## The Centroid

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- Discovering the Area of a Regular Polygon

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The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics-pre-kindergarten through teacher education. The Centroid is published in January and August. Subscribe by joining NCCTM; see the Membership Form on the last page.

## Submission of Manuscripts

We invite the submission of news, announcements, and articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways.

News and announcements (president's messages, award winner announcements, professional development announcements, etc.) must be received by December 1 for the spring issue and by July 1 for the fall issue.

Articles that have not been published before and are not under review elsewhere may be submitted at any time to the address below. Submit one electronic copy via email attachment (preferred) or diskette in Microsoft Word or rich text file format. To allow for blind review, the author's name and contact information should appear only on a separate title page. Manuscripts should not exceed 10 pages double-spaced with one-inch margins. Figures and other pictures should be included in the document in line with the text (not as floating objects). Scannable photos are acceptable and should be large glossy prints mailed to the editor or minimum 300 dpi tiff files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.

Manuscripts should follow APA style guidelines from the most recent edition of the Publication Manual of the American Psychological Association. References should be listed at the end of the article, and should also follow APA style, e.g.,

Bruner, J. S. (1977). The process of education (2nd ed.). Cambridge, MA: Harvard University Press.
National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.
North Carolina Department of Public Instruction. (1999). North Carolina standard course of study: Mathematics, Grade 3. Retrieved October 17, 2005, from $\mathrm{http}: / / \mathrm{www}$. ncpublicschools.org/curriculum/mathema tics/grade_3.html
Perry, B. K. (2000). Patterns for giving change and using mental mathematics. Teaching Children Mathematics, 7, 196-199.
Ron, P. (1998). My family taught me this way. In L. J. Morrow \& M. J. Kenney (Eds.), The teaching and learning of algorithms in school mathematics: 1998 yearbook (pp. 115-119). Reston, VA: National Council of Teachers of Mathematics.

General articles and teacher activities are welcome, as are the following special categories of articles:

- A Teacher's Story,
- History Corner,
- Teaching with Technology,
- It's Elementary!
- Math in the Middle, and
- Algebra for Everyone.


## Editorial Board

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## About the Cover

The Centroid logo is based on the following theorem: The limit of the sequence of midtriangles of a triangle is the centroid of the triangle.

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## From the Editor

Holly Hirst<br>Appalachian State University<br>Boone, North Carolina

We are inaugurating two new features that we hope to run in each issue of The Centroid: The "Professional Development Opportunities for NC Math Teachers" column will list professional development workshop information submitted to the editors. Teachers, faculty, and other educators are welcome to submit information to us by the announcement deadlines for each issue (December 1 for the spring issue and July 1 for the fall issue). We will publish a brief description and web address for any workshop held at a location in North Carolina specifically designed for mathematics teachers at the elementary, middle, secondary or undergraduate levels. See page 20 in this edition! We will also maintain a more detailed list on the Centroid website.

Also new are two activities shared by teachers, one on holding game day to let students practice math and team work and the other on making complex arithmetic drill a little more fun. We would like to include two to four one-page descriptions of activities like these in each issue. Teachers, please share your successful activities!

With this issue we also say THANK YOU to Brian Felkel, my tireless co-editor. Brian will be taking leave from Appalachian for a year to work as an actuary. We wish him luck. My colleague and NCCTM Board member Debbie Crocker will be working with me on future issues.

## NCCTM Spring Conferences

# Western Region Mathematics Conference: Putting the "Excel" in Excellent Teaching 

Saturday, February 23, 2008, 8:30 AM-1:00 PM
Jacobs Fork Middle School, Newton, NC
Eastern Region Mathematics Conference: Capitalizing on Mathematics at the Capitol
Saturday, February 23, 2008, 9:00 AM-12:30 PM
Meredith College, Raleigh, NC

## Central Region Mathematics Conference: Go Wild with Math at the North Carolina Zoo

Saturday, March 8, 2008, 8:00 AM-12:40 PM
North Carolina Zoo, Asheboro, NC
See the NCCTM website for more info [http://www.ncctm.org](http://www.ncctm.org).

## NCCTM State Conference: Oct 30-31

## Call for Speakers Koury Convention Center, Greensboro, NC

The 2008 State Mathematics Conference is a wonderful opportunity to share research, classroom strategies, activities, and resources with your colleagues that make mathematics come alive for your students. Be a speaker at the state conference; the speaker form can be found online [http://www.ncctm.org](http://www.ncctm.org).

## Presidents' Messages

## State President

## Randy Harter

randy.harter@bcsemail.org
In the months ahead NCCTM leaders will continue to partner with other key individuals and organizations in the state to develop new initiatives to improve the teaching and learning of mathematics in North Carolina. On October 10, 2007, those who attended the NCCTM Leadership Pre-session heard from State Superintendent of Public Instruction June Atkinson, Deputy Superintendent J. B. Buxton, NC New School Project President Tony Habit, Chair of the NC Blue Ribbon Commission on Testing and Accountability and President and CEO of the NC Center for Science, Mathematics, and Technology Education Sam Houston, senior scholar at the Friday Institute for Educational Innovation at NC State University Jere Confrey, and others. On November 9, 2007, nearly all of these individuals and others from their organizations met together at the Friday Institute with representatives from the UNC Math and Science Education Network, the NC School of Science and Mathematics, and NCCTM to explore together how we can collaborate to accelerate innovation in the teaching and learning of mathematics in our state. Dates are established for this coalition to meet again in January and February, 2008.

A draft of the recommendations from the NC Blue Ribbon Commission on Testing and Accountability to the State Board of Education published in recent weeks indicates that the State Board is likely to be considering significant changes in current high stakes testing practices. The draft indicated that the Blue Ribbon Commission will be recommending five rather than ten End-of Course Tests in NC high schools and just one in mathematics. Several other tests have been recommended for elimination including tests in writing, science, and computer skills. In the words of Commission Chair Sam Houston, "We're testing more but we're not seeing the results. We're not seeing graduation rates increasing. We're not seeing remediation rates decreasing. Somewhere along the way testing isn't aligning with excellence."

These are exciting times for mathematics educators in North Carolina. This is the first time in my memory that such a diverse and influential group of key individuals and organizations in our state have come together to address the systemic problems of mathematics education. I am hopeful that our coalition of partners can provide leadership on decisions that result in higher achievement for all our students. I hope to see new opportunities for interested teachers and schools to become part of this coalition and become involved in the implementation of innovative curricula and in stimulating professional development initiatives. NCCTM seeks to be actively engaged in promoting these initiatives through upcoming regional and state conferences, through the Centroid, and the NCCTM website. Plan to join us and stay in touch.

## Eastern Region President Rose Sinicrope

SINICROPER@ecu.edu
Greetings from the East!
We are excited about the regional conference at Meredith College in Raleigh on Saturday, February 23, 2008. Elizabeth Murray with Ray Jernigan's help is chairing the program committee. Tim Hendrix is our site coordinator. Kitty Rutherford, Julie Cazin, Katie Stein, Holt Wilson, and Alan Faulkner are working hard to ensure a very exciting and productive conference. We hope that you will join us at the capitol on the morning of February 23 to enjoy sessions and the opportunity to network with teachers of mathematics in the East! We will meet from 9:00 a.m. until about 1:00 p.m. The committee is exploring happenings for the afternoon to share with those of us who will want to enjoy an afternoon in Raleigh.

Congratulations to North Carolina State University! NCSU has a new student chapter of NCCTM. The East now has two student chapters-NCSU and ECU.

The East is getting ready for Math Fairs and Math Contests. We are anticipating new curricula and new curriculum materials. There is so much talent and commitment from teachers of mathematics in the East. We are thankful!

## Central Region President

## Rebecca Caison

rbcaison@mebtel.net
The school year is flying by! I hope everyone had a restful and enjoyable holiday season. I wish to thank all of the fantastic teachers from the Central Region who were session and workshop presenters at the state conference in Greensboro in the fall. Your help in making the conference a success was appreciated by all.
"Go Wild with Math at the NC Zoo" is the theme for our spring conference which will be held on Saturday, March $8^{\text {th }}$, at Asheboro High Zoo School. The Zoo School is located just outside the North American Entrance to the NC Zoo. Ann Crawford has agreed to serve as conference chairperson and is working with the educational staff at the zoo and other teachers to provide us with an exciting program. The conference also will include updates at each grade band from the Department of Public Instruction.

The Central Region math fair will be held at Asheboro High School on March 15th. Thanks to Wendy Rich for always doing an outstanding job as chairperson of the Central Region Math Fair.

I would encourage everyone to check out the NCCTM website. It gives lots of information about the activities of the organization and provides you with a link to many of the benefits of the organization. I would encourage you to look at the opportunities for members such as the mini-grants, scholarships to assist with obtaining a graduate degree, the math fair, and the logo contest as well as information on the spring conference.

I hope the remainder of the school year will be productive for you as we continue to work toward the best math instruction for all of our students. If you have any questions or concerns in the Central Region please contact me.

## Western Region President Debbie Crocker

crockerda@appstate.edu
It is hard to believe that it is already 2008! I hope the New Year is off to a great start for all of you. Mark the dates below on your calendar and participate in the NCCTM activities in the Western Region!
The Western Region Conference will be on Saturday, February 23, 2008 at Jacobs Fork Middle School. We want to thank Jacobs Fork Middle School for agreeing to host this again. It is not too late to get on the program. Contact me via email if you would like to volunteer to speak to either teachers or pre-service teachers at the Western Region Conference this year.

The Western Region Math Fair will be held at Plemmons Student Union on the campus of Appalachian State University on Saturday, April 12, 2008. Get your students started on their projects! Volunteer to judge, help in the student holding rooms, do registration, or be a runner! Look for more information at ncctm.org or contact Cindy Robinson [crobinson@caa.k12.nc.us](mailto:crobinson@caa.k12.nc.us). This spring will be busy in the Western Region of NCCTM! We need all of you to get involved in the activities and participate in NCCTM. You can contact me with questions, concerns, or ideas. I hope to hear from you!

## What is Important in Early Childhood Mathematics? NCTM's Position

The National Council of Teachers of Mathematics affirms that a high-quality, challenging, and accessible mathematics education provides early childhood learners with a vital foundation for future understanding of mathematics. Young children in every setting should experience effective, research-based curricula and teaching practices. Such practices in turn require policies, organizational support, and resources that enable teachers to do this challenging and important work.
[http://www.nctm.org](http://www.nctm.org)

## History Corner

## Completing the Square Historically

William C. Bauldry
Appalachian State University
Boone, North Carolina

## Introduction

Long, long ago, in a school on a mountain far, far away, Mr. Ferguson, my algebra teacher, directed us to "solve the quadratic equation by completing the square." I recall thinking at the time that the square $x^{2}$ looked rather complete already. What more could we do and still have a square? And so I began my quest to be the square who was completed.

As math teachers, we are all familiar with the concept and are used to presenting "completing the square" as an algorithm for bringing quadratics into the useful form $y=a(x-h)^{2}+k$. Adding a bit of history and geometry to the lesson can make it stand out more clearly for our students and show them the origin of the technique. Our students can then make deeper connections between the algebra and geometry. Let's begin with some early mathematics and work our way up to a famous completed square.

## The Egyptians \& Babylonians

The Egyptian mathematicians of about 1500 BC were scribes who presented and taught through specific examples. The scribes had no notation for general cases. The texts that we have, mainly the Rhynd Papyrus and the Moscow Papyrus ${ }^{l}$, show that the scribes only considered practical calculations. The Egyptians had sophisticated examples for areas and volumes, but lacked a method to generalize and didn't appear to study mathematics for its own sake. From our extremely limited sources, we believe the scribes could solve linear equations, but not quadratics.

The Babylonian mathematicians of the same era were much like their Egyptian counterparts.
Mathematics was done by scribes as a necessary part of business and government; their study focused on practical uses. The Egyptian and Babylonian scribes did not have variables and equations or even negative numbers or zero; they taught with descriptions of processes by giving examples. Unlike the Egyptians, the Babylonians did not stop with completely practical problems, but invented esoteric exercises as challenges for students. One well-known, rather intricate Babylonian problem is:

A trapezoidal field. I cut off a reed and used it as a measuring reed. While it was unbroken, I went 1 three-score steps along the length. It's sixth part broke off for me. I let follow 72 steps on the length. Again $1 / 3$ of the reed and $1 / 3$ cubit broke off for me; in 3 three-score steps I went through the upper width. I extended the reed with that which [in the second instance] broke off for me, and I made the lower width in 36 steps. The surface is 1 bur. What is the original length of the reed? (Berlinghoff \& Govêa, 2002)
The Babylonians were able to solve problems that we would use quadratic equations to answer. The method the scribes used is equivalent to what we now call completing the square. Their method had a geometric basis that justified the name.

## The Algebra of Al-Khwarizmi

About 830 AD, the Arab scholar Abu Ja'far Muhammad ibn Musa al-Khwarizmi wrote the text Al-kitab almuhtasar fi hisab al-jabr w'al-muqabala (The Condensed Book of Calculation by Rejoining and Balancing). AlKhwarizmi was a member of the Bayt al-Hikma or "House of Wisdom" in Baghdad, an academy that today we would call a research institute (O'Connor \& Robertson, n.d.). Al-Khwarizmi's book titles gave us the terms algebra, from al-jabr, and algorithm, from Algoritmi de numero Indorum, which is Latin for "Al-Khwarizmi on the Hindu Art of Reckoning" (the original Arabic title isn't known) ${ }^{2}$. Although credited with the first text on

[^0]algebra, al-Khwarizmi did not use variables, negative values, or zero coefficients; his formulas were stated in prose. Thus, he found it necessary to break the general monic quadratic into six cases devoting a chapter to each:

1. Squares equal to roots
2. Squares equal to numbers
3. Roots equal to numbers
4. Squares and roots equal to numbers
5. Squares and numbers equal to roots
6. Roots and numbers equal to squares

In modern times we use $x^{2}$ for al-Khwarizmi's "squares" and $x$ for his "root." Examples of equations matching the six cases are:

1. $x^{2}=10 x$
2. $x^{2}=39$
3. $x=39$ (the coefficient of $x^{2}$ is 0 )
4. $x^{2}+10 x=39$ (now a famous problem treated below)
5. $x^{2}+39=10 x$
6. $10 x+39=x^{2}$

Today we see these all as special cases of $a x^{2}+b x+c=0$. An exercise for students is to show that all six cases are needed in order to avoid negatives and zeroes as coefficients and numbers.

In texts as recent as Loomis's popular Treatise on Algebra (Loomis, 1873), quadratic equations were still broken into the cases "Incomplete Equations," $x^{2}=q$, and "Complete Equations," $x^{2}+p x=q$, the latter solved only via completing the square.

## The First Completed Square

Possibly the most famous example problem used to introduce the technique of completing the square was posed by al-Khwarizmi in his preface (al-Khwarizmi, 830). Specifically, "one square, and ten roots of the same, amount to thirty-nine dirhems"; that is to say, what must be the square which, when increased by ten of its own roots, amounts to thirty-nine? Since the square is $x^{2}$ and 10 roots is $10 x$, we get the modern equation $x^{2}+10 x=39$.

This problem was used by nearly every algebra author up through the 16 th century. "This example runs like a thread of gold through the algebras for several centuries, appearing in the algebras of Abu Kamil, AlKarkhi, and Omar al-Kayyami, and frequently in the works of Christian writers," according to Karpinski in the introduction to his text Robert of Chester's Latin Translation of the Algebra of Khowarizmi (Karpinski, 1925). To whet our interest, al-Khwarizmi describes the method that answers this problem that he posed.

The solution is this: You halve the number of the roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this, which is eight, and subtract from it half the number of the roots, which is five; the remainder is three. This is the root of the square which you sought for; the square itself is nine.

After discussing all six cases, al-Khwarizmi turned to geometry to justify his procedure. We will look at a simplified version of his argument for the famous problem next.

## The Geometrically Complete Square

Consider al-Khwarizmi's example quadratic equation $x^{2}+10 x=39$. Draw a square of side $x$ having area $x^{2}$ :


[^1]Add a rectangle of length $x$ by 10 with area $10 x$ (we don't know the size of $x$ so it is important to remember that this picture has no scale):


This rectangle has area 39 according to our equation. Split the added rectangle in two:


Move one of the halves to the bottom of the square:


Add a small $5 \times 5$ square to the lower right corner to "complete" the square.


The last figure shows that we now have a square of side, which is has area $39+25=64$, and hence, $x+5=8$ or $x=3$. Al-Khwarizmi's root is 3 and the square is 9 . The problem is solved.

By the time of Loomis's Treatise on Algebra in 1873, the standard exposition had become completely symbolic, or, as we would say, completely algebraic. Loomis first gave a terse derivation to show

$$
x=-\frac{p}{2} \pm \sqrt{q+\frac{p^{2}}{4}} \text { solves } x^{2}+p x=q
$$

He then went on to describe his calculation in the form of an algorithm (Loomis, 1873).
$1^{\text {st }} \quad$ Reduce the given equation to the form of $x^{2}+p x=q$.
$2^{\text {nd }}$ Add to each member of the equation the square of half the coefficient of the first power of $x$ :

$$
x^{2}+p x+\left(\frac{p}{2}\right)^{2}=q+\left(\frac{p}{2}\right)^{2}
$$

$3^{\text {rd }}$ Extract the square root of both members, and the equation will be reduced to one of the first degree $x+\left(\frac{p}{2}\right)= \pm \sqrt{q+\frac{p^{2}}{4}}$, which may be solved in the usual manner.

## Conclusion

The technique of completing the square has a long and interesting history that can be used to advantage in teaching algebra. The early Babylonians could solve specific problems that we would write as quadratic equations by using geometric reasoning. However, they lacked the language of algebra for generalization. The Arab scholar al-Khwarizmi wrote of completing the square in his text on algebra, the first book on the subject. Al-Khwarizmi was able to generalize, but still wrote equations in words without using zero or negative numbers. It's intriguing that he avoided zero, since he also wrote a book on Hindu numeration and the use of zero in
decimal notation. Modern students often miss the connection between the algebraic procedure for completing the square and the underlying geometry. We can enhance our students' learning by helping them to discover this elegant connection between algebra and geometry.

## References

Al-Khwarizmi, M. (830). The author's preface. In The algebra of Mohammed Ben Musa. Rosen, F. (Ed. and Trans.). (1831). London: Oriental Translation Fund. (Also available from the Internet Archive: http://www.archive.org/details/algebraofmohamme00khuwrich)<br>Berlinghoff, W. \& Govêa, F. (2002). Math through the ages. Farmington, ME: Oxton House Publishing.<br>Karpinski, L. (1915). Robert of Chester's Latin translation of the algebra of Khowarizmi. New York: Macmillan.<br>O’Connor, J. \& Robertson, E. (n.d.). Abu Ja'far Muhammad ibn Musa Al-Khwarizmi. In The MacTutor History of Mathematics archive. Retrieved January 1, 2008, from http://www-groups.dcs.standrews.ac.uk/ history/Biographies/Al-Khwarizmi.html<br>Struik, D., Ed. (1969). A sourcebook in mathematics, 1200-1800. Princeton, NJ: Princeton Univ. Press.

## Mini-grants

The North Carolina Council of Teachers of Mathematics through its mini-grant program, provides incentive funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research, which will enhance the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. Available to current members of NCCTM, the mini-grants are awarded by each of the three regional organizations to members within their geographic boundaries (If you incorrectly identify with the wrong region, your proposal will be ineligible for funding). A total of $\$ 15,000$ is available each year for the state's mini-grants, with each region awarding approximately $\$ 5000$ in grants to its members. In recent years, approximately $30-35$ proposals have been partially or fully funded, for an average grant of just less than $\$ 800$.

Grant proposals must be postmarked by September 15 (or emailed on that date), and proposals selected for funding will receive their funds as soon as possible after the state conference. You will receive an email confirmation of receipt once your proposal has been received. If you do not receive a confirmation within one week, it is your responsibility to follow-up with the Mini-grant Coordinator.

## Directions

The directions and application are available on the NCCTM website [http://www.ncctm.org](http://www.ncctm.org). Please read all directions carefully and fill out application and cover sheet completely, as directed. Failure to correctly list the NCCTM region and membership number will cause your application to not be considered. Be sure that your NCCTM membership is current and active for the 2007-2008 school year! Be sure that it is a NCCTM membership and not NCTM or some other organization. Each year we have applications that cannot be considered because of the membership requirement.

## Are you wondering what you could ask for from the mini-grant program?

Possible projects for consideration include: Math clubs, field days, contests, math activities and laboratories, research projects/topics, family math, and parent workshops, etc. This list is not meant to limit you to these ideas. Creativity is encouraged!

For a specific example of a funded project, and the impact it had on both middle grades and high school students, see the mini-grant report on the Brunswick County Early College High School River Project on the next page.

## Mini-grant Report

The River Project

Rebecca S. Willis<br>Brunswick County Early College High School Bolivia, North Carolina

Brunswick County Early College is a new school that served about 80 ninth graders in the 2006-2007 school year. With the funds we received from the NCCTM mini-grant we were able to purchase Vernier probeware that allowed the Earth and Environment teacher to incorporate the use of the graphing calculator in the science classroom. One way she was able to do this was through the River Project. The River Project was a lesson in water pollution that connected the students to their community. In this project, the students worked in groups to study different aspects of water quality and the types of pollution including the effects of this pollution on animals, wastewater, and groundwater.

The students took two field trips to the aquaculture department at Brunswick Community College, where they learned how to use various tests to measure for nitrate, phosphate, dissolved oxygen, pH , hardness, and chloride. The students took water samples from the creek on campus and used the Vernier probeware to measure the pH and temperature. Since macroinvertebrates and vegetation play a large role in determining water quality, samples were taken to the lab for further testing and for examination under a microscope. The students were amazed by their findings but were pleased to surmise that the water seemed to be of fair to good quality.

The final step to completion of the River Project was sharing our new-found knowledge with fifth graders from Supply Elementary School. We invited them to our school and shared our findings, and then we gave them an opportunity for hands-on sampling. We had water samples available for them to test and to examine under the microscopes. With the aid of our students they were able to see the organisms found in the water and understand their effect on water quality. Prior to their leaving, the fifth graders watched a movie created by one of our students on pollution with the purpose of educating people on what not to do to insure the preservation of our water quality.

This project offered a wonderful opportunity to educate our students and the fifth graders from Supply on the effects of pollution on water quality. The probeware, purchased with the NCCTM mini-grant, made this project possible.


Using the Vernier Probeware, students measure to see what the temperature and pH of the creek is.


Back at the aquaculture lab, students perform various water quality tests on the water samples


A high school student, shows elementary students what organisms are in the pond water


High school students show the elementary students how to use the Vernier probeware to measure the temperature and pH of the water.

## Middle Grades Activity: Game Day

## Submitted by Nancy Ruppert, UNC-Asheville, Asheville, North Carolina

When I was teaching middle school I sometimes used Fridays as "Game Day." What it does for kids is allows them to move around, interact with their peers, and engage in cooperative learning. Those who are stronger in mathematical problem solving can help those who need reinforcement. This approach also allows students to practice the process skills of reasoning, problem solving, and communication.

## Set-up

First, put the students into teams. Mix up the abilities of the students if you can. Provide them with the rules to the game: Each round consists between one and five questions, and each round focuses on a different way of doing mathematics. The teams work together to agree upon one answer. When all teams have an answer, each team shows their answer and points are given for accuracy (not speed - until the speed round). I also share that sportsmanship is very important and if poor sportsmanship is displayed or if people are not participating their groups can lose a point at a time.

## The Rounds

Round 1 - Vocabulary: Either provide a word and ask the teams to write the symbol or write down a symbol and ask them for the word such as "in the fraction $3 / 4$ what is the 3 called?" Use five words or symbols. Each group can get up to 5 points - 1 point each.

Round 2 - Computation: Place a problem on the board such as $1 / 2-1 / 3$. Have the students work the problem on their paper, discuss it and come up with one answer. When all teams have an answer, have them show it and give them points. Then I suggest one of your students show how they got the answer. Give a total of 5 problems like this, and again up to 5 points.

Round 3 - Word Problem: Such as: Ervin was working the fields and had $3 / 4$ of a bushel of corn picked. His little brother Spike came over and took $1 / 8$ of the bushel of corn from his bushel. How much of the bushel of corn was left? When all teams have an answer, have them show it and give them points.

Round 4 - Speed Test: This is a fun one; students are given a large piece of paper that has a table with rows and columns. Together they are to figure out what goes where. They have 2 minutes to take this test. In the table below, I have put + in the upper left indicating they should add, but it could be comparative (row $<.>$ or $=$ column, or $\times$ or $\div$ ). If you put it on a $11 \times 17$ paper, they can all work problems at once and put answers in. You then call time and take up the papers. For each cell that is correct they get a point.

| + | $1 / 3$ | $1 / 4$ | $2 / 5$ | $3 / 8$ |
| :---: | :---: | :---: | :---: | :---: |
| $4 / 3$ |  |  |  |  |
| $3 / 8$ |  |  |  |  |
| $2 / 5$ |  |  |  |  |
| $1 / 4$ |  |  |  |  |
| $1 / 2$ |  |  |  |  |
| Round 4 Table |  |  |  |  |

Round 5 - Final Round: This should be a thought provoking or challenging problem. You may want to use problems from Math Counts [http://www.mathcounts.org/](http://www.mathcounts.org/) or other enrichment problems. Here is one that I like:

Using the digits 1-9, arrange the numbers so that each of the rows, columns and the two main diagonals add to 15 . You may only use each number once and you must use all 9 digits.

This round can count up to 25 points or whatever it takes to ensure that even the last group could win if they get this one and the other groups don't get it.

Add the scores up and choose a winner. Take their picture as a group and put them on the Wall of Champions of Game Day. You don't have to give them candy or prizes. Reward their expertise and their sportsmanship.


Grid for Round 5 Challenge Problem

## College Algebra / Algebra II Activity: Complex Number Puzzle Submitted by: Solomon Willis, Cleveland Community College, Shelby, North Carolina

During my nine years of teaching, I have been a regular user of the Algebra with Pizzazz series (Marcy, 1983). Whether I am teaching Algebra I to $7^{\text {th }}$ graders or basic algebra to adults, all students seem to really enjoy using the worksheets to help build their math skills. They often have fun figuring out the answer to some corny riddle. I now teach Intermediate Algebra to college students on a regular basis, and for five years I never could find a good worksheet to use with complex numbers. I finally decided to write my own, and my students know that I am a big Dolly Parton fan so I decided to incorporate one of Dolly's songs into the activity. My developmental math students had a blast with it, and I am sure that I will use it for years to come. I will have to admit that writing such a worksheet was much harder than I ever expected. I had to edit the worksheet three times before it was perfect. I hope that it might be something other teachers can use in their classroom for a quick activity to review complex number concepts.

## Complex Numbers \& Dolly

Simplify or evaluate each expression below. Find your answer at the bottom of the page and write the letter of that exercise in the blank above it. If an answer appears more than once then write the same letter for each answer. Once you are finished you will have the name of one of Dolly Parton's many songs. This particular song is one she wrote about a childhood memory.
A. $(5-i)^{2}$
C. $(5-\sqrt{-25})-(11-\sqrt{-36})$
F. $\sqrt{16}+\sqrt{-25}$
L. $(10-\sqrt{-32})+(9-\sqrt{-18})$
M. $(-3+4 i)(6-8 i)$
N. $(3-6 i)-(7+3 i)$
O. $(-5+4 i)-(6-2 i)$
R. $\sqrt{-300}$
S. $(1-8 i)(7+6 i)$
T. $\sqrt{-180}$
Y. $4 i(8-3 i)$

| $\overline{-6+i}$ | $\overline{-11+6 i}$ | $\overline{24-10} i$ | $\overline{6 i \sqrt{5}}$ | $\overline{-11+6 i}$ | $\overline{4+5 i}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\overline{14+48} i$ | $\overline{24-10} i$ | $\overline{-4-9 i}$ | $\overline{12+32} i$ |  |  |
| $\overline{-6+i}$ | $\overline{-11+6} i$ | $\overline{19-7 i \sqrt{2}}$ | $\overline{-11+6} i$ | $\overline{10 i \sqrt{3}}$ | $\overline{55-50} i$ |

## Reference

Marcy, S. (1983). Algebra with pizzazz! Practice exercises for first year algebra. New York: Wright Group/ McGraw-Hill.

# Discovering the Area of a Regular Polygon 

Kimberly Smith Burton<br>Fayetteville State University<br>Fayetteville, North Carolina

Kimberly Young<br>Gray's Creek High School<br>Hope Mills, North Carolina

Stephanie Duncan<br>West Caldwell High School<br>Lenoir, North Carolina

The purpose of this article is to present an activity that uses the precepts of discovery learning to facilitate high school geometry students' understanding of the area formula for regular polygons. The lesson was facilitated by particular teaching methods and guided learning principles that incorporated problem solving strategies useful in finding the area of a regular polygon. For the purpose of this article, the term discovery learning will refer to the comprehension of the individual student, teaching and instructional strategies designed by the teachers and the environmental setting when such strategies are employed.

## Discovery Learning

Jerome Bruner insists that meaningful learning in mathematics requires the student to search actively for solutions. Discovery learning is "an approach to instruction through which students interact with their environment-by exploring and manipulating objects, wrestling with questions and controversies, or performing experiments" (Ormrod, 1995, p. 442). A major theme in Bruner's framework is that learning is an active process in which the learner relies on past experience and prior knowledge to construct new ideas and concepts (Bruner, 1967).

It is the responsibility of the teacher to encourage students to discover principles by themselves. The teacher and student should engage in active discourse. The task of the teacher is to prepare activities and lessons that will guide the student through an exploration of a mathematical concept in a format that is appropriate to the student's current knowledge level. The activity should be organized in a spiral manner so that the student can continually build upon a prior skill set. As teachers with 20 years combined experience at the high school level, we have found that discovery learning has proved far more beneficial for our students than learning based on memorization and conditioning. Discovery learning allows students to gain a greater understanding of the objectives being explored than memorization and conditioning.

Bruner (1966) states that a theory of instruction should address four major aspects: (1) predisposition towards learning, (2) the ways in which a body of knowledge can be structured so that it can be most readily grasped by the student, (3) the most effective sequences in which to present material, and (4) the nature and pacing of rewards and punishments. Good methods of structuring knowledge should result in simplifying, generating new propositions and increasing the manipulation of information.

One of the objectives in the current geometry curriculum is to teach students the process for finding the area of a regular polygon (NCDPI, 2003). To a mathematics teacher, this objective seems relatively simple; however, in the eyes of many students it is usually a difficult concept to master. Many students soon develop a heightened level of frustration when they realize that they must perform nearly ten steps to find the area of a regular polygon. In an effort to simplify this complex concept for our geometry students, we, as high school geometry teachers, developed a discovery activity to guide our students through the exploration of finding the area of regular polygons.

## Objective

The goal of this discovery activity was for students in a high school geometry class to investigate various ways of finding the area of regular polygons prior to knowing the formula Area $=\frac{1}{2} a \cdot P$, where $a$ is the apothem of the regular polygon and $P$ is the perimeter of the figure. It was our hope that through individual reflection, students would be able to develop a rule or a pattern for finding the area of all regular polygons. This hands-on activity proved to be the perfect motivation for students to collaborate in their efforts to reach a common mathematical goal.

## Description of the Activity

The activity that was designed for the students consisted of three parts. In section one (see Figure 1), students use a ruler to calculate the area of a triangle, square, pentagon, and hexagon. In all the regular polygons except
the triangle, students must find the area twice. The first time, in part a, they found the area by separating the polygon into smaller familiar figures, of which they could successfully determine the area. For example, a pentagon can be easily divided into an isosceles trapezoid and a triangle. In solving for the area in part $b$, students were required to use only triangles for separating the polygons. This portion of the activity was completed at home.

## Exploring the Area of Regular Polygons

I. use a ruler and calculator to fine the area of each figure. All segments should be measured in centimeters. Follow the directions for each part.
Show all of your work.

1. A regular triangle
a. Find the area using your own method and describe your work.

2. A regular quadrilateral (square)
a. Find the area using your own method and describe your work.

b. Find the area by separating into smaller triangles of your choice. Draw the triangles in the figure below and describe your work.

3. A regular pentagon
a. Find the area using your own method and describe your work.

b. find the area by separating the figure into smaller triangles of your choice. Draw the triangles in the figure below and describe your work.

4. A regular hexagon
a. Find the area using your own method and describe your work.

b. Find the area by separating the figure into smaller triangles of your choice. Draw the triangles in the figure below and describe your work.


Figure 1: Section I of the discovery lesson
In section two of the discovery lesson (see Figure 2), students were strategically placed in small groups during a 45 -minute class period and again asked to find the area of regular polygons; a triangle, square, pentagon, hexagon, and decagon. In order to complete this section, students were not allowed to use rulers to determine the necessary lengths for computing area. The directions provided information stating that the length of each side was 2 inches long. A second requirement of section two was to create congruent triangles in each figure. Using the center as a point of reference, the students were to draw $n$ congruent triangles in a polygon with $n$ sides. Thus, in a regular triangle three auxiliary lines were drawn to form three smaller and congruent triangles. In order to determine the area of the regular triangle, students were required to find the area of one of the smaller triangles and multiply by the number of triangles created.

## Exploring the Area of Regular Polygons

II. Using a calculator and your knowledge of right triangles, find the area of each figure. Do not use a ruler. One side of each polygon measures 2 inches. In this section, you must divide each polygon into smaller triangles to solve the problem. Use the center of each polygon as a point of reference for drawing the triangles. For a polygon with $n$ sides, you must draw enough auxiliary lines to form $n$ congruent triangles.

1. Find the area of a regular triangle.
*Remember you must use 3 equal triangles to solve the problem in section II.

2. Find the area of a regular quadrilateral.
*Remember you must use 3 equal triangles to solve the problem in section II.

3. Find the area of a regular pentagon.

4. Find the area of a regular hexagon.

5. Find the area of a regular decagon. (no diagram provided)

Figure 2: Section II of the discovery lesson

## Student Work for Section 2

Students created $n$ smaller triangles in a regular $n$-gon. As previously stated, they calculated the area of one of the smaller triangles and multiplied their answer by the number of sides. In order to find the area of one of the smaller triangles, students were asked to solve for the height of the individual triangle. In their discussions, students discovered the necessity of finding angle measurements. Groups chose to either find the measure of the central angle or the interior angle of the polygon to use as their reference angle. Using this angle measure and their knowledge of the special right triangle rules and the trigonometric functions, students computed the base and the height of the smaller triangle (see Figure 3). At this point, students were able to find the area of the smaller triangle, multiply by the number of sides and thus find the area of the whole regular polygon.


Figure 3: Examples of student work

## Student Reflection

Section three of the activity was a reflection through a writing assignment. Students were to answer certain questions that required them to explain why or why not each person would have arrived at the same answer for section one (measuring and finding area). Most students recognized that due to measuring errors there might be various answers among their peers. However, they affirmed that all answers should be relatively close regardless of human error or differences in separating the figures. Second, students were asked if all groups would have the same answers for section two (dividing into $n$ congruent triangles) and if the method of computing would be exactly the same. The students agreed that the only differences in their method would be the use of trigonometric functions versus special right triangle rules. Students discovered that the answers to the area questions should be the same. The final two questions asked students to review their work and think about the process for finding the area. They were asked to examine their calculations to look for patterns among their work. Students were then expected to develop a rule or formula that may work to find the area for all regular polygons. While approximately $70 \%$ of the students were unable to discover the official formula for finding the area of a regular polygon, some were, in fact, able to create a valid rule (see Figure 4).


Figure 4: Section III of the discovery lesson and student response

## Teacher Reflection

This hands-on activity was a great teaching tool for our students. Overall they expressed the fact that they enjoyed the discovery activity. It was rewarding to see the students working to incorporate many of the concepts that we have used throughout the year into one new concept. As the activity was facilitated in our individual high school geometry classrooms, we were surprised by what we heard. Students were using mathematical language in communication about the activity with each other. This mathematical communication should be a continued goal of all mathematics teachers. Within a group, one student would formulate a hypothesis about how to find the area in section two of the activity. Others in their group would try the suggestion, and revise it until they were able to agree upon the best, most accurate method of solving for area. It was our hope that, as high school geometry teachers, we had created an opportunity for educational exploration within our classrooms and that through this activity; students were able to better understand the meaning behind the pieces of a regular polygon identified to find the area. For example, although at the time the activity was presented, students were not provided with the proper terms, such as apothem, it was easily recognized when the formula was revealed in the follow-up lesson. While the activity proved successful in our geometry classrooms, modifications may be
needed for students at various levels of geometry, whether standard level or honors level. For example, in section two of the activity it may be necessary for the teacher of students at a lower level to demonstrate the process for drawing $n$ congruent triangles in a polygon with $n$ sides. The students may also need a brief reminder of how to use special right triangle formulas and trigonometric functions as they work to find the area. The activity proved to be a successful learning opportunity for both teachers and students. We will continue to use this discovery lesson in the future.

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## Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in the State. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

If you have nominated someone in the past who has not received the award to date, or if you would like to nominate someone new, please submit as much of the following information as possible! Nominations are accepted at any time.

Please submit the following information. Use as many typewritten pages as needed. If possible, attach a vita of the nominee.

- Name of the nominee
- Current position
- Your relationship to the nominee (e.g. principal, co-worker, etc.)
- The nominee's contributions to mathematics education, NCTM, NCCTM, etc. (Please include information on specific offices held and honors received by the nominee.)
- Any information about contributions to the community, teaching, and education that would be of value to the Rankin Award Committee in its deliberations
- Other relevant information
- Letters of endorsement from other colleagues may be included.
- Date of nomination

Nominator* Name
Current position; Business or educational institution
Preferred mailing address; Preferred telephone number
*The Rankin Award Committee reserves the right to use portions of nomination information in the presentation of the award if the candidate is selected.

Send to: Ms. Jan Wessell
23 Shore Drive
Wrightsville Beach, NC 28480

## Awards

NCCTM Rankin Award Recipients<br>Benjamin G. Klein, Barbara M. McGill, and William E. "Bill" Scott<br>Reported by Jan Wessell<br>Wrightsville Beach, North Carolina<br>(Photos by Janice Richardson-Plumblee)

Dr. Benjamin G. Klein is in his $40^{\text {th }}$ year of service as a teacher of mathematics, the last 36 of which have been in the state of North Carolina. His contributions are primarily as an outstanding teacher of mathematics at Davidson College. As a teacher Dr. Klein has twice received outstanding teaching awards from Davidson. He has been honored as the North Carolina Professor of the Year by the Council for Advancement and Support of Education. He has also received the Distinguished Teaching Award from the Southeastern Section of the MAA.

In addition to teaching, Dr. Klein has a distinguished body of


Dr. Klein (right) with NCCTM President Randy Harter scholarly work in both mathematics and in mathematics education. This work includes authoring numerous journal articles, editing and reviewing professional manuscripts, and making presentations at professional conferences. He has been a leader at Davidson College by serving as a Mathematics Department Chair for a number of years and also as Vice President for Academic Affairs for a brief period of time. He has served as chair of the institution's Teacher Education Committee for nearly 30 years and also was named Chair of the Department of Education for one year.

During the first decade of service to our profession in this state, Dr. Klein's expertise as an outstanding mathematician and mathematics educator was demonstrated by a continuing involvement with the Advanced Placement Program of the College Entrance Examination Board. He continues to serve the College Board, and over the years this service has included such roles as reader, table leader, question leader, test author, question author, and consultant. During the 1980's the North Carolina Department of Public Instruction called upon him to help establish college or university teacher education requirements for prospective teachers of both secondary and middle school mathematics. At the same time DPI asked him to chair the Secondary Mathematics Curriculum Committee. During the 1990's he was asked once again to serve on this committee.

Dr. Klein continues to be involved with NCCTM, including service on the NCCTM Board of Directors and as Regional Vice President for Colleges. He is a frequent and valued speaker at annual regional and state meetings of our organization. Also, the NCCTM Mathematics Contest Committee has asked him to serve as author or co-author of the Comprehensive State Mathematics Contest for nearly half of the 29 contests that have been held. His service has won accolades from the entire contest committee.

Related service to mathematics education includes speaking to school mathematics clubs, judging school mathematics fairs, making presentations at meetings of the National Council of Teachers of Mathematics, the National Council of Supervisors of Mathematics, and the North Carolina Association for Advanced Placement Mathematics Teachers. His involvement at the highest levels of the Mathematics Association of America (as Governor, Chair of the Southeastern Section, and Executive Board Member) has consistently been marked by efforts to strengthen connections between the MAA organization and high school mathematics teachers.

The file of documents provided to our committee supporting Dr. Klein's candidacy for this award is marked by an impressive collection of letters of support from a variety of sources. These included a high school teacher, colleagues and administrators from his own college, and from colleagues located at other universities. Dr. Klein is characterized as a most humble person, an outstanding mathematician and mathematics teacher, and a sincere and dedicated human being.

The second recipient of the W.W Rankin Award for Excellence in Mathematics Education, Barbara M. McGill, has taught mathematics to students at the middle school, high school, and college levels for over 30 years. Her contributions to mathematics and mathematics education include: Mathematics Department Chair, workshop presenter for high school and middle school math teachers, and leader in the NC Council of Teachers of Mathematics at both the regional and state levels

Ms. McGill has served NCCTM as the state Conference Co-


Ms. McGill with President Harter Chair and State Conference Chair, regional Vice President and State Vice President. She is also a member of the National Council of Teachers of Mathematics, NC Association of Educators, and Phi Delta Kappa. She is currently on the Advisory Board for the Math Science Education Network. In addition to her teaching, she has served as a curriculum facilitator, mentor, and team leader at the schools where she has served. Her positive relationships with students, parents, and faculty are the keys to her success. One colleague comments: I have found her to be very hard-working and conscientiousboth as a classroom teacher and as an officer in NCCTM. She is always willing to go the extra mile. If there was a need, I knew I could ask her.

Regarding Ms. McGill's teaching, it is noted that she is always willing to teach challenging students who had been "turned off" to learning mathematics. She is able to get down on their level and convince them that they can and will succeed in her class. She was that type of teacher whom others turned to and the students loved. Ms. McGill was recognized by student vote as the outstanding teacher in the Greensboro Area Math and Science Education Center. It is also no surprise that she was also the first recipient of a Mathematics Excellence Teaching Award at the middle school level in 1991. In the words of another colleague: "She is one of the finest teachers and role models I have had the pleasure to work with."

The final recipient of the W.W. Rankin Award for Excellence in Mathematics Education, William E. "Bill" Scott, has been a mathematics educator for over 30 years. In this time, he has taught at the middle school, the high school, and the community college levels and has provided in-service and professional development training to teachers all across the State of North Carolina.

Mr. Scott's contributions to mathematics and mathematics education include service as a K - 12 resource teacher, a curriculum developer of standards-based content for mathematics Grades 4-8 and Algebra 2, co-author of algebra materials and resources for the State of North Carolina, program evaluator, NCCTM conference speaker, North Carolina Algebra Project presenter, and TI calculator workshop leader.

He has served the North Carolina Council of Teachers of Mathematics and the National Council of Teachers of Mathematics with distinction. The list of service activities consists of: membership on the NCCTM Board of Directors, Publicity Chair for the 1999 NCTM Southern Regional Conference, Publicity Chair for the 2002 Carolinas' Conference, and Program Chair for the 2003 and 2006 NCCTM State Mathematics Conferences.

A recent president of the North Carolina Council of Teachers Mathematics noted that his "service to the organization has been greatest in the difficult roles..." for which Mr. Scott has stepped up and undertaken. In its time of need, Mr. Scott has always


Mr. Scott (left) with President Harter been there for NCCTM. Another prominent leader remarked that our honoree "has been a godsend to NCCTM," describing him as "...the ultimate program chair," and the "go-to" person when answers are needed.

Having served as the Webmaster for the Association of State Supervisors of Mathematics and the "unpaid but busy webmaster for NCCTM," Mr. Scott is known for the "countless hours spent: (a) working the kinks out of each part of the NCCTM website's administrative system, (b) uploading and modifying text, (c) fine-tuning data sets, and (d) writing/rewriting, editing, and monitoring information necessary for the website to run smoothly."

Not surprisingly, Mr. Scott is no stranger to excellence...having been named the Rutherfordton-Spindale Jaycees Outstanding Young Educator, a regional nominee for the Governor's Business Awards Program for Excellence in Teaching Mathematics, and the recipient of the Distinguished Service Award for Contributions to Excellence in Mathematics Education by the Department of Mathematics, Science, and Technology Education at NC State University, his alma mater. Mr. Scott is a former Mathematics Consultant at the Department of Public Instruction and current K-12 Mathematics Curriculum Specialist for the Charlotte-Mecklenburg Schools. He has always been a leader who promotes the professional growth of North Carolina's math teachers...and he continues helping and sharing to this day.

## Professional Development Opportunities for NC Math Teachers

Below is a list of upcoming professional development workshops being held in NC for math teachers. This information is provided as a service to teachers; the Centroid editors have not reviewed the workshop contents. Being on this list does not imply endorsement by The Centroid or by NCCTM, and teachers should seek more information before applying. Be sure to check for costs, financial support, availability of academic or CEU credit, and application deadlines!

See the Centroid website [http://www.mathsci.appstate.edu/centroid/](http://www.mathsci.appstate.edu/centroid/) for a more complete, up-to-date listing of opportunities. Know of a workshop not on this list? Email Holly Hirst [HirstHP@appstate.edu](mailto:HirstHP@appstate.edu) to add your workshop to the list on the Centroid website.

- 2008 Algebra through Function Academy [http://www.math.ohio-state.edu/~elaughba/](http://www.math.ohio-state.edu/~elaughba/): June 22-27 workshop for teachers led by Debbie Crocker (Appalachian) and Ed Laughbaum (Ohio State) in Duck, NC
- Duke University [http://howardhughes.trinity.duke.edu/outreach/teachers](http://howardhughes.trinity.duke.edu/outreach/teachers): July 13-18 workshop on integrating the quantitative and life sciences for AP teachers in mathematics, statistics and the life sciences
- Mathematics Education Leadership Training (MELT) [http://www.melt.appstate.edu](http://www.melt.appstate.edu): workshops held at Appalachian in Boone, NC; topics: algebra, geometry, TI Inspire, Integrated Math, discrete math
- Meredith Mathematics and Science Institutes [http://www.meredith.edu/math/team2/](http://www.meredith.edu/math/team2/): two-day math and science content focused workshops for teachers in grades K-12 sponsored by Meredith College
- NC Math and Science Education Network (MSEN) [http://www.unc.edu/depts/msen/](http://www.unc.edu/depts/msen/): Dates vary. Check your local MSEN frequently for postings.
- NC MSEN Statewide Institutes for Teaching Excellence (SITE) [http://education.uncc.edu/cmste/SITE/](http://education.uncc.edu/cmste/SITE/): one week summer institutes for K12 math and science. No 2008 daates posted yet.
- NC-PIMS [http://ncpims.northcarolina.edu/index.php](http://ncpims.northcarolina.edu/index.php): professional development for teachers in 12 school districts sponsored by the North Carolina MSEN, NCDPI, and several UNC schools
- NC Science House [http://www.science-house.org/teacher/index.html](http://www.science-house.org/teacher/index.html): math and science workshops for K12 teachers sponsored by NC State


## Problems to Ponder

# Spring 2008 Problems 

Gregory S. Rhoads

Appalachian State University
Boone, North Carolina

Grades K-2 John and Mary are playing a game of bowling. They have 3 chances to knock down as many of the 18 pins as they can. John knocks down 8 with his first ball, 4 with his second ball and 1 with his third. Mary knocks down 5 with her first ball, 5 with her second and 4 with her third. How many pins do each person leave standing? Who knocked down more pins?

Grades 3-5 The student council is making and selling pizzas as a fundraiser. Each pizza can be cut into 8 slices and they will sell each slice for $\$ 1.50$. If the cost of the ingredients for 50 pizzas is $\$ 100.00$ and if the student council sells every slice in the 50 pizzas, how much money will they make?

Grades 6-8 A total of $11,540,000$ people visited Myrtle Beach during 2005. In 2006, there were 12,182,000 visitors to Myrtle Beach. The number of visitors in 2007 increased by the same percentage over the previous year as did the number of visitors in 2006 over its previous year. How many people visited Myrtle Beach in 2007?

Grades 9-12 Suppose the lengths of the sides of a trapezoid are 5, 5, 5, and 11. What is the area of the trapezoid?

## Directions for submitting solutions

1. Neatly print the following at the top of each solution page:

- Your full name (first and last)
- Your teacher's name
- Your grade
- Your school

2. Submit one problem per page.

Students who submit correct solutions will be recognized in the next issue of The Centroid. We wish to publish creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission. Keep in mind that proper acknowledgement is contingent on legible information and solutions.

## Send solutions by 15 April 2008 to:

Problems to Ponder
c/o Dr. Greg Rhoads
Dept. of Mathematical Sciences
Appalachian State University
Boone, NC 28608
As these problems are intended to stimulate independent thinking, it is expected that a submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

## Grades K-2 Fall 2007 issue

Susie started writing some numbers down in a pattern. She spilled some water on her paper and some of the numbers washed away. She was left with

```
1
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where the blanks are missing numbers. Write down the entire list of numbers.
Solution: By Steven Lopez, $1^{\text {st }}$ grade, Porter Ridge Elementary (Teacher: Ms. Elizabeth Ryan).


Editor's Note: The pattern was that the sequence alternated between numbers repeating and increasing by 3 . This was a difficult problem for students of this age.

Correct Solutions were received by: Hugo Arteaga of East Elementary, Steven Lopez of Porter Ridge Elementary, and Emin Abeitma, Saheem Allen, Brittney Blount, Jamya Faulkner, Quintavius Garris, Elias Gomez, Shelby Haws, Payton Hillock, Malik McClain, Jacob Medlin, Daniella Riquelme, Ricardo Sanchez, Shawn Sizemore, Daniel Velasquez, Dulce Velasquez, Bricara Wilson of Wingate Elementary.

## Grades 3-5 Fall 2007 issue

Water runs out of your outdoor hose at a rate of 1 gallon every 15 seconds. If your outdoor pool holds 180 gallons, how many minutes will it take to fill the pool using your hose?

Solution: By Daniel Donaldson, $5^{\text {th }}$ grade, Fairview Elementary (Teacher: Mrs. Donna Thomas).
Correct Solutions were received by: Matt Bozarth, David Burns, Patrick Carr, Jeremy Cummings, Clay Dugo, Joel Green, Lucy Hansen, Cheyanne Helms, Kyle Johnson, Rachel Lowery, Julia McChesney, Jilian Paris, Ashley Pike, Jared Reninger, Sarah Rhyne, Sammy Sheaffer, Tiffany Shreves, Patrick Snyder, Zach Steele, Sam Tracy, Derrick Wood, Cassidy, Collin, and Max from Antioch Elementary, Bradley Blumer, Nick Cramer, Daniel Davidson, Jeffrey Dedrick, Justin Kerr, David Kusnitz, Michaela Larrison, Joseph Romaine, and Julia Wall from Fairview Elementary, Mustafa Abdeh, Shelby Beard, Hannah Davis, Kai Dueminler, Amie Hewitt, Anthony Pappas, Stephen Pound, Savannah Reed, Sasha Rivera, Elizabeth Sinnett, and Vlad Tupehiy from Hemby Bridge Elementary, Caroline Bradley, Robert Britt, Austin Chaney, Julian Coley, Dalton Davis, Kayla Funderburk, Charity Hamilton, Taylor Helms, Allyson High, Kyle Lilly, Montreal Mclendon, Melissa Silsby, Shaun Silsby, Anna Smith, Rachel Wallace, and Ny'Juan Williams from Marshville Elementary, Emily Crouse, Jack Ruth, and Akeyta from Waxhaw Elementary, Caitlin Palmer and Taylor Smith from Weddington

Elementary, and Christian Gose, Tarian Harrell, Monei Patterson, Ricardo Sigmon Jimenez, Amber Theroux, and Troylan Toledo from Wingate Elementary.


Editor's Note: Most students took this approach - multiply the number of gallons by 15 seconds to find the number of seconds to fill the pool, then divide by 60 seconds/minute to find the number of minutes.

## Grades 6-8 Fall 2007 issue

Harry walks twice around a circular track with diameter 56 feet. Jill walks three times around a square track where each side has length 40 feet. Who walks further and by how much?

Solution: By Donyetta Monique McBride, $6^{\text {th }}$ grade, Hallsboro Middle School (Teacher Mrs. Josie McKoy).


Editor's Note: In addition to her pictures and calculations, Donyetta also gave a very good step-by-step explanation of her solution in sentences.
Correct Solutions were received by: Takonti Askew, Brandon Barnes, Jazmine DelaGorza, Shakera Eley, Lyiah Harmon, DraQuan Harrell, Teneisha Hayes, Tyshaun Hayes, Trevon Jefferson, Laarni Lapat, Reginald Lee, Kenneka Moore, Stephanie Moore, Candice Parker, Tyreus Pugh, Joella Quiroz, Willisha Riddick, Jacqui Stokes, Branden Taylor, Carmen Villamor, Jaurice White, Khadyah White and Keyonne Williams from Bertie Middle, Will Beasley, Layla Booker, Whitney Brooks, R.J. Casey, Jesse Coen, Allyson Como, Brittany Deal, Krista Dennis, Chris Farr, Hayden Fletcher, Alexis Ford, Amanda Grandy, Shanesah Grizzle, Ian Halferty, Jennifer Jackson, Summer Rieve, Alex Sample, Angel Saunders, Emma Tate, and Chad White from Currituck Middle, Cierra Coleman, Chelsea Goodman, Caleb Graham, Douglas Hayward, Lillie Malpass, Donyetta McBride, Roland Moore, Madison Patrick, and Richard Roby from Hallsboro Middle, Damian Agers, Ethan Averette, Kimberly Cipriano, Sarah Crews, Samantha Denny, Alex Elliott, Taylor Harris, Brian Hart, Liz Hester, Allie Hobgood, Kayla Mize, Ashlee Morgan, Hannah Moss, Lauren Parham, Tyler Parrott, Andrew Pruitt, Kristen Rollins, and Megan Sullivan from Northern Granville Middle, and Courtney Beck, Catherine Braxton, Daniel Hinshaw, Jessica Johnson, and Mary Triplett from Southeast Middle.

## Grades 9-12 Fall 2007 issue

Everyone is voting for either Spot or Rover for class president. After 65\% of the votes are counted, Rover has $60 \%$ of the votes and Spot has $40 \%$. What percentage of the remaining votes must Spot get in order to catch Rover?

Solution: The submitted solutions were difficult to scan clearly, so here is a solution by the editor, which is similar in nature to the method used by the correct submissions.

Spot has $40 \%$ of $65 \%$ of the votes cast so he has $26 \%$ of the votes cast so far. He needs $24 \%$ of the remaining votes to catch Rover. If $x$ is the percentage of the remaining votes Spot needs, then

$$
x \times 35 \%=24 \% \text {. }
$$

Solving for $x$, we see

$$
x=68.57 \% \text {. }
$$

So Spot needs $68.57 \%$ of the remaining votes.
Correct Solutions were received by: Ermin Bibic and Denzell Faison from Ragsdale High.

## Websites with Math Activity Ideas

Check out this list of websites; you may find some interesting ideas to use with your students.

- Mega-Mathematics [http://www.ccs3.lanl.gov/mega-math/welcome.html](http://www.ccs3.lanl.gov/mega-math/welcome.html): Los Alamos National Laboratory and the University of Idaho project that includes activities on topics at the "frontier of mathematics" such as graphs, knots, algorithms, and infinity.
- MSTE [http://www.mste.uiuc.edu/stat/stat.html](http://www.mste.uiuc.edu/stat/stat.html): University of Illinois-Urbana Champaign Office of Math, Science and Technology Education database of standards based activities for K12 and college-level math and science.
- National Science Digital Library [http://nsdl.org/](http://nsdl.org/): National Science Foundation funded online library for education and research in science, technology, engineering, and mathematics (STEM).


# 2007 Outstanding Mathematics Education Award Winners Eastern Region: Cara Nicole (Nicki) Hull; Meredith College Central Region: Lindsey L. Bakewell; University of North Carolina - Greensboro Western Region: Polly Megan Rudoff; Western Carolina University 

Reported by Bampia A. Bangura<br>North Carolina Agricultural and Technical State University Greensboro, North Carolina

Each fall, NCCTM sponsors the selection of three Outstanding Mathematics Education Students. The Special Awards Committee identifies the most Outstanding Mathematics Education Student from each of the three regions in North Carolina. Nominations are requested from all Colleges and Universities in North Carolina with teacher preparation programs.

From the Eastern Region of NCCTM, Cara Nicole (Nicki) Hull is a senior at Meredith College, completing a bachelor's degree in mathematics with licensure requirements in secondary mathematics. In addition, she is completing a bachelor's degree in Spanish. She is also a Teaching fellow and an honor student on campus. Nicki has been an active student leader at Meredith College in a variety of ways. She serves as the scholarship weekend Co-Chair for the honor's program, she is a member of Pi Mu Epsilon, she has attended faculty meetings as the main student representative to the Mathematics/Computer Science Department at Meredith College, she has tutored in the Meredith College Learning Center since the spring of her freshman year on campus, and she has been recognized as an outstanding tutor. Nicki also participated in the 2006 NCCTM State Annual Meeting. It was obvious from the beginning that Nicki was an excellent student in Mathematics. As one of the top tutors of Mathematics in the Learning center, Nicki is demonstrating that she will become an excellent secondary school teacher of mathematics. She maintains an active presence in co-curricular activities and other social activities on campus.

From the Central Region of NCCTM, Lindsey Bakewell is a senior at the University of North Carolina in Greensboro. She is pursuing a BS Degree in the Mathematical Sciences with a Teacher licensure. Lindsey is a North Carolina Teaching Fellow. She is active in the Department of Mathematics and Statistics at UNCG. In addition to her activities in the UNCG secondary mathematics teacher education program, Lindsey is involved in several campus activities including: the Sigma Alpha Lamda Honors Organization, the Alpha Lamda Delta Freshman Honor Society, the Kappa Delta Pi Education Honor Society, and the UNCG School of Education Marshals. Lindsey is a recipient of the Deberry Scholarship at UNCG 2004-2005 and the Grogan Scholarship in the UNCG Mathematics Department 2005-2006. Her community and civic activities are exceptional. She has been involved with the children's Festival, the Young Writer's Conference, Fundraiser for Mary's house, and Caring for Katrina's Kids. Lindsey has tutored mathematics in several places including the UNCG Teaching and Learning Center, Western Piedmont Community College in Morganton, NC, Walter Hines Page and Ben L. Smith high schools in Greensboro, NC. Lindsey is not only an excellent scholar but also a fine teacher candidate who is truly an outstanding mathematics education student.

From the Western Region of NCCTM, Polly Megan Rudoff is a senior at Western Carolina University pursuing a bachelor's degree in mathematics education. Polly has been involved in many aspects of the Mathematics and mathematics education programs at Western Carolina. She served as vice-president of the NCCTM student affiliate organization for the 2006-07 school year and is currently serving as president of that organization. She has attended all the NCCTM State Fall Conferences since the fall of 2004. Polly is a North Carolina Teaching Fellow and a member of the Western Carolina University's Honors College. As a measure of the recognition Polly has earned as a consequence of her strong academic record, she has been nominated for the following honors and awards: All American Scholars award, National Collegiate Mathematics award, National Dean's List for America's Outstanding College students, National Collegiate Education Award, and many more. Polly has served in other campus activities; she is a member of the College of Education Field and Clinical Experience Committee, a member of the WCU Teaching Fellows Leadership Committee, and a member of the University Teacher Education Partnership. Overall, Polly is a mature mathematics educator who will contribute immensely to the lives of young people.

## Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The purpose of this award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state. Further, this program must have been sustained for a period of at least three years. A number of organizations have made significant contributions to mathematics education in North Carolina; the Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations by sending in the form below. Nominations will be retained in the active file for at least three years.

## NOMINATION FORM

Name of Nominee: $\qquad$
Present Position: $\qquad$
Outstanding contributions to mathematics education in North Carolina which serves as the basis for this nomination:

Additional information that would be of value to the selection committee:

Signature: $\qquad$ Date: $\qquad$
Name (print/type): $\qquad$
Position: $\qquad$
Business or Institution: $\qquad$
Address: $\qquad$
Phone: Business $\qquad$ Home: $\qquad$
Email: $\qquad$
Send to: John Parker
316 West Soundside Road
Nags Head, NC 27959

## NCCTM Trust Fund Scholarship

North Carolina Council of Teachers of Mathematics
$\$ 600$ scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction.

Applicants must be:

- Currently employed as a pre-K - 12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting this application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are:

## - March 1

- October 1

| Send completed applications to: | Direct inquiries to: |
| :--- | :--- |
| NCCTM Trust Fund Chairperson | John R. Kolb, Chairperson |
| 6520 West Lake Anne Drive | phone: (919) 787-8116 |

(Please print all information.)

## PERSONAL INFORMATION:

Name: $\qquad$
Home address: $\qquad$
$\qquad$ NC $\qquad$
Home phone: $\qquad$ Home e-mail: $\qquad$
NCCTM membership number: $\qquad$
EMPLOYMENT INFORMATION:
How many years of teaching experience? $\qquad$
Currently employed in what school system? $\qquad$
School name: $\qquad$
School address: $\qquad$
School phone: $\qquad$ School e-mail: $\qquad$
Current teaching assignment: $\qquad$
Principal's name: $\qquad$

## COURSE INFORMATION: (One course only)

Institution of higher education: $\qquad$
Graduate degree program in which you are currently enrolled: $\qquad$
Course name: $\qquad$ Course number: $\qquad$
Dates of enrollment: (circle one) Fall semester
Spring semester Summer session Year: $\qquad$
Name of course instructor: $\qquad$

PROFESSIONAL ACTIVITIES WITHIN PAST 5 YEARS WITH EMPHASIS ON ACTIVITIES RELATED TO
MATHEMATICS EDUCATION:

## BRIEF STATEMENT OF FUTURE PROFESSIONAL GOALS:

## REQUIRED SIGNATURES:

Applicant signature: $\qquad$
Principal's signature: $\qquad$
Instructor signature (if currently enrolled): $\qquad$
Date: $\qquad$ Thstructor signature (if currenty enrolled).

## REQUIRED ATTACHMENTS:

Please attach a copy of

1. A letter of acceptance to an accredited graduate program in North Carolina;
2. Official verification of enrollment in the graduate course described in the COURSE INFORMATION above if the course is currently being taken, OR official transcript containing the grade awarded to the applicant if the course described in the COURSE INFORMATION above has been completed.

NOTE: Applications must be complete to be considered. If your application is approved, an official course grade report must be submitted to verify successful completion of the course before scholarship funds will be issued.

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Name: $\qquad$ Home Telephone: $\qquad$ ) - $\qquad$
Address: $\qquad$ School Telephone: $\qquad$ ) - $\qquad$
City: $\qquad$ State: $\qquad$ Zip: $\qquad$ E-mail:

School System:

## POSITION

## LEVEL

Teacher

- K-3

D Department Chair
Supervisor/Administrator
Full-time College Student
Retired
O Other $\qquad$ .

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$\square$ New Former/Renewing Member \# $\qquad$

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- Senior High

2-Year College/Technical
4-Year College/University
$\square 1$ year:
3 years:
$\square$ Full-time Student:
Contribution to Trust Fund:
Total Payment Enclosed:
Payment by $\square$ Check $\square$ Visa MasterCard
Card \#
Exp. Date
Signature
$\$ 20.00$ $\qquad$
$\$ 50.00$ $\qquad$
$\$ 10.00$ $\qquad$
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$\qquad$

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P.O.Box 4604

CARY, NC 27519
Payments by credit card may be mailed or faxed to
919-859-3342


|  |
| :---: |


[^0]:    ${ }^{1}$ See, e.g., http://www.mathsisgoodforyou.com/topicsPages/egyptianmaths/rhynd.htm for pictures and worksheets.
    ${ }^{2}$ The Oxford English Dictionary states that the original meaning of al-jabr was "the reuniting of broken parts," including the setting of fractured bones. The term carried both meanings until recently. The dictionary also states that algorithm,

[^1]:    "while certainly derived from the surname of al-Khwarizmi, is actually a form of the old word algorism which passed through many pseudo-etymological perversions" originally meaning the Hindu-Arabic decimal system of numeration. Of course, al-Khwarizmi is an English translation of the Latin transliteration of the Arabic translation of the original namewhich is anyone's guess.

