Lee Browne's own words:

> *The purpose of this paper is to set forth some topological properties of and relations between certain classical groups. While much of the material included here may be known to a few, the main interest of this paper lies in the simplicity of the proofs of some important, though obscured, results.*

Throughout her career, she tried to make mathematics accessible to as many people as possible, both in her research and her teaching. She received numerous grants, honors and awards, including the first W. W. Rankin Memorial Award for excellence in mathematics education from the North Carolina Council of Teachers of Mathematics.

Editor's Note: The answer key is available at the Centroid website
<http://www.mathsci.appstate.edu/centroid/>

Awards

2005 NCCTM Rankin Award Winners

Reported by Jeanette Gann and Bob Jones

Betty Long and Bill McGalliard

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Betty Long and William "Bill" McGalliard have received the W.W. Rankin Memorial Award for Excellence in Mathematics Education and Service from NCCTM. Dr. Long and Dr. McGalliard are professors in Appalachian State University's Department of Mathematical Sciences where they teach mathematics and education classes.

Betty Long is the NCCTM Representative to the National Council of Teachers of Mathematics, past president of the NCCTM Western Region, and has organized many NCCTM math fairs. For more than 30 years, she has given of her time and talent in an unselfish manner to the quest for excellence in mathematics education. Betty has taught math at the secondary and college level and has been a major player in preparing young people to teach mathematics. She has taken on numerous leadership tasks in NCCTM, and carried them out in exemplary fashion.

The enthusiasm with which Betty approaches a task is contagious. Colleagues say things like, "If you want something done and done well, ask Betty," or "Betty is enthusiastic about all matters in mathematics education," or "Betty is an informed and organized person." Audiences respond enthusiastically to this gifted teacher's presentations.

Bill McGalliard has been actively involved in the teaching of mathematics for almost 40 years. His contributions to mathematics education include being a secondary mathematics teacher and a college mathematics professor. As a professor, he not only taught perspective and current teachers, but went into classrooms and taught with teachers.

Bill has also been an active member of NCCTM for many years; he has served in many capacities, including State President, President of the Western Region, and NCCTM representative to the National Council of Teachers of Mathematics

As a colleague so aptly stated: "Bill has worked a lifetime trying to improve the quality of mathematics education that our children receive. Bill is a wonderful role model for the many elementary, middle school and secondary mathematics teachers in our state."

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in the State. Presented in the fall at the State Mathematics Conference, the award, named in memory of W.W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

> If you have nominated someone in the past who has not received the award to date,
> or if you would like to nominate someone now,
> please submit as much of the following information as possible.
> Nominations are accepted at any time.

Nominations should include the following information. Use as many typewritten pages as needed. If possible, attach a vita of the nominee.

Name of the nominee

Current position

Your relationship to the nominee (e.g. principal, co-worker, etc.)

The nominee's contributions to mathematics education, NCTM, NCCTM, etc. (Please include information on specific offices held and honors received by the nominee.)

Any information about contributions to the community, teaching, and education that would be of value to the Rankin Award Committee in its deliberations

Other relevant information

Letters of endorsement from other colleagues may be included.

Date of nomination

Nominator* Name
Current position
Business or educational institution
Preferred mailing address
Preferred telephone number

*The Rankin Award Committee reserves the right to use portions of nomination information in the presentation of the award if the candidate is selected.

Send to: Dr. Ralph DeVane
P. O. Box 1762
Cullowhee, NC 28723

Awards

2005 Innovator Award Winner

Reported by Phillip Johnson

Gail Stafford

Since 1994, NCCTM has recognized innovative contributions to the organization and to mathematics education in the state with the Innovator Award. The purpose of the award is to recognize and reward individuals or groups who have made an outstanding or noteworthy contribution by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographical region of the state or the entire state.

This year's individual Innovator Award winner is Gail Stafford, Associate Professor of Mathematics at North Carolina Wesleyan College. Gail has been on the faculty at North Carolina Wesleyan since 1990, and she chaired the Division of Mathematics and Sciences from 1998-2001. After receiving her BS in Mathematics Education from North Carolina State University, she taught in high school for twelve years before returning to East Carolina University to pursue an MA Ed in Mathematics Education. After graduating from East Carolina, she joined the East Carolina faculty as a Lecturer of Mathematics until assuming her present position at North Carolina Wesleyan.

In 1990, Gail took on the responsibility of coordinating the Eastern Regional Runoffs in Algebra I, Geometry, and Algebra II. More than 100 students compete annually in the Eastern Region runoff site, many more than at each of the other two sites. In addition to coordinating the contest for the Eastern Region, Gail writes one of the three tests each year.

The contest coordinator has many duties, including writing one of the three tests, corresponding with local contest sites, securing and preparing award materials, administering and grading tests, announcing winners, and corresponding with participating schools following the contest. The contest coordinator also reports results to the person who has agreed to compile results for the entire state and identify the top ten finishers in each category in the state. For five years, Gail compiled the results for the state and reported these to members of the Executive Contest Committee so statewide results could be posted on the contest website and so the top five finishers could receive plaques for their accomplishments.

Gail is presently working on solutions for the 2003 Algebra I, Geometry, and Algebra II tests so they can be posted on the contest website. She is also working on a Wesleyan Math Contest website that will be linked with the State Contest site.

Awards

2005 Innovator Award Winner

Reported by Phillip Johnson

Summer Ventures in Science and Mathematics

For the first time since its inception in 1994, the Innovator Award was presented to an organization. The Summer Ventures in Science and Mathematics Program was presented the award during the 35th Annual NCCTM State Mathematics Conference. Groups have always been eligible for the award, and there are several groups in North Carolina deserving of such recognition. Dr. Sally Adkin, State Coordinator of Summer Ventures in Science and Mathematics, deserves primary credit for this article on the Summer Ventures Program.

History of the Program

Summer Ventures in Science and Mathematics (SVSM) completed its twenty-first summer of operation in 2005. The North Carolina General Assembly enacted legislation in 1984 that created SVSM. The program was conceived as a summer enrichment program for high-ability high school students potentially interested in science-and-mathematics-based careers. Accordingly, SVSM was placed under the administration of the North Carolina General Administration with coordination and supervisory roles delegated to the North Carolina School of Science and Mathematics.

Five campuses initiated programs in the summer of 1985: Appalachian State University, East Carolina University, North Carolina Central University, The University of North Carolina at Charlotte, and Western Carolina University. Each campus was empowered to operate programs under a director appointed by the local Chancellor. The overall program and the admissions process were coordinated and evaluated through the North Carolina School of Science and Mathematics. In 1987, the program was expanded when The University of North Carolina at Wilmington joined as the program's sixth campus. Currently the funding supports approximately 480 students statewide to participate in a four-week residential program.

Program Description

Admission to SVSM is open to any rising high school junior or senior whose parents live in North Carolina. Applications are distributed through the local high school counselor's office. The application includes information about the student's interest in mathematics and science, his or her high school transcript, and a teacher recommendation. An admissions committee, made up of professionals in science and mathematics education from across North Carolina, selects program finalists. Each finalist is assigned to a specific campus based on the availability of the student's preferred academic program.

SVSM provides an innovative match of the public school and university summer calendars and teaching staff. The structured programs include daylong and weekend activities. Students live in dormitories under the supervision of qualified residential advisors who provide guidance and plan social, athletic, cultural and co-curricular activities. At each campus, university professors open their labs to the summer students and share their research interests. Master high school teachers are also recruited to work in cooperation with university professionals. This contributes to a sound instructional program, helps link the university with its cohorts in secondary education, and provides learning experiences for both the high school teachers and the university professors.

The academic offerings of SVSM vary from one campus to another and are tailored to best suit the campus's unique resources and geography. For example, Marine Science is a popular assignment at UNC Wilmington. During the summer program, students learn experimental design, laboratory skills, instrumentation, mathematical modeling, and strategies in mathematical problem solving and exploratory data analysis. Moreover, the students learn these basics while engaged in specific scientific and mathematical topics of interest to them. Students also learn about computer applications, careers in science and mathematics, social issues related to science, and communication skills for mathematics and science competitions. The SVSM program culminates with student presentations on the focus of their research efforts. These presentations are conducted in the style of a scientific professional meeting.

The strong focus on "learning through doing" is reflected in the basic evaluation instrument, which consists of twenty-six skills basic to science and mathematics investigation. Students assess their skills in these areas before attending Summer Ventures and after completing the program. In addition to this skill-growth evaluation, the program also conducts a faculty survey, a residential life survey, document review, and site visits. The Summer Ventures program values the collection and analysis of data to help gauge program impact and improve the program.

Impact

The SVSM State Coordinator's Office collects and analyzes a significant amount of data from all six campuses. Results from alumni respondents indicate that 86.3% have enrolled in an in-state institution of higher education, 85.2% have pursued graduate studies, 64.1% have majored in mathematics, science, or technology, and 73% are paying North Carolina income tax.

Beginning in 2003, the SVSM Directors suggested that SVSM programs institute recognition of outstanding participant research. The SVSM Directors proposed that up to three projects per campus be recognized each summer via the SVSM Catalyst Awards. Students representing all six of the SVSM campuses present mathematics and technology research at the October meeting of the North Carolina Council of Teachers of Mathematics. The SVSM's State Coordinator's Office coordinates these presentations and collects, publishes, and disseminates selected research papers.

SVSM has served over 10,700 high school students since its inception. The benefits of SVSM to the citizens of North Carolina include:

- Innovative research opportunities for North Carolina high school students.
- A model and impetus to jump-start math/science research in all high schools across North Carolina.
- An early college residential experience and the associated benefit of new friendships and important connections.
- Synergistic relationships resulting from exchange among exemplary university professors, master high school teachers, and the State's exceptional high school students.
- An opportunity for tomorrow's leaders to gain respect for and loyalty to a UNC campus.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The purpose of this award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state. Further, this program must have been sustained for a period of at least three years. A number of organizations have made significant contributions to mathematics education in North Carolina; the Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations by sending in the form below. Nominations will be retained in the active file for at least three years.

Nomination Form

Name of Nominee____

Present Position____

Outstanding contributions to mathematics education in North Carolina which serves as the basis for this nomination:

Additional information that would be of value to the selection committee:

Signature:____Date:___________________

Name (print/type):____

Position:____

Business or Institution:____

Address:____

Phone: Business:________________ Home: _________________Email:___________________

Send to: Phillip Johnson, Math and Science Education Center,
ASU Box 32091 Boone, NC 28608-2091

NCCTM Trust Fund Scholarship

$500 scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction.

Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting this application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Currently enrolled in a mathematics or mathematics education course, or have completed a mathematics or mathematics education course within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are:

- **March 1**
- **October 1**

Send completed applications to:
NCCTM Trust Fund Chairperson
6520 West Lake Anne Dr.
Raleigh, NC 27612

Direct inquiries to:
John Kolb, Chairperson
Phone: (919) 787-8116
E-mail: JKolb1@nc.rr.com

(Please print all information.)

PERSONAL INFORMATION:

Name:______________________________

Home address:______________________________

Home phone:____________________ Home e-mail:____________________

NCCTM membership number:______________________________

EMPLOYMENT INFORMATION:

How many years of teaching experience?______________________________

Currently employed in what school system?______________________________

School name:______________________________

School address:______________________________

School phone:____________________ School e-mail:____________________

Current teaching assignment:______________________________

Principal's name:______________________________

____
____

COURSE INFORMATION:

Institution of higher education:____

Graduate degree program in which you are currently enrolled:____

Course name:____ Course number:______________

Dates of enrollment: *(circle one)* Fall semester Spring semester Summer session Year:__________

Name of course instructor:____

PROFESSIONAL ACTIVITIES WITHIN PAST 5 YEARS:

BRIEF STATEMENT OF FUTURE PROFESSIONAL GOALS:

REQUIRED SIGNATURES:

Applicant's Signature:____Date:________________

Principal's Signature:____Date:________________

Instructor's Signature (if currently enrolled):_____________________________Date:________________

REQUIRED ATTACHMENTS:
Please attach a copy of verification of acceptance and enrollment in accredited graduate program in North Carolina.

NOTE: Applications must be complete to be considered. If your application is approved, an official course grade report must be submitted to verify successful completion of the course before scholarship funds will be issued.

NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

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Name: ____ Home Telephone: (____) - ____________

Address:____ School Telephone: (____) - ____________

City: ____________________ State: ________ Zip: _________ E-mail: ______________________________

School System: ____________________________________

MEMBERSHIP STATUS
❑ New ❑ Former/Renewing Member # _________

POSITION
- ❑ Teacher
- ❑ Department Chair
- ❑ Supervisor/Administrator
- ❑ Full-time College Student
- ❑ Retired
- ❑ Other ___________________

LEVEL
- ❑ K-3
- ❑ 4-6
- ❑ Junior High/Middle School
- ❑ Senior High
- ❑ 2-Year College/Technical
- ❑ 4-Year College/University

MEMBERSHIP DUES

❑ 1 year:	$10.00	____________
❑ 3 years:	$25.00	____________
❑ 10 years:	$75.00	____________
❑ Full-time Student:	$5.00	____________
❑ Contribution to Trust Fund:		____________
Total Payment Enclosed:		____________

Payment by ❑ Check ❑ Visa ❑ MasterCard

Card # ______________________________

Exp. Date ______________________________

Signature ______________________________

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CARY, NC 27519
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