The Centroid

Makes Wonders

In This Issue:

- **Functions as Graphs: Depth Versus Volume in a Vase**
- **The Shape of an Ellipse**
- **An Interview with Author Joan Holub**
- **2011 Outstanding Mathematics Education Students**
- **2011 Rankin and Innovator Award Winners**

NCCTM

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The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through teacher education. *The Centroid* is published in January and August. Subscribe by joining NCCTM; see the Membership Form on the last page.

Submission of Manuscripts

We invite the submission of news, announcements, and articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways.

News and announcements (president's messages, award winner announcements, professional development announcements, etc.) must be received by December 1 for the spring issue and by July 1 for the fall issue.

Articles that have not been published before and are not under review elsewhere may be submitted at any time to the address below. Submit one electronic copy via e-mail attachment (preferred) or diskette in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page. Manuscripts should not exceed 10 pages double-spaced with one-inch margins. Figures and other pictures should be included in the document in line with the text (not as floating objects). Scannable photos are acceptable and should be large glossy prints mailed to the editor or minimum 300 dpi tiff files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.

Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*. References should be listed at the end of the article, and should also follow APA style, e.g.,

Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, Grade 3*. Retrieved October 17, 2005, from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics, 7*, 196–199.

Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology*,
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

About the Cover

The Centroid logo is based on the following theorem: The limit of the sequence of midtriangles of a triangle is the centroid of the triangle.

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The Centroid

OFFICIAL JOURNAL OF THE NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS
VOLUME 38 • NUMBER 1 • SPRING 2012

Articles

News & Information

NCCTM 2012 Conference: October 25-26

Koury Convention Center, Greensboro

The 2012 State Mathematics Conference is a wonderful opportunity to share research, classroom strategies, activities, and resources with your colleagues that make mathematics come alive for your students. Take time to be a part of this annual professional development opportunity by becoming a Speaker.

To present at the 2012 Conference, fill out the online speaker proposal form, available on the NCCTM site. If you have questions contact either Sheila Brookshire <sheila.brookshire@bcsemail.org> or Kelly Delong <kdelong@ncmcs.org>. Watch the website or more information.

<http://www.ncctm.org>

NCCTM Spring Leadership Seminar

Implementing the Common Core State Standards for Mathematics

Greensboro-High Point Marriott Airport
9:30 AM to 3:15 PM, March 23, 2012
Keynote Speaker: Dr. Cathy Seeley

Dr. Seeley's keynote address will focus on content in the Common Core State Standards for Mathematics (CCSSM) as well as assessment as it relates to the CCSSM. Those who attend will have a clearer understanding of how the Common Core State Standards for Mathematics will affect their teaching, their assessments, and their students.

There will be four afternoon breakout sessions (K-2, 3-5, 6-8, 9-12), each led by a team of discussion leaders. Each team will include a DPI staff member, a higher education faculty member, and an LEA representative.

To register, go to http://www.ncctm.org

NCCTM Membership Changes

New Definition of the "Membership Year" – July 1 through June 30

Currently, the NCCTM membership year is a non-uniform member specific twelve-month period. For example, if you joined for one year on December 1, 2010, your membership expired November 30, 2011. On October 26, 2011, the NCCTM Board of Directors voted to change to a fixed membership year running from July 1 to June 30 effective July 1, 2012.

During the transition period, if you join or renew your membership between November 1, 2011, and June 30, 2012, your membership year will be extended. The extended expiration dates will be as follow

- One-year membership will expire June 30, 2013
- Three-year membership will expired June 30, 2015
- Five-year membership will expired June 30, 2017

The extended membership year(s) is only for this transition period. After this transition period, all memberships will run from July 1 to June 30 for a one-year, three-year or five year period ending on June 30.

Presidents' Messages

State President
Betty Long
longbb@appstate.edu

First of all I would like to take this opportunity to express my appreciation to everyone who helped to make this year's State Math Conference a huge success. There were 2,504 people in attendance this year, despite the tight economy. A special thank you goes to Drew Polly and Amanda Northrup (Program Co-Chairs) and Elizabeth Murray and Donna Thomas (Conference Co-Chairs) along with the chairs of the various conference committees that include Vincent Snipes (Audio Visuals), Rebecca Hoover and Becky Caison (Commercial Exhibits), Jan Wessell and Pat Sickles (First Timers), Ryan Dougherty, Ana Floyd and Michelle Gray (Publicity and Signs), Tina McSwain (Troubleshooting), Shana Runge and Melissa Wilson (Marketplace), Kathy Jaqua (NCTM Materials), Deborah Crocker (Student Exhibits), C.E. Davis (Registration Volunteers), Marilyn Preddy (Conference Services), and Rebecca Hoover (Management Services). Also, I want to thank all of the presenters who shared their knowledge and expertise in the many sessions and workshops as well as the NCDPI's Mathematics Consultants for their part in helping us get ready to implement the Common Core State Standards for Mathematics (CCSSM). There were many, many more volunteers that assisted at the conference by working at the registration desk, handing out workshop tickets, working at the NCCTM booth (organized by Tracie Salinas and Emily Elrod), helping with the Awards Celebration and Reception and assisting me with whatever needed attention at the moment. I sincerely appreciate everything that everyone did to make this conference a worthwhile professional development experience.

Now that the State Math Conference is behind us, we turn our attention to the many exciting NCCTM events that will occur in the spring. Since the majority of the folks attending last fall's Leadership Seminar expressed an interest in having another one in the spring, the first-ever NCCTM Spring Leadership Seminar will be held on Friday, March 23 at the Greensboro-High Point Marriott Airport at 1 Marriott Drive in Greensboro. The keynote speaker will be Dr. Cathy Seeley, author of *Faster Isn't Smarter* -- *Messages About Math, Teaching, and Learning in the 21st Century*. Dr. Seeley's keynote address will focus on content in the Common Core State Standards for Mathematics as well as assessment as it relates to the CCSSM. Additional topics of discussion will be how to provide administrative support for the implementation of the CCSSM and how to help classroom teachers with the challenges they will face during this transition. There will be four afternoon break-out sessions (K-2, 3-5, 6-8, 9-12), each led by a team of discussion leaders including DPI staff members, higher education faculty members, and LEA representatives. For more details and to register, please visit the NCCTM website (NCCTM.org).

Other NCCTM spring activities include the three regional mathematics conferences, math contests, math fairs, logo contest, Trust Fund Graduate Scholarships, and the election for the state and regional presidents. For more information on all of these activities, go to NCCTM.org. If you have questions or want to help with any of these activities, please contact me via email or by phone <828-262-2372>.

At the last Board meeting, a motion was passed to change the NCCTM membership year to a fixed year of July 1 to June 30 for all members. There is a transition plan in place for those folks whose current membership extends beyond May 31, 2012. The details of this plan are on the NCCTM website.

Once again, I want to thank all the wonderful folks (Board members, committee chairs, committee members, and other volunteers) who spend countless hours serving NCCTM and mathematics education as they work toward expanding the ways NCCTM can better serve mathematics teachers and students. I believe that by working together we can make a real difference in mathematics education. If you would like to become more involved in NCCTM, please contact me.

Central Region President
Pat Sickles

pat@sickles.org

The Central Region is planning for another super Spring Mini-Conference. Holt Wilson, Vice President for Colleges is the chairperson of the conference, assisted by Melissa McKeown, Vice President for Elementary Schools; Amy Travis, Vice President for Middle Schools and Beth Layton, Vice President for Secondary Schools. The primary audience for the Central Region conference will be prospective and beginning middle and secondary mathematics teachers. Invited speakers will offer sessions on instructional tasks and discourse strategies to support teachers in preparing for the Common Core State Standards in Mathematics.

Awards! Congratulations to Central Region's Marilyn Preddy, winner of the 2011 Rankin Award. We are so lucky to have Marilyn's expertise in many areas but especially in helping to make the NCCTM Annual Conference run smoothly. Congratulations, too, to the NCSSM faculty, winners of the 2011 Innovator Award, for their contributions to mathematics education both in NC and nationally.

Voting! Voting will soon be open for State and Regional Presidents for 2013-2015. The Central Region Nominees for President are: Sylvia Davis (Randolph County Schools) and Vincent Snipes (WSSU). You will be able to vote online—and please do vote!

Get involved! NCCTM provides so many opportunities for members to be involved in mathematics education in our state. The Annual Conference and Leadership Seminar in October offered a tremendous opportunity for us to hear speakers, to network with other educators and to volunteer our services with the organization. NCCTM also offers members Mini-grants for worthwhile mathematics projects and also awards scholarships to help teachers who are pursuing an advanced degree. The Central Region has $6000 available each year for mini-grants, so please write and send in a proposal. The NCCTM website has information about deadlines and application forms.

Get your students involved! On March 31, 2012 the Central Region Math Fair will be held at the North Asheboro Middle School. Chairperson Melanie Burgess can be reached at <mburgess@randolph.k12.nc.us>. Winning projects will then be a part of the NCCTM State Math Fair in May at the NCSSM in Durham.

The NCCTM Logo Contest is yet another way to involve students in thinking about mathematics. Congratulations to Central Region's Jonathan Schwartz, this year's logo winner, a student at East Chapel Hill High School whose teacher was Beth Neill. His winning logo could be seen at the conference on t-shirts and posters.

Western Region President
Katie Mawhinney

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As the 2012 portion of the school year has begun, I hope that each of you is well and off to a great start! I also hope that any changes in your practice that you have made this year, in order to address the Standards for Mathematical Practice, have been successful. As with any profession, teaching is formative in nature. We are constantly changing what we do in the classroom, reflecting on effectiveness, and then altering what was new in order to make it even better. If a new task or activity or other classroom structure you put to action did not succeed as you hoped, it may be that the idea needs only minor changes in order to be improved. This is one of the many places where collaboration with colleagues can be useful. And I encourage you all to seek out and utilize your colleagues in mathematics education, especially as we transition into the new content standards presented within the Common Core State Standards for Mathematics.

Upcoming opportunities for sharing ideas with colleagues and preparing for the Common Core include NCCTM's Spring Leadership Seminar of March 23 (in Greensboro) and within our Western region of the state, the NCCTM Western Regional Conference which will be held on Saturday, March 24, at Charles D. Owen High School in Swannanoa, NC. More information about both days may be found at the NCCTM website (www.ncctm.org), and submissions for talks at the Western Regional Conference may be sent to me at mawhinneykj@appstate.edu. I hope to see you there!

Functions as Graphs: Depth versus Volume in a Vase

Holly Hirst, Appalachian State University

Many mathematics textbooks contain a problem similar to the following:

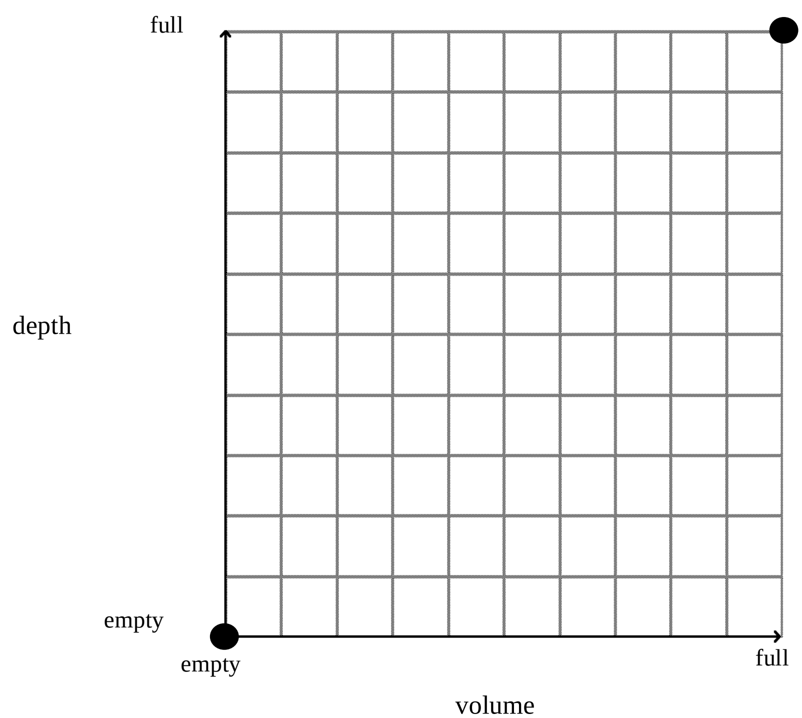
> *Given the vase in the picture to the right, sketch a graph of the depth of water in this vase as a function of the volume of the water.*

This problem is suitable for many levels of students; in fact, a similar activity or problem is contained in the middle grades mathematics curriculum *Connected Mathematics 2* ([CMP], Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006), as well as in the college calculus text *Calculus from Graphical, Numerical, and Symbolic Points of View, 2nd Edition* (Ostebee & Zorn, 2001). Even in its simplest form this problem can pose difficulties for advanced students because the independent variable is volume rather than time.

The vase problem can be expanded into an extended activity providing a rich investigation of the notion and properties of functions. Depending upon how advanced the students are, topics from basic measurement and graphing to concavity and rates of change can be studied through a hands-on, engaging exploration of this functional relationship. This article presents a group activity that culminates in a contest to see who can most accurately predict the graph of the relationship between volume and depth of water in a vase.

The Activity

Preliminaries: For each group you will need: a ruler; a vase; copies of the blank graphs printed the same size on paper and a transparency; markers; rice, sand or water for filler; a measuring device for the filler. The measuring device can be sophisticated (a graduated cylinder borrowed from a friendly chemistry teacher) or simple (a small paper cup like those supplied for condiments in fast food restaurants). Water and graduated cylinders stress the point of careful and accurate scientific measurements, but rice and condiment cups also work.



Coordinate System – For Paper and Transparency Handouts

Phase 1. Estimate the curve: Give each group a vase, a paper copy of the coordinate system, and a marker. Ask the groups to think about and then sketch on the graph their best estimates of the shape of the curve representing the relationship between volume and depth in the vase. Depending upon the students, you may need to work through one together to point out the ideas to think about: concavity, slope, etc. Guiding questions (with suitable word choices depending upon the level of student):

- If v represents the volume of the water and d represents the depth, explain what $d(0)$ is in terms of depth and volume. How do we interpret the notation $d(v)$?
- Will the curve ever be higher than the (full, full) data point? lower than the (empty, empty) data point? What are the domain and range for this function?
- Will the curve be continuous? What would a discontinuous curve mean in terms of volume and depth?
- How non-linear do you expect the curve to be? What would a linear curve mean in terms of volume and depth? What would be the interpretation of the slope of such a linear curve?
- Will the curve increase for some values of the volume? Decrease? What is the interpretation of the curve increasing or decreasing in terms of volume and depth?
- Will the curve be concave up for some values of the volume? Concave down? What is the interpretation of the concavity of the curve in terms of volume and depth? What would an inflection point say about the shape of the vase?
- Will the curve relate in any obvious way to the outline of the vase?

Phase 2. Find the curve: Collect the graph estimate papers from the groups. Give each group a transparency of the coordinate system (the same size as the paper copy), a ruler, some filler, and a measuring device. Ask the students to collect data by pouring small amounts of filler into the vase, recording the volume poured in and the resulting depth in the vase. Once they are satisfied that they have good data points, have them plot the data and then sketch a curve through the points. Questions to think about, in addition to those from phase 1:

- What should one unit on each of the axes represent, since there are 10 units shown between empty and full?
- What inaccuracies are introduced through your measurement technique, if any?

These questions are very interesting when using the condiment cup measurement approach. Unlike with graduated cylinders, which are typically subdivided in milliliters, students must grapple with the inaccuracies inherent in parts of cups and come to some conclusion as to how much these inaccuracies distorted their graphs. Good results are possible with the condiment cup, but students are much more convinced of the accuracy of their graphs when using graduated cylinders.

Phase 3. Comparing results: Collect the curves traced using the data and ask the class to vote on who came closest by aligning the two graphs from each group. Ask the groups to reflect on and explain why their graphs were inaccurate. Some of the questions from phase 1 might be helpful to get the groups started thinking.

Phase 4. Challenge (best done several class periods later): If you had different vases for the different groups, take the curves produced from the data, labeled with numbers, and the vases, labeled with letters, and place them on a table. Ask the students to match the curves to the vases, writing an explanation for their choices using the vocabulary of graphs.

Phase 5. Ultimate challenge 1: Give the students back their vases and ask them to draw a graph of the *rate of change of the depth as a function of time* assuming that liquid is being poured into the vase at a constant rate.

Phase 6. Ultimate challenge 2: Give the students a graph of depth as a function of volume and ask them to sketch the vase that would produce the graph.

Placement in the Curriculum

Variations on this activity have been used with middle school and high school inservice teachers, as well as calculus students and undergraduate faculty. As a calculus exploration, the vase activity as presented here was especially effective as a first group activity to help students review the terminology of functions and change. Used in this way, the activity has the added benefit of allowing students to get acquainted with each other. A variation on the activity presented here could be used at several places in the middle and high school curriculum as outlined in the new Common Core State Standards for Mathematics (Common Core State Standards Initiative [CCSSI], 2011):

From the eighth grade content standard on functions (8F):

> Use functions to model relationships between quantities. . . .5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSSI, 2011, p. 55)

From the high school content standard on interpreting functions (F-1F):

> Interpret functions that arise in applications in terms of the context. . . .4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (CCSSI, 2011, p. 69)

In the current North Carolina Standard Course of Study (2003), these notions are explored in courses in Advanced Functions and Modeling, Precalculus, and to some degree in both Technical Mathematics and Algebra II.

Some Final Observations

Middle school, high school, and college teachers and students who have tried this activity are routinely surprised by several things:

- Approaching the estimation stage carefully (with the intent of winning the contest!) forces one to think hard about slope, concavity, and inflection.
- A very curvy vase does not produce a graph that is as different from linear as one might expect.
- Accurately estimating the curve for a vase is surprisingly difficult, even after practicing on a few different vases.

If the goal of the mathematics curriculum is to get students to think carefully about concepts, this activity may be just the ticket!

References

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Common Core State Standards Initiative. (2011). *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/

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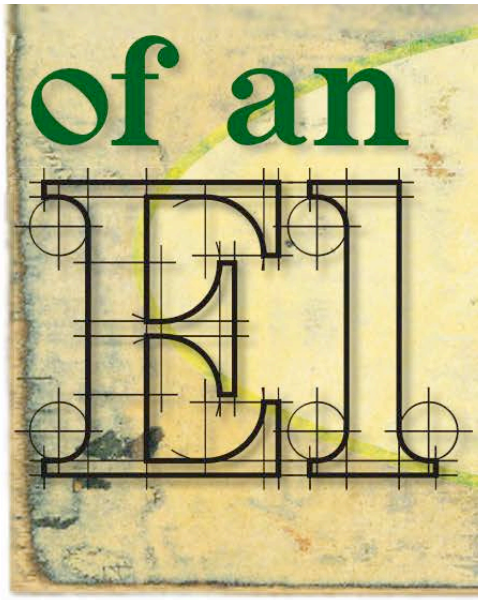
The Shape of an



Ellipses vary in shape from circular to nearly parabolic. An ellipse's eccentricity indicates the location of its foci, but its aspect ratio is a direct measure of its shape.

Gregory D. Foley

Conic sections have a long and rich history. Although conics had been studied by others before him, Apollonius of Perga (ca. 250–175 BCE) went far beyond his predecessors in his set of eight books entitled *Conics*. In this comprehensive work, Apollonius recognized that ellipses, hyperbolas, and parabolas were cross sections of a cone and represented the hyperbola with two branches. Interest in these curves was revived 1800 years later when Galileo Galilei (1564–1642) showed that the trajectories of projectiles are parabolas and Johannes Kepler (1571–1630) found that the paths of planets are ellipses. (See Katz [2009], Kline [1972], and Sagan [1980] for details.)

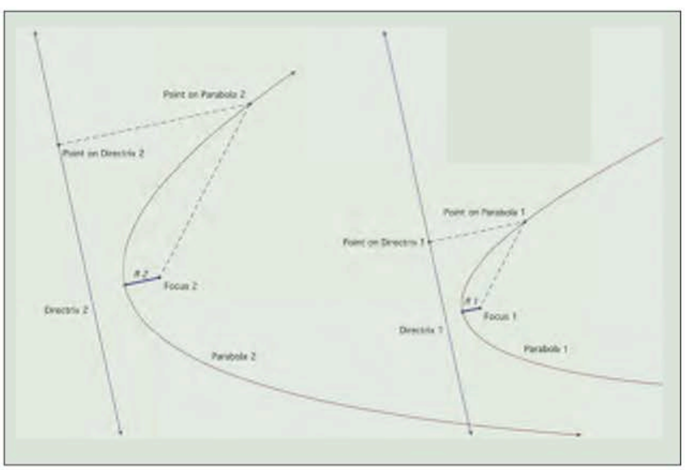


Fig. 1 The focal length of parabola 2 (fl_2) is twice that of parabola 1 (fl_1).

Although currently somewhat out of fashion, conic sections are neither useless nor outdated. Conic sections are the paths of nature: Any free-moving object in a gravitational field follows the path of a conic section. We can enliven the study of conic sections in general and ellipses in particular by focusing on their geometry and in the process use technology, mathematical reasoning, measurement, data analysis, modeling, communication, multiple representations, and connections to astronomy.

An ellipse's shape is usually associated with its eccentricity, but, as we shall see, eccentricity—although associated with shape—is not a direct measure of shape. When high school and college mathematics students study conic sections, they often learn little about their shapes and may even form misconceptions. To be sure, understanding the shape of conics is not a trivial matter.

CONIC COUSINS

Let's begin with a quick look at the shape of an ellipse's closest conic cousins—the circle and the parabola. Most students know that all circles have the same shape or, more precisely, that any two circles are similar in the geometric sense: Their ratio of similitude is the ratio of their radii. But few students know that any two parabolas are similar: Their ratio of similitude is the ratio of their focal lengths (vertex-to-focus distances). For example, in **figure 1**, parabola 2 is a dilation of parabola 1 by a factor of 2. The second parabola has then been translated to a new position; therefore, parabola 2 is similar to parabola 1. Because parabolas extend to infinity, we can never depict entire parabolas. Some partial parabolas appear to be wider than others. So it is

surprising—but true—that all parabolas are similar in the geometric sense and thus all have the same shape.

Ellipses, on the other hand, vary in shape from circular to nearly parabolic. Planets, asteroids, comets, and other bodies that orbit the sun follow elliptical paths. Some elliptical orbits are nearly circular, like the orbits of Neptune and Venus; others are significantly elongated, like that of Hale-Bopp, the great comet of 1997.

This article takes a careful look at the shape of an ellipse and offers practical suggestions and specific activities to deepen students' understanding of the geometry of an ellipse. Using the Common Core (2010) practices and NCTM (2000, 2009) Standards, the activities link an ellipse's shape to algebra, data, and measurement and make connections between mathematics and astronomy.

THE VOCABULARY OF AN ELLIPSE

A study of the geometry of an ellipse should start with some terminology and notation to facilitate classroom discourse. Carefully worded definitions illuminate geometric relationships and can serve as the basis for precise communication, reasoning, proof, and other aspects of doing mathematics (Stein et al. 2009). Whereas the locus of points on a circle is based on a point (center) and a distance (radius), an ellipse is determined by two points (foci) and a combined distance. An *ellipse* is the set of all points in a plane whose distances from two fixed points in the plane have a constant sum (see **fig. 2**).

After the term has been defined, the key parts of an ellipse should be defined, displayed, and labeled. These parts include the focus, center, focal axis, vertex, chord, major axis, and minor axis. Geometry software can be used to construct an ellipse dynamically and display the interaction among its parts (see **fig. 3**). Definitions are available in standard textbooks (e.g., Demana et al. 2011).

Incorporating such "intentional language instruction and support" helps all students, especially English language learners (Bay-Williams and Herrera 2007, p. 46). In addition to Latin plurals (e.g., *axis-axes*, *focus-foci*, *vertex-vertices*), other linguistic surprises should be pointed out to students. For example, the word *axis* is used in three ways:

- The focal axis of an ellipse is a line.
- The major and minor axes are line segments.
- The semimajor and semiminor axes are numerical measures, denoted as a and b.

ROBIN BLANKENSHIP

Fig. 2 Vocabulary building should be tied to other representations and activities.

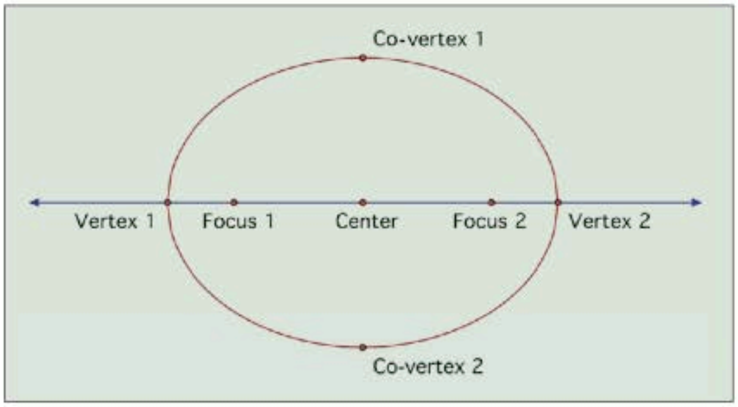


Fig. 3 Students should recognize five special points on the focal axis and the two points that are the co-vertices of an ellipse.

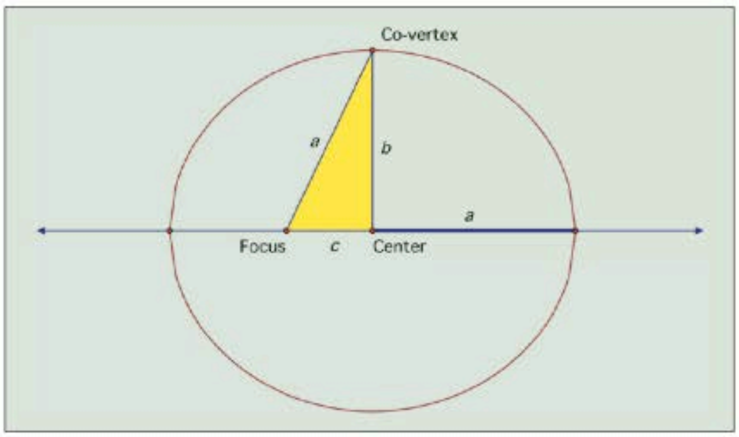
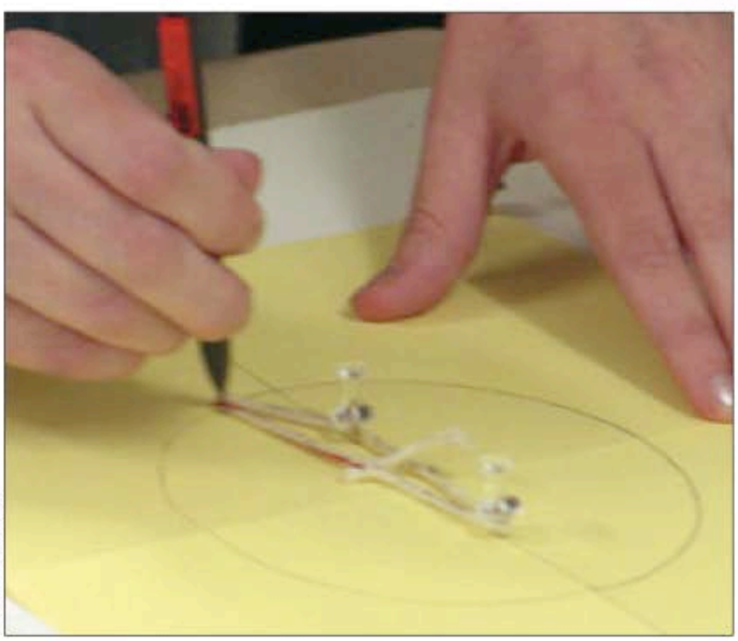


Fig. 4 The numerical measures *a*, *b*, and *c* are the side lengths of a right triangle.



ROBIN BLANKENSHIP

Fig. 5 Using the definition and basic tools, students can build an ellipse.

A third numerical measure—the distance between the center of an ellipse and either focus—is denoted as c. **Figure 4** suggests several important relationships involving these measures:

- $a > c \geq 0$. The constant sum of distances in the ellipse definition is $2a$, the length of the major axis. (Can you see why?)
- The constants a, b, and c have a Pythagorean relation that can be written as $a^2 = b^2 + c^2$, $b^2 = a^2 - c^2$, or $c^2 = a^2 - b^2$. (This is not the usual $a^2 + b^2 = c^2$ Pythagorean relationship because, for ellipses, a is the hypotenuse.) See **fig. 4**.

The *eccentricity* of an ellipse is the ratio

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}.$$

Students may find this notation confusing. Alert them to the fact that the eccentricity e is *not* the natural base e used for exponential and logarithmic functions.

The concept of eccentricity applies to all types of conics:

- Circles, having a single shape, have a single eccentricity: $e = 0$.
- Likewise, because all parabolas have a common shape, they all have a common eccentricity: $e = 1$.
- By contrast, hyperbolas have a range of eccentricities: $e > 1$.
- Because $a > c \geq 0$ and $e = c/a$, ellipses have a range of eccentricities: $0 \leq e < 1$.

Elongated ellipses have eccentricities close to 1; for example, the orbit of the comet Hale-Bopp has eccentricity $e = .995$. Nearly circular ellipses have eccentricities close to 0; for instance, Venus's orbit has an eccentricity of approximately .0068. When the eccentricity of an ellipse is 0, the semimajor and semiminor axes degenerate to a common value, which is the radius of the resulting circle.

CONSTRUCTING ELLIPSES TO LEARN ABOUT THEIR SHAPE

Nearly everyone at some point in life uses a compass to construct a circle in a manner consistent with the geometric definition of a circle. In an analogous way, using the definition that the sum of the distances from the foci to each point on an ellipse is a constant, we can construct an ellipse using a pencil, a piece of string with its ends tied together to form a loop, and two pushpins (see **fig. 5**). Put the loop around the two pins, which serve as the foci; pull the string taut with a pencil point; and, keeping the string taut, move the pencil around to trace out the ellipse. The string's length is actually $2a + 2c$. (Take a moment to convince yourself of this.)

To enact this ellipse construction as a small-group activity, divide the class into groups of three or four students and provide each group with a pencil, unlined paper, a 20-cm loop of string or unwaxed dental floss, pushpins, a ruler, and a foam board (used poster board works well). Have each group do the following:

- Place a sheet of unlined paper on the foam board and, with care and accuracy, place the two pushpins 2 cm apart near the center of the paper. Construct an ellipse using the loop of string and a pencil.
- On separate sheets of paper, repeat this construction process three more times, placing the pushpins 4, 6, and 8 cm apart.

Have students label each of their four ellipses as shown in **figure 4** and measure a, b, and c. Using these data, students can compute the corresponding values for e and the ratio b/a. To organize these data and look for patterns, they can create a table with five columns to record the resulting values of a, b, c, and e and the ratio b/a for their four ellipses.

- Ask students to write their observations about the ratio b/a as the eccentricity ratio e increases. Which of these two ratios measures the shape of the ellipse? Which measures how off-center the foci are?
- Have students plot the ordered pairs $(e, b/a)$, determine a formula for the ratio b/a as a function of the eccentricity e, and overlay this function's graph on the scatter plot.
- Ask students to determine how far apart the pushpins would need to be for the ellipse to have a b/a ratio of 1/2. Then have them construct the ellipse with this b/a ratio.

You and your students may find the results of this activity surprising. **Table 1** lists the theoretical results. The actual results will vary from group to group, but the key relationships should be as follows:

- The ratio b/a decreases slowly as $e = c/a$ increases rapidly.
- The ratio b/a is the minor-to-major axis ratio, which measures the shape of the ellipse. When b/a is close to 1, the ellipse is nearly circular; when b/a is close to 0, the ellipse is elongated.

Consequently, the ratio b/a can be considered the *aspect ratio* of an ellipse. This ratio is positive and less than or equal to 1 and is the aspect ratio of

Table 1 Geometric Parameters of Ellipses Constructed Using a 20-cm Loop of String

a (cm)	b (cm)	c (cm)	$e = c/a$	b/a
9	$\sqrt{80} = 8.944$	1	$1/9 = .111$	$\sqrt{80}/9 = .994$
8	$\sqrt{60} = 7.746$	2	$2/8 = .250$	$\sqrt{60}/8 = .968$
7	$\sqrt{40} = 6.325$	3	$3/7 = .429$	$\sqrt{40}/7 = .904$
6	$\sqrt{20} = 4.472$	4	$4/6 = .667$	$\sqrt{20}/6 = .745$

the associated rectangle or its reciprocal. (Aspect ratios of rectangles are width-to-height ratios used to measure the shape of television and computer screens as well as visual images. Common aspect ratios of television and movie screens are 4:3, 16:9, and 1.85:1. Portraits and writing paper typically have aspect ratios of less than 1, such as 8 × 10-in. photographs and 8.5 × 11-in. paper.)

Eccentricity measures how far off-center the foci are. The word itself is derived from *ex-centric*, meaning "out-of-center" or "off-center" (Schwartzman 1994). A circle with eccentricity $e = 0$ is perfectly "centric": It is an ellipse with both foci at the center. As the foci move off the center and toward the vertices, the ellipse becomes more eccentric as measured by the ratio $e = c/a$. (In everyday life, we say that a person is eccentric if his or her behavior is off-center—that is, if it deviates from the norm or central tendencies of behavior.)

Figure 6a shows the plot of the $(e, b/a)$ ordered pairs. Because

$$b = \sqrt{a^2 - c^2},$$

the aspect ratio b/a can be expressed as a function of the eccentricity e as follows:

$$\frac{b}{a} = \frac{\sqrt{a^2 - c^2}}{a} = \sqrt{\frac{a^2}{a^2} - \frac{c^2}{a^2}} = \sqrt{1 - e^2}$$

If this equation were written in terms of x and y, it would be

$$y = f(x) = \sqrt{1 - x^2}.$$

So it is interesting that the points $(e, b/a)$ lie on the unit circle. Because a, b, and c are nonnegative, **figure 6b** restricts the graph of

$$\frac{b}{a} = \sqrt{1 - e^2}$$

to the first quadrant portion of this circle.

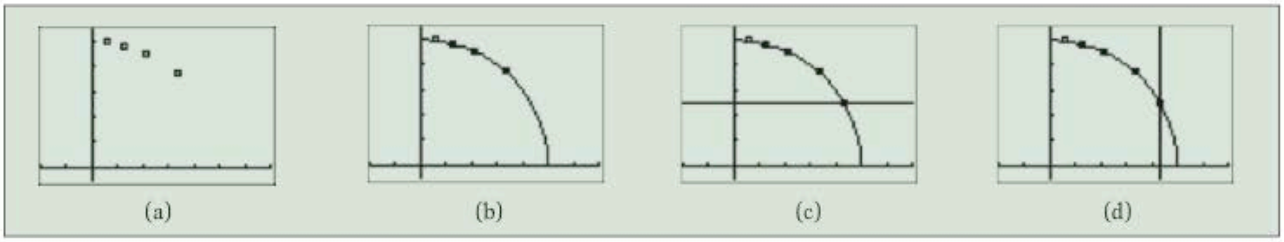


Fig. 6 The aspect ratio b/a as a function of the eccentricity e. Screen shots show a scatter plot of the $(e, b/a)$ data in the window [–0.4, 1.4] by [–0.1, 1.1] (**a**); the graph of $b/a = \sqrt{1-e^2}$ interpolating the $(e, b/a)$ data (**b**); the same graph with the added line $b/a = 1/2$ (**c**); and the graph of the line $e = \sqrt{3}/2$ (**d**).

Figures 6c and **6d** illustrate that for the ellipse to have an aspect ratio of 1/2, the eccentricity must be $\sqrt{3}/2 = 0.866$. Using this eccentricity value and the fact that our loop of string is $2a + 2c = 20$ cm yields the following system of equations:

$$\begin{cases} 2a + 2c = 20 \\ c/a = \sqrt{3}/2 \end{cases}$$

Solving this system reveals that to produce an ellipse with a 1:2 aspect ratio using our 20-cm loop of string, the pushpins must be separated by a distance of

$$2c = \frac{20\sqrt{3}}{2+\sqrt{3}} = 9.282 \text{ cm}.$$

This distance is nearly half the string's 20-cm length. The pushpins, and hence the foci, are only 0.718 cm from the vertices of the ellipse but 4.641 cm from its center—more than five times as far! This seems a bit extreme for an ellipse that is only twice as long as it is wide. As a further example, the orbit of Halley's comet is roughly 36 AU (astronomical units) long and 9 AU wide—a 1:4 aspect ratio—and its eccentricity is about 0.97. This analysis leads us to the following conclusion: The foci of an ellipse must be extremely off-center for the ellipse to be significantly elongated.

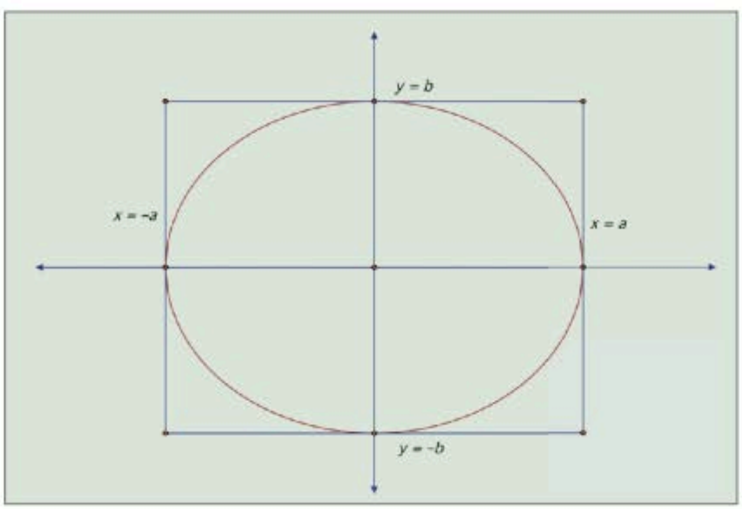


Fig. 7 Every ellipse has an associated rectangle with the same aspect ratio.

The pushpin-and-string construction activity helps students get a feel for the geometric definition of an ellipse. It helps them gain insight into the geometric and numerical relationships among a, b, c, e, and the aspect ratio b/a. This hands-on, intuitive work should be done before considering various possible equations for an ellipse.

SKETCHING AN ELLIPSE BY USING A GUIDING RECTANGLE

A quick activity that can help students draw accurate, well-rounded ellipses involves simple curve sketching. This sketching can be done on unlined paper or on graph paper, with the aid of a straightedge. To move students one step closer to studying the coordinate equations for an ellipse, have them begin by creating a set of coordinate axes. Next, they should carefully draw a $2a \times 2b$ rectangle using segments of the line $x = \pm a$ and $y = \pm b$ (see **fig. 7**). Last, have them draw a pleasingly plump ellipse inscribed in and tangent to this guiding rectangle.

Ask students what would happen to the diagram in **figure 7** if $a = b$. What would we get if we uniformly stretched (dilated) a circle in the direction of one of its diameters? At this point, discuss with students how the standard derivation of the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

can be used to establish algebraically this important transformational relationship between the unit circle $x^2 + y^2 = 1$ and the ellipse $(x/a)^2 + (y/b)^2 = 1$. Students who are familiar with the transformation of the graphs of functions may find it easier to compare

$$y = \pm\sqrt{1-x^2} \text{ and } y = \pm b\sqrt{1-(x/a)^2}.$$

It follows from this dilation argument that the area enclosed by an ellipse is given by $A = \pi ab$. Moreover, just as a circle encloses $\pi/4$ of the area of a circumscribing square, an ellipse encloses $\pi/4$ of the area of its associated rectangle.

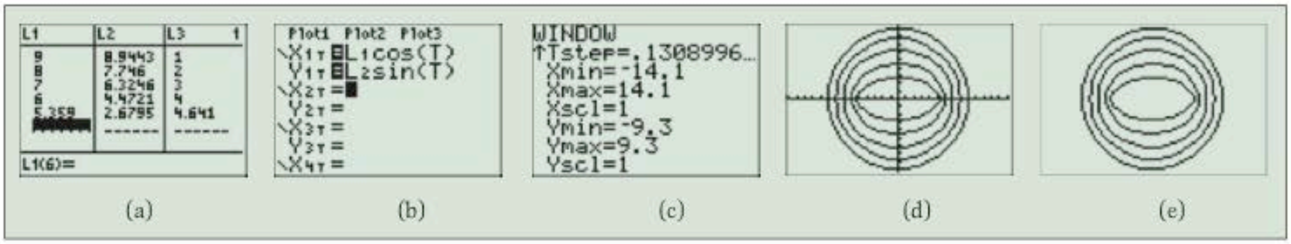


Fig. 8 This sequence of screen shots shows the setup **(a–c)** and the results **(d–e)** of graphing ellipses using parametric equations.

REPRESENTING AN ELLIPSE WITH PARAMETRIC EQUATIONS

The parametric equations for an ellipse provide another nice way to help students see ellipses as generalized circles. Using graphing calculators set in parametric mode and a decimal viewing window, students can graph the following where a and b are various positive constants and where $0 \le t \le 2\pi$:

- $x = \cos t$, $y = \sin t$ to obtain the unit circle
- $x = a\cos t$, $y = a\sin t$ for a general circle
- $x = a\cos t$, $y = b\sin t$ for a general ellipse

As a tie-in to the pushpin construction activity, use the data from **table 1** together with the data for the ellipse with a b/a ratio of 1/2 to model the set of ellipses produced earlier by hand. As suggested by **figures 8a–c**, have students enter the values for a and b as lists $\mathbf{L_1}$ and $\mathbf{L_2}$ and then graph the equations $x = L_1\cos t$ and $y = L_2\sin t$ where $0 \le t \le 2\pi$ and the **t-step** is $\pi/24$. The resulting graphs show that the ellipses we constructed earlier form a nested set of increasingly eccentric ellipses.

Polar coordinates provide another approach to the geometry of ellipses—the approach most often used by astronomers. This extension is explored in an appendix posted at www.nctm.org/mt004.

Although of ancient origin, conic sections are as useful as ever. The study of ellipses is dead only if we teach it in a lifeless way.

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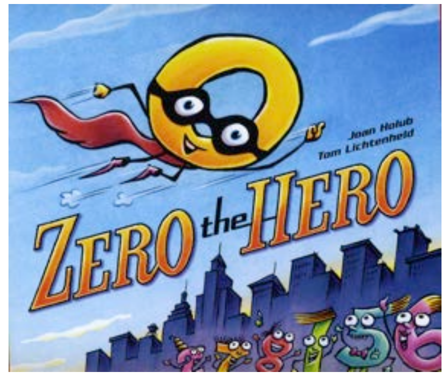
GREGORY D. FOLEY, foleyg@ohio.edu, is the Robert L. Morton Professor of Mathematics Education at Ohio University in Athens. A mathematics educator and textbook author, he advocates for inquiry-based methods and innovative uses of technology for mathematics teaching and learning at all levels, especially in grades 6–14.

For more background on ellipses and polar coordinates by Foley, go to **www.nctm.org/mt004.**

Download one of the free apps for your smartphone. Then scan this tag to access **www.nctm.org/mt004.**

An Interview with Author Joan Holub

Debbie Crocker, Appalachian State University



Joan Holub is an author and illustrator who lives in Raleigh, North Carolina. Her new picture book, *Zero the Hero,* is due to be released in late February. She was available to speak with *The Centroid* about the book and how she became an author of children's books with mathematics themes.

Editor: Tell the reader a little bit about how you became an author, and your background in children's books, particularly ones with a mathematics theme.

Joan Holub: My first clue that my destiny might be different than anticipated is when my high school math SAT score proved to be the same as my English/reading score. I'd always considered myself a writer and artist. How could math possibly figure into that equation? Turns out that Writing + Math = *Zero the Hero*.

I got my start in publishing as an associate art director in children's trade books at Scholastic, where I worked with some wonderful editors, authors, and illustrators, and learned how books are put together. Since 1995, I've written and/or illustrated about 135 books, including one other math picture book, *Riddle-iculous Math*.

Editor: Where did you get your idea for the book *Zero the Hero*?

Joan Holub: On a visit to a first/second grade classroom in the San Francisco area. The teacher is a friend, who mentioned that she used the Zero the Hero concept to teach place-holding to her students. I immediately visualized Zero as an underdog math superhero, complete with cape. But it took me about five years to actually finish writing a book to go with the title *Zero the Hero*.

I owe a lot to illustrator Tom Lichtenheld, who really brought Zero and his number pals to life as characters. Early on, he gave me this birdhouse that he made with his niece and painted with a *Zero the Hero* theme. This was the first color sketch I had seen of the Zero character (which evolved to look a little different in the book) and I was really wowed.

Editor: Can you give a brief summary of *Zero the Hero*?

Joan Holub: I keep thinking of Seinfeld's "show about nothing" because in a sense, this is a book about nothing. As in *Zero. Zip. Zilch. Nada.* That's what all the other numbers think of Zero. He doesn't add anything in addition. He's of no use in division. And don't even ask what he does in multiplication. (Hint: Poof!) But Zero knows he's worth a lot, and when there's big trouble, he swoops down just in time to prove that his talents are innumerable.

Editor: How would you like the book to be used in classrooms?

Joan Holub: To entertain teachers and kids! Oh, and also to teach place-holding and arithmetic operations like addition and subtraction. I also slipped in stuff like rounding up and down, even/odd numbers . . . and perhaps most importantly, I noted how much the number eight looks like a snowman.

Editor: When will the book be available and how can people purchase it?

Joan Holub: *Zero the Hero* will zoom into bookstores February 28, 2012. It's published by Henry Holt and Company. It's also a Junior Library Guild selection, and will be available in stores, both traditional and online.

Editor: Is there any other information you would like teachers to know about the book?

Joan Holub: There's an short about *Zero the Hero* that's lots of number fun. Tom and I wrote the script for it, our publisher had it produced, and we hope everyone will take a look! It will be available soon.

Editor: Do you have plans to write another children's book with a mathematics theme?

Joan Holub: Gr**eight** idea! After all, there are nine more numbers, who all have their own stories to tell.

If you would like a copy of *Zero the Hero*, you can find it at amazon.com, bn.com, or through an independent bookseller <http://www.indiebound.org/indie-store-finder>. You can visit Joan Holub at

- her blog <http://joanholub.blogspot.com/>,
- website <http://www.joanholub.com/>,
- or on Facebook <http://www.facebook.com/people/Joan-Holub/1380038793>.

References:

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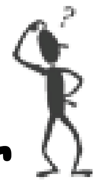
Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. The mini-grants are awarded by each of the three regional organizations to members ***within their geographic boundaries***. A total of $15,000 is available each year for mini-grants, with each region awarding approximately $5000 in grants to its members. In recent years, approximately 30-35 proposals have been funded, for an average grant of just less than $800.

Directions

The application is available on the NCCTM website <http://www.ncctm.org>. Read all directions carefully, and fill out the application and cover sheet completely. Failure to correctly list the NCCTM region and membership number will cause your application to not be considered. Grant proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. You will receive an email confirmation of receipt of your proposal. If you do not receive a confirmation within one week, follow up with the Mini-grant Coordinator. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement.

Problems to Ponder



Spring 2012 Problems

Holly Hirst, Appalachian State University

Grades K–2: Madison has earned 5 credits for completing her chores. Her dad lets her trade her credits for things each month: 1 credit – one candy bar; 2 credits – one song from iTunes; 3 credits – one milkshake; 4 credits – one video game rental. What are all the different combinations of things can Madison get using all 5 credits?

Grades 3–5: Byron's mom parked her car in the town's pay parking lot from 11:30 AM to 6:45 PM. The parking lot sign is shown below. How much will she pay?

PARKING RATES	
First hour	$ 1.50
Each additional half hour	$ 0.50
Maximum daily rate:	$10.00

Grades 6–8: Here is a math riddle: Every birthday of my life, my cake contains my age in candles. Starting on my fourth birthday, I have always blown out all my candles. Before that age, I averaged a 50% total blowout rate. So far, I have blown out exactly 375 candles. How old am I?

Grades 9–12: For how many values of a does the system $\{x^2 - y^2 = 0, (x-a)^2 + y^2 = 1\}$ have exactly 3 solutions?

Directions for submitting solutions

1. Neatly print the following at the top of each solution page:
   - Your full name (first and last)
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page.

Students who submit correct solutions will be recognized in the next issue of The Centroid. We wish to publish creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission.

Proper acknowledgement is contingent on legible information and solutions.

Send solutions by 1 June 2012 to:

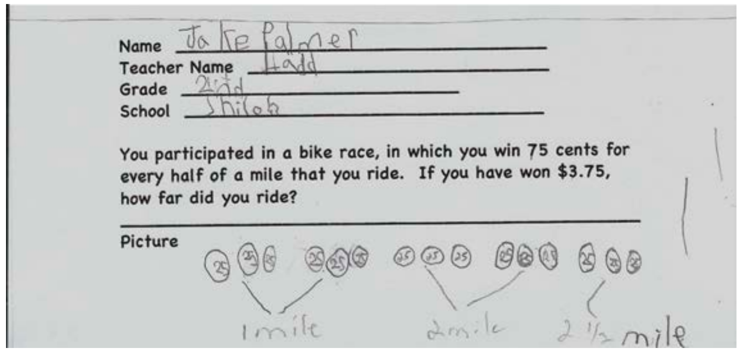
Problems to Ponder, c/o Dr. Holly Hirst
BOX 32068 Appalachian State University
Boone, NC 28608

As these problems are intended to stimulate independent thinking, it is expected that a submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

SOLUTION: Grades K–2 Fall 2011 Issue

You participate in a bike race, in which you win 75 cents for every half mile you ride. If you have won $3.75, how far did you ride?

Example Correct Solution by Jake Palmer (Grade 2 at Shiloh Elementary (Mrs. Ladd)).



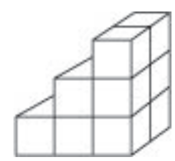
Correct Solutions were submitted by

- **New Salem Elementary School** (Mrs. McGarvie): Will Brooks; (Mrs. Maulden): Dylan Little; **New Towne Elementary School** (Mrs. Tebou): Aiden Boyd; **Shiloh Elementary School** (Mrs. Ladd): Aiden Gallagher, Ashley Gordillo, Dakota Arnold, Genaro Macchiavello, Gina Marcell, Isaiah Robinson, Jake Palmer, Morgan Goodwin, Riley Fecker

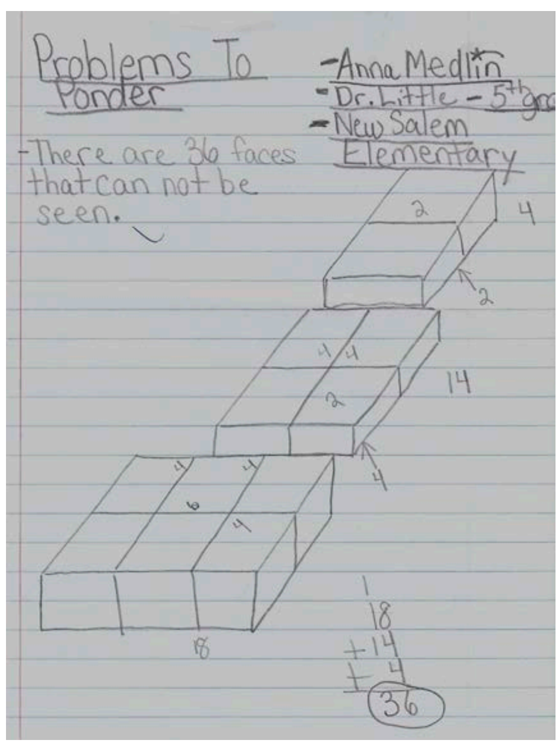
Editor's Note: Some students reported that the answer was five half-miles, while others were able to convert that answer to the more usual way of stating the distance.

SOLUTION: Grades 3-5 Fall 2011 Issue

Twelve cubes are used to create the shape pictured. How many of the cubes' faces can NOT be seen from outside of the shape?



Example Correct Solution by Anna Medlin (Grade 5 at New Salem Elementary (Dr. Little)).



Correct Solutions were submitted by:

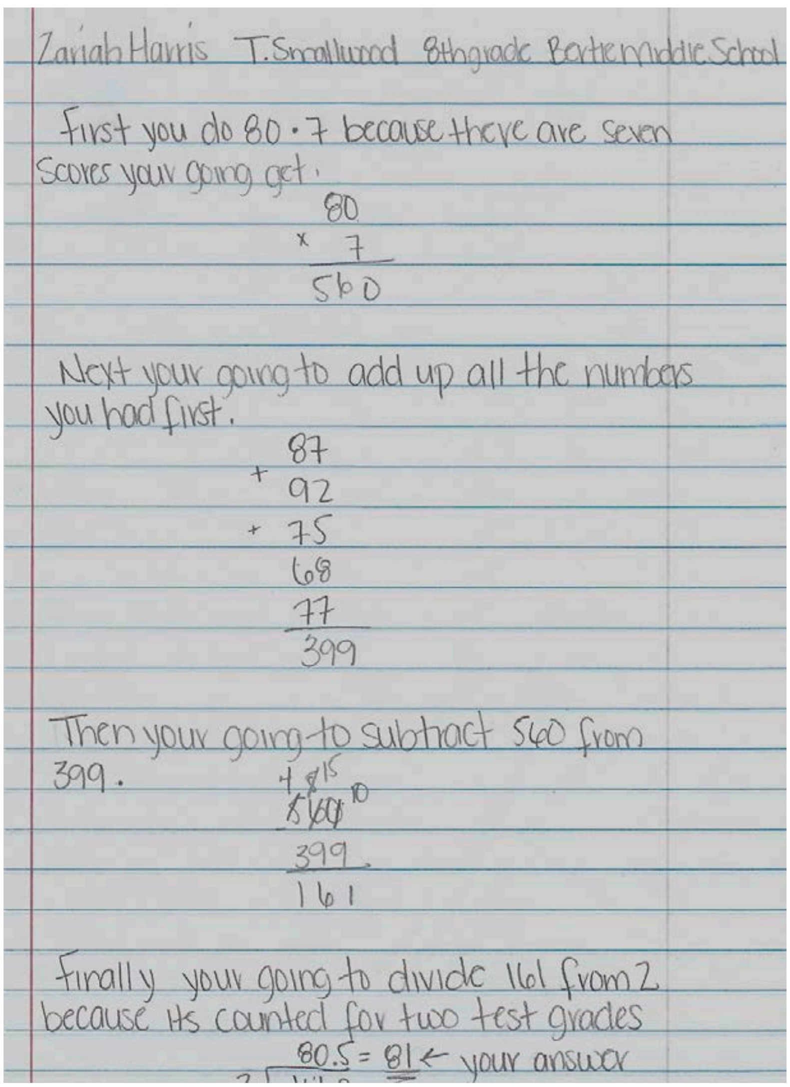
- **Indian Trail Elementary School** (Mrs. Murphy): Matthew Li; **Kensington Elementary School** (Dr. Baron): Patrick Larrik; **New Salem Elementary School** (Dr. Little): Anna Medlin; **Ravenscroft Elementary School** (Mrs. Baccus and Mrs. Byrne): Stephen Butters; **Wesley Chapel Elementary School** (Ms. Hafliger): Akul Patel, Ava Mikeal, Jonathan Nicholson

Editor's Note: I intended the solution pictured above. However, in examining all of the submissions, I found that the notion of face should have been more carefully explained. There are 18 hidden faces if you think of the face "between" two blocks as being one face. Thus I accepted both 18 and 36 as the correct solution.

SOLUTION: Grades 6-8 Fall 2011 issue

You have earned five grades so far in your math class, and you have one more major test remaining. Your grades are 87, 92, 75, 68, and 77. You want to receive a final grade of 80 (B) and the final test counts twice in your average. What grade to do you need to make on the final test?

Example Correct Solution by Zariah Harris (Grade 8 at Bertie Middle School (Mrs. Smallwood)).



Zariah Harris T.Smallwood 8thgrade Bertie Middle School

First you do 80 · 7 because there are seven
Scores your going get.

$$\begin{array}{r} 80 \\ \times\ 7 \\ \hline 560 \end{array}$$

Next your going to add up all the numbers
you had first.

$$\begin{array}{r} 87 \\ +\ 92 \\ +\ 75 \\ 68 \\ 77 \\ \hline 399 \end{array}$$

Then your going to subtract 560 from
399.

$$\begin{array}{r} 560 \\ 399 \\ \hline 161 \end{array}$$

Finally your going to divide 161 from 2
because its counted for two test grades

80.5 = 81 ← your answer

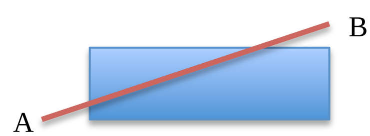
Correct Solutions were received from

- **Bertie Middle School** (Mrs. Bayley): Davon Tucker, Jason Cowan; (Mrs. Carlton): Andrew Hoggard, Ashlynn Lee, Chris Cherry, Zephia NewKirk; (Mrs. Jefferson): Marcus Philyaw; (Ms. Lowman): Jeana Jernigan; (Mr. Orbita): Armani Speller, Brianna Sessoms, Dyasia Smallwood, Emma Hughson, Justin Robinson, La'Shanna Norman, Ricardo Rodriguez, Sandrika Freeman, Shakira Bond, Tauheeah Jawharah; (Mr. Ruffin): Carlisa Spivey, Josh Villamor; (Ms. Sauls): Courtney Pugh; (Mrs. Smallwood): Ahija Spivey, Daijha Perry, Deandre Watson, Jharnell Cofield, Kiana Morgan, Laneice Phillips, Maria Villamor, Tahjanay Purvis, Tionna Ryan, Zanah Harris; (Mrs. Tyson): Bryce Orbita, Daniel Parker, Takia Outlaw;
- **South Asheboro Middle School** (Mr. Cagle): Alexis Currie, Amber Moon, Caleb Cranford, Emily Johnson, Jasmin Far, LaDasha Howie, Perla Pinales, Reagan Thompson, Savannah Myrick, Shawn Allen; (Ms. DeCoeur): Candace Church, Jonathan Beach, Maria Castro, Simon Foy, Wesley Chilton; (Mr. Hynd): Allison Kauffman, Amar Singh, Aranza Gallegos, Carlos Chavez, Dylan Allen, Dylan Hoffman, Emma Nunn, Maia Antoniou, Matthew Swaney, Matthew White, Moses Speight, Jr., Samantha Gimenez, Tatiana Chavez, Vivian Spencer; (Ms. Runnfeldt): Averi Ridge, Erin Devilbiss, Jaden Skelly, Logan Brinkley; (Ms. Salamone): Alicia Rianno-Martinez, Katie Davis, Luke Lamason, Megan Cornwall, Noah I.; (Ms. Thomas): Abbie Warsham, Abigail McBride, Adam Gross, Alicia Peterson, Bailey Allgood, Connor Criscoe, Hannah Ferguson, Hunter Morgan, Josiah Clark, Katie Cernava, Lindsey Farmer, MacKenzie Hammer, Parris Brown, William Aquino

Editor's Note: Many students reported 80, and appear to have found this solution by the correct means, but improper rounding. Those who checked their answer by averaging the final seven grades caught their mistakes.

SOLUTION: Grades 9-12 Fall 2011 Issue

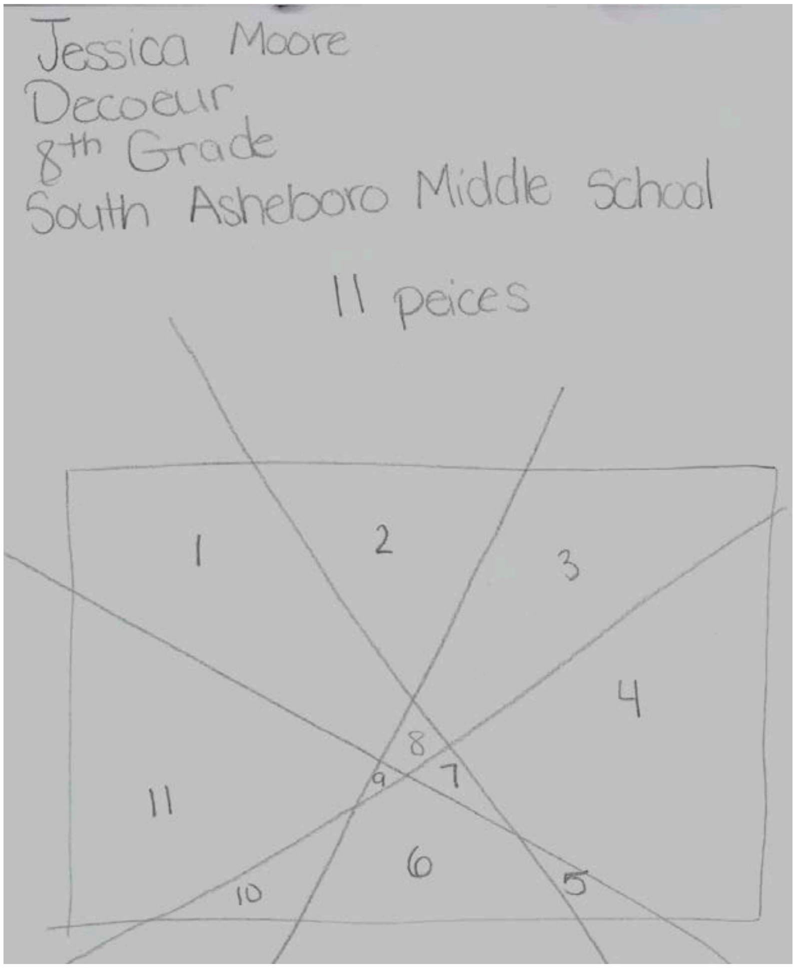
In the rectangle below, the segment AB divides the rectangle into two pieces. What is the maximum number of pieces that can be produced with four line segments drawn through the rectangle?



Example Correct Solution by Jessica Moore (Grade 8 at South Asheboro Middle School (Ms. DeCoeur)).

Correct Solutions were received from

- **South Asheboro Middle School** (Ms. DeCoeur): Jessica Moore; (Ms. Thomas): Hannah Brown



Editor's Note: Both of these middle school students provided the answer by showing the cutting line segments; I challenge all students to try to explain why there is not a clever way to move one of the line segments in the picture above to get more than 11 pieces.

NCTM High School Math Institute

With today's economic and workforce challenges it's more important than ever that students get the right tools to succeed; still, many of our students lack the crucial reasoning and sense-making skills necessary to apply mathematics to other contexts. NCTM's Interactive Institute on High School Mathematics, is a new event designed to give you strategies for creating a high school classroom rich with reasoning and sense making. The institute will kick off in Los Angeles, California, July 24-26, 2012, followed by year-long online professional development. For more information visit us online at:

<http://www.nctm.org/reasoning>

Awards

2011 Outstanding Mathematics Education Award Winners

Reported By: Bampia A. Bangura, North Carolina A&T State University

Each Fall NCCTM sponsors the selection of outstanding mathematics education students. The Special Awards Committee requests nominations from all North Carolina colleges and universities with teacher preparation programs. All nominees receive a certificate and a one-year membership in NCCTM. Top award winners are recognized at the Awards Program at the State conference in the fall. The winners also receive a plaque and a check for $100.00. The recipients of this year's awards are: ASHLEY R. HARRELL from East Carolina University in the Eastern Region, KATHRYN HUFFMAN from Elon University in the Central Region, and CAITLIN YENCHA from Western Carolina University in the Western Region. The recipients are pictured with Richard Haworth, Professor Emeritus, Elon University.

ASHLEY R. HARRELL is a double major with a BS in Mathematics Education and a BA in Mathematics at East Carolina University. Miss Harrell is very active in mathematics and mathematics education activities at ECU. She is a member of the NCCTM student chapter and co-presented at the 2011 NCCTM Fall Conference. She plans to graduate in three years, in May 2012. Ashley is an ECU Mathematics Teaching Fellow. In her essay "Why I want to be a High School Mathematics Teacher," Ashley stated that she chose mathematics because she loved it. She has a passion for mathematics. She liked Mathematics because it is one of the few things in life that will always make sense.

In addition to her many mathematics related activities, Ashley also participated in several community/ Civic activities: She works as a Resident Advisor for ECU Campus Living; she serves as Team Captain for Relay for Life at ECU; she tutors students in multiple subjects; and she volunteers at the Greenville Community Shelter. One of her professors described her as a truly amazing young woman who is committed to becoming a great Mathematics Teacher. CONGRATULATIONS ASHLEY.

KATHRYN HUFFMAN is a senior at Elon University who is pursuing a BS in Middle Grades Mathematics and Social Studies, on track to graduate in May 2012. Like many before her at Elon, Kathryn has been involved in many aspects of the mathematics and mathematics education program. Kathryn is member of the NCCTM Student Affiliate at Elon; She is member of the NCTM, and attended the NCCTM 2010 Fall Conference. Kathryn served as a Teaching Assistant in ELON 101, a freshman seminar, for two years. She served as a math tutor at Western Alamance Middle School and served as facilitator in the Alamance Youth Leadership. Kathryn is Dean's List Recipient.

Kathryn also has extensive involvement in community/civic activities at Elon. In 2010 she was Co-President of the Methodist fellowship, and she is currently the President of the Methodist Fellowship. Kathryn is described by one of her professors as a very positive and reflective personality; a role model for all teachers in that she loves learning, teaching, and service to others. Her leadership skills are exemplary. CONGRATULATIONS KATHRYN.

CAITLIN YENCHA is a senior at Western Carolina University completing a BS in Mathematics with a concentration in Education. Caitlin is very active in mathematics and mathematics education activities at Western Carolina. She served as office assistant for the WCU Math and Computer Science Department for a year; she assisted in the WCU Annual High School Mathematics Contest for two years; she presented on the history of mathematics at the Smoky Mountain Undergraduate Research Conference in

2010. Caitlin is a member of the NCCTM student chapter and served as president for a year. She has attended the NCCTM Fall for three years.

In addition to her Mathematics related activities, Caitlin has participated in several other campus activities including events with the WCU North Carolina Teaching Fellows Program, Leadership Council, Class Representative, WCU Open Houses on Saturdays, Dean's Student Advisory Committee, and the WCU Catholic Campus Ministry. Caitlin is described as a mature and scholarly mathematics student who will contribute much to the lives of young people who she will teach. CONGRATULATIONS CAITLIN.

Awards

2011 W. W. Rankin Award Winner: Marilyn B. Preddy

Reported by: Lee Stiff, North Carolina State University

At its 41st Annual State Mathematics Conference held in Greensboro, NC on 27-28 October 2011, the North Carolina Council of Teachers of Mathematics presented Marilyn B. Preddy, formerly a Guilford County Schools Mathematics Teacher and currently an Adjunct Professor of Mathematics Education at the University of North Carolina at Greensboro, with the W. W. Rankin Memorial Award for Excellence in Mathematics Education. The Rankin Award is the highest honor that NCCTM can bestow upon an individual.

Ms. Preddy was recognized for her 30+ years of service as an elementary teacher, a mathematics facilitator, a national consultant, a college instructor, and an author of mathematics books. She has been recognized as Teacher of the Year for Guilford County Schools, Winner of the North Carolina Award for Excellence in Science and Mathematics Teaching, Winner of the Presidential Award for Science and Mathematics Teaching Excellence, and a Fulbright Memorial Teacher.

Her service to NCCTM as a Board Member, Regional and State Secretary, and Convention Services Director along with her contributions to the National Council of Teachers of Mathematics, were key reasons why Ms. Preddy was honored by NCCTM.

Of Ms. Preddy it was said, "She is a consummate professional. Her enthusiasm is contagious, her knowledge is rich, and her integrity is valued by all who know her."

NCMATYC Spring Conference

2012 NCMATYC Spring Conference will be held at Cape Fear Community College – Wilmington, NC

Cape Fear Community College will be hosting the 2012 NCMATYC Spring Conference March 15th – 16th at the downtown campus at 411 North Front Street, Wilmington, NC 28401. For more information:

http://www.cfcc.edu.

Awards

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in the State. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

If you have nominated someone in the past who has not received the award to date, or if you would like to nominate someone new, please submit as much of the following information as possible! Nominations are accepted at any time.

Please submit the following information. Use as many typewritten pages as needed. If possible, attach a vita of the nominee.

- Name of the nominee
- Current position
- Your relationship to the nominee (e.g. principal, co-worker, etc.)
- The nominee's contributions to mathematics education, NCTM, NCCTM, etc. (Please include information on specific offices held and honors received by the nominee.)
- Any information about contributions to the community, teaching, and education that would be of value to the Rankin Award Committee in its deliberations
- Other relevant information
- Letters of endorsement from other colleagues may be included.
- Date of nomination

Nominator* Name
Current position; Business or educational institution
Preferred mailing address; Preferred telephone number

*The Rankin Award Committee reserves the right to use portions of nomination information in the presentation of the award if the candidate is selected.

Send to: Lee V. Stiff
326-D Poe Hall, Box 7801
North Carolina State University
Raleigh, NC 27695-7801
Sending information in the form of emails is okay: lee_stiff@ncsu.edu

Awards

2011 NCCTM Innovator Award Winner: The Mathematics Department at the North Carolina School of Science and Mathematic

Reported by: Pat Sickles, Durham

For over 25 years the Mathematics Department at NCSSM has dedicated considerable time and resources to offering a variety of professional development opportunities for high school teachers across North Carolina. The faculty have led workshops and conference sessions for other teachers in the areas of Pre-calculus, AP Calculus, Advanced Functions and Modeling, Algebra 2, and AP Statistics. Many of these workshops have spanned two to three years with follow-up components that support teachers throughout the school year. They have presented sessions and workshops at regional, national, and state conferences and are recognized nationally for their work. They have developed a variety of materials for math teachers and have shared these through websites. They have also participated in distance learning opportunities for both teachers and students across North Carolina. This has helped NC teachers develop their own mathematical content knowledge and innovative teaching strategies.

NCSSM Faculty accepting the award at the annual NCCTM meeting in October 2011

With a grant from the Carnegie Corporation of New York in 1984 to develop a comprehensive course that would go beyond the conventional "preparation for calculus," they attempted to help students with preparation for a variety of other coursework in mathematics by study topics such as probability, recursion, and finance, including the use of technology using the graphing calculator. In addition to developing a syllabus for the study of these topics, they took on the additional task of developing supporting teacher materials, including data analysis, geometric probability, matrices, and recursion. Along the way, NCTM published three monographs in 1988. Then in 1991, the widely used textbook, *Contemporary Precalculus through Applications*, was published, with new editions following.

Next, during the Calculus reform movement of the 1990's, they teamed with math professors from Duke University to develop a calculus course that included modeling and applications along with new technology. After initial collaboration, they developed their own teaching materials, obtained their own funding from the National Science Foundation, and published the textbook, *Contemporary Calculus through Applications.*

Between 2002 and 2004, in collaboration with representatives from UNC, UNC Charlotte, and the NC Department of Public Instruction they developed the syllabus for the Advanced Functions and Modeling course and followed that up by developing teaching materials and professional development activities for AFM teachers. For the next 5 years, they led summer workshops for AFM teachers in North Carolina using materials they had developed.

Through these publications and curriculum development efforts, the NCSSM faculty have influenced the mathematics high school curriculum taught in North Carolina and the nation.

Awards

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The purpose of this award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state. Further, this program must have been sustained for a period of at least three years. A number of organizations have made significant contributions to mathematics education in North Carolina; the Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations by sending in the form below. Nominations will be retained in the active file for at least three years.

NOMINATION FORM

Name of Nominee: ____

Present Position: ____

Outstanding contributions to mathematics education in North Carolina, which serves as the basis for this nomination:

Additional information that would be of value to the selection committee:

Signature: ____ Date: ____________

Name (print/type): ____

Position: ____

Business or Institution: ____

Address: ____

Phone: Business ______________________ Home: ____________________________

Email: ____

Send to: John Parker
316 West Soundside Road
Nags Head, NC 27959

Donating to the Trust Fund

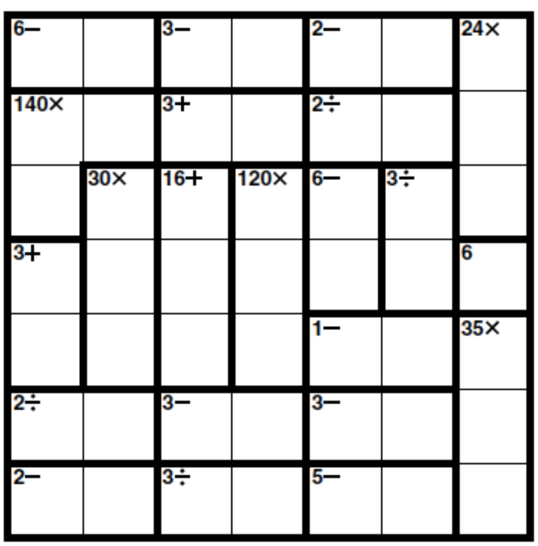
If you wish to memorialize or honor someone important to you through a donation to the NCCTM Trust Committee, please send your donation to:

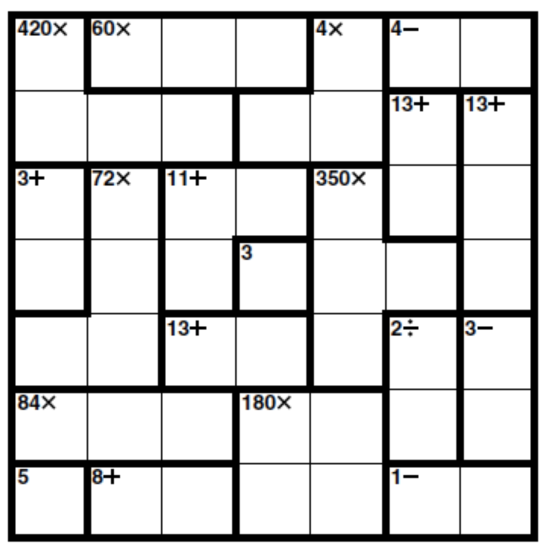
Rebecca Hoover, NCCTM Business Manager
P.O. Box 4604
Cary, NC 27519

Contributions (checks) should be made payable to Pershing LLC for the NCCTM Trust Fund. Please provide the name of the person being honored or memorialized through the donation and the name and address of the person that NCCTM should notify of your gift. For more information, contact John Kolb, Trust Fund Chair.

Puzzles

KenKen: Invented by a Japanese teacher to help students practice arithmetic, the goal is to fill the grid with each digit once in each row and column. Each outlined set – called a cage – lists a target number and the operation to use to obtain the target. Here are two for you to try that use the digits 1 through 7. Notice that each has a one-by-one cage to help you get started.





Solutions are posted on the Centroid page.

NCCTM Trust Fund Scholarship

North Carolina Council of Teachers of Mathematics

$600 scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction.

Applicants must be:

- Currently employed as a pre-K - 12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting this application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are:

- **March 1**
- **October 1**

Send completed applications to:
NCCTM Trust Fund Chair
1302 Oakview Dr.
Greenville, NC 27858

Direct inquiries to:
Robert Joyner, Chair
phone: (252) 756-6803
e-mail: rjoyner3@suddenlink.net

(Please print all information.)

PERSONAL INFORMATION:

Name: ____

Home address: ____
Street

_______________________________, NC _______________
City Zip

Home phone: ______________________ Home e-mail: ____________________

NCCTM membership number: ____

EMPLOYMENT INFORMATION:

How many years of teaching experience? ____________________________

Currently employed in what school system? __________________________

School name: ____

School address: ____

School phone: ______________________ School e-mail: ___________________

Current teaching assignment: ____

Principal's name: ____

COURSE INFORMATION: (One course only)

Institution of higher education: ____

Graduate degree program in which you are currently enrolled: ___________________________________

Course name:____ Course number: ____________

Dates of enrollment: *(circle one)* Fall semester Spring semester Summer session Year: _______________

Name of course instructor: ____

PROFESSIONAL ACTIVITIES WITHIN PAST 5 YEARS WITH EMPHASIS ON ACTIVITIES RELATED TO MATHEMATICS EDUCATION:

BRIEF STATEMENT OF FUTURE PROFESSIONAL GOALS:

REQUIRED SIGNATURES:

Applicant signature: ____ Date: ________

Principal's signature: ____ Date: ________

Instructor signature (if currently enrolled): ___________________________________ Date: ________

REQUIRED ATTACHMENTS:

Please attach a copy of

1. A letter of acceptance to an accredited graduate program in North Carolina;
2. Official verification of enrollment in the graduate course described in the COURSE INFORMATION above if the course is currently being taken, **OR** official transcript containing the grade awarded to the applicant if the course described in the COURSE INFORMATION above has been completed.

NOTE: Applications must be complete to be considered. If your application is approved, an official course grade report must be submitted to verify successful completion of the course before scholarship funds will be issued.

Internal Revenue Information for Grant Recipients: Please be aware that NCCTM is required to report all grants of $600.00 or more to the Internal Revenue Service. In such a case you will receive an IRS Form 1099-MISC from NCCTM. However, you should be able to avoid the payment of any income tax on this. NCCTM has been advised that, if you receive one of the NCCTM grants, you must include the grant proceeds in income unless you made a binding commitment to have the proceeds paid directly to the sponsoring school.

NCCTM Board

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Centroid Ads	Ray Jernigan, jernigan@suddenlink.net
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Math Fair	Debbie Crocker, crockerda@appstate.edu
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NCSSM Representative	Ryan Pietropaolo, rpietropaolo@moc.edu
NCTM Representative	Debbie Crocker, crockerda@appstate.edu
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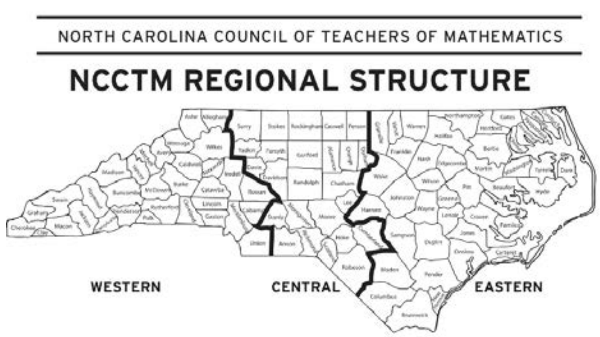
President NC MATYC	Nancy Rivers, njrivers@waketech.edu
Rankin Award	Lee Stiff, lee_stiff@ncsu.edu
Special Awards	Bampia Banguria, babangur@ncat.edu
Student Affilliates	Lisa Carnell, lcarnell@highpoint.edu
Trust Fund	Robert Joyner, rjoyner3@suddenlink.net

Becoming a Member

Follow the “Membership Information” link on the ncctm.org website,
or go directly to: http://www.ncctm.org/members/register.cfm



North Carolina Council of
Teachers of Mathematics
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