The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

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Volume 40, Issue 2 • Spring 2015

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

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Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:

  Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:

  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.

  Chapters or sections of books:

  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:

  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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Thanks to All Who Participated in the 2014 State Math Conference Conference

The North Carolina Council of Teachers of Mathematics 44th Annual State Mathematics Conference was a success! We want to thank everyone who attended, participated, and/or volunteered for helping to make it a wonderful experience.

The following exhibitors provided sponsorship:

- Carnegie Learning, Benefactor Donor
- ETA Hand-to-Mind, Sponsor of keynote speaker David Schwartz
- Houghton Mifflin, Sponsor of keynote speakers Dr. Juli Dixon and Dr. Thomasenia Lott Adams
- McGraw Hill for neck wallets
- Pearson, Benefactor Donor
- Texas Instruments, Sponsor of keynote speaker Wade Ellis

NCCTM's 45th Annual State Math Conference

November 5th and 6th 2015
Koury Convention Center in Greensboro, NC.
'*Principles to Actions*' in Action

Visit http://ncctm.org for more information!

President's Message

State President Debbie Crocker
Appalachian State University, Boone, NC
crockerda@appstate.edu

I hope the spring semester is going well for all of you! I use the term "spring" loosely. I am writing this during a snow day! I know that many of you have experienced this weather as well. I would like to encourage you, your colleagues, and your students to take advantage of all of the events supported by the North Carolina Council of Teachers of Mathematics this spring. Our spring events include the Logo Contest, the Regional and State Math Fairs, the Regional and State Math Contest, the Regional Conferences, and the Spring Leadership Seminar. The dates and deadlines for these events can be found on the NCCTM web site at ncctm.org.

I especially want to encourage you to attend the Spring 2015 Leadership Seminar on March 27th at the Marriott Greensboro Airport. We are fortunate to have Dr. Jo Boaler, Professor of Mathematics Education at Stanford University, as our keynote speaker. We will also have informative breakout sessions, by grade band, facilitated by the Mathematics Consultants from DPI. I know the day will be useful and rejuvenating for all! I hope to see you there!

As the end of April approaches, I will be finishing my two-year term as President of NCCTM. I want to thank everyone who has attended events over the last two years, volunteered their time for the organization, and supported me as I tried to keep the organization moving forward. I cannot name everyone here. I do want to say a special thanks to a few people, without whom I would not have been able to do this. I want to thank Dr. Betty Long, the previous President, for providing me starting information, support, and friendship over this two years. Her contributions and work are too numerous to list. I appreciate her support and input so much. I want to thank Marilyn Preddy for keeping me on schedule and taking care of all the facility arrangements for our events over the last two years. I value her friendship and support. I also want to thank Ray Jernigan, the chair of the Finance Committee for all of his hard work and support. Ray supports NCCTM in so many ways as a volunteer and has been an invaluable friend during my time as President. I also want to thank the Conference Co-Chairs, Dr. Tracey Howell and Dr. Carol Seaman; and the Program Co-Chairs, Dr. Lisa Carnell and Amy Travis. The work and time they contributed to the NCCTM Annual Fall Conference in 2014 and are contributing to the Annual Fall Conference in 2015 is appreciated so much. I want to thank the members of the Board of Directors who served with me during 2013-2015. You have all supported me and helped the organization in so many ways. You were always willing to volunteer your time. I cannot list all of the Committee Chairs and Conference Committee Chairs who have volunteered their time and energy over the last two years. I want to thank all of you for what you have done and continue to do for NCCTM. It has been a pleasure to work with each and every one of you.

This is my last column as the President of NCCTM. I can't believe how quickly the two years passed! It is difficult to express how much I have enjoyed serving as President of such a wonderful professional organization. The position of President comes with a lot of work, but also with a lot of gradification. I have enjoyed all of the interactions with all of you over the two years. I know you will enjoy working with the new President, Dr. Ron Preston. I am sure the next two years will be strong ones for NCCTM and I know that Ron will be a wonderful leader during this time. I wish all of you the best and thank you for all of the support and contributions you have made over the past two years. This is your professional organization. Your support and work make it strong. I hope you will continue to volunteer and participate in the activities of NCCTM! It has been a wonderful two years!

How Two Becomes One: A Recurrence and a Trig Identity

William Bauldry, Appalachian State University, Boone, NC
Matthew Jobrack, Appalachian State University, Boone, NC

The authors investigate an expression containing continuing nested square roots, which leads to a recurrence formula that relates to a trigonometric identity.

The identity provides an intriguing way to evaluate the recurrence. The article includes sample student projects that could be used in precalculus or calculus at the high school and college levels.

"Interdisciplinary math" — in the sense of combining techniques from different areas of mathematics — is very powerful. Trigonometric identities can appear in unusual places and be unexpectedly helpful. An iterated trigonometric identity can produce a recurrence equation. The generated recurrence equation is often then quite easy to understand. We'll look at an example that combines three concepts: continuing nested roots, recurrence equations, and a simple trigonometric identity.

The Formula

Let's investigate the continuing nested square root expression formed with infinitely many $\sqrt{2}$'s.

$$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}} \qquad (1)$$

There are a number of ways to search for evidence concerning the value represented by expressions like (1). One of the first things to do for insight is to gather some numbers. Start by making a table of values.

Table 1: Successive Values of a_n

n	a_n	*Decimal Approximation*
1	$\frac{1}{2}\sqrt{2}$	0.707107
2	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	0.923880
3	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}$	0.980785
4	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$	0.995185
5	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}$	0.998795
6	$\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}$	0.999699

Continuing the pattern in Table 1 would show that $a_{10} \approx 0.999999$. The numbers suggest we're converging to 1, but how can we prove this?

The Trig Connection

Studying the forms in Table 1 leads to a recurrence relation for a_n:

$$a_1 = \tfrac{1}{2}\sqrt{2} \text{ and } a_n = \tfrac{1}{2}\sqrt{2 + 2a_{n-1}}, \text{ for } n > 1 \qquad (2)$$

Notice that a_n looks very much like the half-angle identity: $\cos\left({}^{\theta}/_{2}\right) = \sqrt{\frac{1+\cos(\theta)}{2}} = \frac{1}{2}\sqrt{2 + 2\cos(\theta)}$

Let's iterate the identity: $\cos\left({}^{\theta}/_{4}\right) = \frac{1}{2}\sqrt{2 + 2\cos(\theta/2)} = \frac{1}{2}\sqrt{2 + \sqrt{2 + 2\cos(\theta)}}$

$$\cos\left({}^{\theta}/_{8}\right) = \tfrac{1}{2}\sqrt{2 + \sqrt{2 + 2\cos(\theta/2)}} = \tfrac{1}{2}\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos(\theta)}}}$$

When $\theta = {}^{\pi}/_{4}$, we have $2\cos\left({}^{\pi}/_{4}\right) = \sqrt{2}$. Then we see from the half angle identity that:

$$a_2 = \frac{1}{2}\sqrt{2 + \sqrt{2}} = \frac{1}{2}\sqrt{2 + 2\cos({}^{\pi}/_{4})} = \cos\left({}^{({}^{\pi}/_{4})}/_{2}\right)$$

Continuing to match terms using the iterated identities with $\theta = {}^{\pi}/_{4}$ will yield:

$$\begin{aligned} a_1 &= \cos\left({}^{\pi}/_{4}\right) \\ a_2 &= \cos\left({}^{({}^{\pi}/_{4})}/_{2}\right) = \cos\left({}^{\pi}/_{2^2}\right) \\ a_3 &= \cos\left({}^{({}^{\pi}/_{4})}/_{2^2}\right) = \cos\left({}^{\pi}/_{2^4}\right) \\ &\vdots \\ a_n &= \cos\left({}^{({}^{\pi}/_{4})}/_{2^{n-1}}\right) = \cos\left({}^{\pi}/_{2^{n+1}}\right) \end{aligned}$$

Since ${}^{\pi}/_{2^n}$ goes to zero as n goes to infinity and cosine is a continuous function at $\theta = 0$, we see that a_n must approach $\cos(0) = 1$. The nested roots of 2 have become 1!

The Usual Algebraic Approach

If we assume that a_n does have a limit L as n goes to infinity, then we can use the recurrence formula to calculate the value. Replace a_n and a_{n-1} in (2) with their limit L.

$$L = \tfrac{1}{2}\sqrt{2 + 2L},$$

which generates the quadratic equation

$$4L^2 - 2L - 2 = 0 \qquad (3)$$

This quadratic has solutions $L = 1$ and ${}^{-1}/_{2}$. Since L must be positive, the limit must be 1. While this method appears to be much simpler, we would still need to prove our assumption that the limit exists for the computation to be valid. That's a nontrivial exercise that requires a powerful theorem! (See Project 3.)

Student Projects
We'll close with a set of activities for students.[1]

Project 1*: Experiment with a calculator or spreadsheet to find the first n so that a_n rounds to 1.0.*

Project 2: *What happens when θ is changed to another special angle, such as $\pi/3$?*

Project 3 *(Monotone Project): Prove that the limit of the recurrence (2) exists. (So that we can use the algebraic approach described above.) The Bolzano-Weierstrass Theorem implies that if an increasing sequence is bounded from above, then it must have a limit.*

1. *Show the recurrence is increasing with an induction argument. Base case: $a_1 < a_2$.*

   Induction: If $a_{n-1} < a_n$, then $a_n < a_{n+1}$. (Hint: Multiply $a_{n-1} < a_n$ by 2, add 2, then take square roots.)
2. *Show the recurrence is bounded with an induction argument. Base case: $a_1 < 1$.*

   Induction: If $a_n < 1$, then $a_{n+1} < 1$. (Hint: Multiply $a_n < 1$ by 2, add 2, then take square roots.)

Therefore, by the Bolzano-Weierstrass theorem, a_n has a limit.

Project 4 *: Investigate other trigonometric identities to see if iteration leads to nested roots or recurrence equations.*

Project 5 *: Choose $b > 0$. Experiment with the recurrence relation $x_1 = 1/\sqrt{b}$ and $x_n = \sqrt{\frac{1+x_{n-1}}{b}}$ for $n > 1$, connecting it to the nested square roots:*

$$\frac{\sqrt{b + \sqrt{b + \sqrt{b + \sqrt{b + ?}}}}}{b}$$

1. *Do these continuing nested roots have a limit?*
2. *Are there special values of b that lead to rational limits?*
3. *Is there a pattern?*

Project 6: *Form the product P_N of the a_n's from (2), i.e., $P_N = \frac{1}{2} a_1 a_2 a_3 \cdots a_N$, $N \geq 1$. Using a data table, study the values of P_N and $1/P_N$. Relate your experiments to Viète's infinite product formula.*

Project 7*: Report on MathWorld's article on nested radicals. (http://mathworld.wolfram.com/NestedRadical.html)*

Project 8*: Find the mistake in the following argument. Let $a > 0$, and set $x = \sqrt{a + \sqrt{a + \sqrt{a + \cdots}}}$.*

1. *Show that the recurrence relation for x is $x_1 = \sqrt{a}$, and $x_n = \sqrt{a + x_{n-1}}$, for $n > 1$.*
2. *Using the "Usual Algebraic Technique" from above, assume the sequence of x_n's has limit L, then solve the resulting quadratic for L in terms of the nonnegative root.*
3. *Substitute $a = 0$ into the expression for L obtaining the value $L = 1$.*
4. *Substitute $a = 0$ into the expression for x_0, and conclude that each x_n is 0, so L must be 0.*
5. *Aha! We have $0 = 1$. A surprising new fact! Or …*

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[1] *A spreadsheet to investigate the recurrences is available at http://mathsci.appstate.edu/~wmcb/Recurrences.*

The Racing Game – A Probability Exploration

Holly Hirst, Appalachian State University, Boone, NC

This article provides a brief overview of the sources of curriculum standards related to teaching probability at the K-12 level.

Also included is an activity teachers can adapt for their own use that allows students to experiment with a simple race both by rolling a number cube and through a web-based simulation.

The activity can be used to teach experimental versus theoretical probability and the way probabilities of independent events combine.

For the last three decades, teachers, administrators, mathematicians, statisticians, professional societies, and politicians have been discussing standards for teaching concepts related to probability. The National Council of Teachers of Mathematics (NCTM) addressed the issue in its publication *Curriculum and Evaluation Standards for School Mathematics* (1989) and again in the updated *Principles and Standards for School Mathematics* (2000). The PSSM Data Analysis and Probability standard states in part, "Instructional programs from prekindergarten through grade 12 should enable all students to . . . understand and apply basic concepts of probability" (NCTM, 2000, p. 11).

In 2007, the American Statistical Association (ASA) released the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report,* which built on the NCTM documents by providing a "conceptual framework for K-12 statistics education" (p.5). In 2010, the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA Center) provided specific guidance for probability content via the *Common Core State Standards*, including the following notions in grade 7:

- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. . . .
- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency. . . .
- Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. (§ "Grade 7")

These documents and others provide advice on teaching probability, but many teachers still struggle with developing lessons, especially with regard to helping students observe "long-run relative frequency." The activity that follows can be used to introduce or reinforce the content standards listed above and includes using simulation and drawing tree diagrams to analyze multiple independent events. Suggestions for teachers are embedded in the document in red text. Teachers are welcome to freely adapt the activity for use in their classes. An editable, word document version is available at: http://www.mathsci.appstate.edu/~hph/racing.html

References

ASA. (2007).*Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k-12 curriculum framework.* Alexandria, VA: Author. Retrieved from http://www.amstat.org/education/gaise/

CCSSO & NGA Center. (2010). *Common core state standards initiative: Preparing America's students for college and career. Statistics and probability.* Retrieved from http://www.corestandards.org/Math/Content/SP/

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NCTM. (2000). *Principles and standards for school mathematics: An overview.* Reston, VA: Author.

Activity: Racing on Down the Track

In this activity we will learn about the probability of independent events. We will practice fraction, decimal, and percent skills along the way.

Step One: Playing the game on paper.

Here are the rules for the race game: The class will be divided into teams. You and your team-mates will have a blue and red car to move along the track below. You will roll a standard six-sided number cube.

Race 1: The red car will move when a 1, 3, or 5 is rolled, and the blue car will move when a 2, 4 or 6 is rolled.

Questions:

- Does one car have an advantage over the other?
- Can you tell before rolling who will get to move?
- If red moves first, can you tell what the next move will be?

Let's start. Below is a track. The race is over in one move.

Play this game 12 times and tally who wins each time: Red:_____ Blue:_____

Questions:

- Which car won first? Which car won more often? Was this what you expected?
- Do you think the result would be different if red moves on 1, 2, 3 and blue on 4, 5, 6?
- What fraction represents the proportion of the time the blue car won?

Race 2: Red moves on 1, 3, 5, 6 and blue moves on 2, 4.

Play this new game 12 times and tally who wins: Red:_____ Blue: _____

Questions:

- How often did the blue car win? Was this what you expected?
- What fraction represents the proportion of the time the blue car won?

Stop and share.

Discussion starters:

- Ask each team to share which car won the first race, which car won the most races, and how many times out of 12 the blue car won.
- Help students to notice that the teams did not all get the same results. Introduce the idea of **variability.**
- Combine the team results and discuss the idea that more **trials** leads to more predictable results.

Consider the games below, all with one move to win. Can you make a prediction about how many times out of 12 blue will win on average? Then play the games 12 times and record your actual results.

Race	*Blue Wins – Prediction*	*Blue Wins - Results*
3: Blue moves on 1, 2, 3, 4, 5 and red moves on 6.		
4: Blue moves on 2, 4, 5, 6 and red moves on 1, 3.		

Questions:

- How close were your predictions and results?
- Is it important that blue moves on the particular rolls mentioned or just that blue moves on a given number of rolls?

Stop and share.

Discussion starters:

- Introduce some basic probability ideas:
  - The six different rolls are **equally likely.**
  - The **theoretical probability** is the **expected** fraction of the time, or **likelihood**, an event (such as "blue wins") should occur on average when a large number of trials are conducted. The theoretical probability of an event can often be determined by thinking about all the ways the event could occur.
  - The **experimental probability** is the **actual** fraction of the time an event occurs when we run the experiment.
  - As the number of trials gets much larger, the experimental probability should more consistently approximate the theoretical probability.
  - These numbers can be reported as fractions, decimals or percents.
  - These numbers will always be between 0 and 1, since the least we can win is none of the time (0 out of 12) and the most we can win is all of the time (12 out of 12).

Report your result for the event "blue wins" below as a fraction, a decimal and a percent.

Race	Fraction	Decimal	Percent
3: Blue moves on 1, 2, 3, 4, 5			
4: Blue moves on 2, 4, 5, 6			

Step 2: **Playing the game with lots of trials.**

Now we will use the web version of the racing game to test your guesses on what will happen in races 1 through 4 by running the race 1000's of times. Start the Racing Game with One Die.

http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/

You should see the race on the screen when the applet finishes loading. Practice using this activity by trying the following:

1. Set the race segments to 1.
2. Let red move on 1, 3, 5 and blue move on 2, 4, 6.
3. Press Roll Die, and see who wins. Then press Restart. Try this 5 or 6 times.

Now we are ready to run the race using the multiple trials panel. Be sure the race is set to one segment, and test each of the races using 100, 1000, and 10000 trials, recording the percent of the time that blue wins. Clear the stats between each run.

Race	Expected %	% with 100 trials	% with 1000 trials	% with 10000 trials
1: Blue wins on 2,4,6				
2: Blue wins on 2,4				
3: Blue wins on 1,2,3,4,5				
4: Blue wins on 2,4,5,6				

Stop and share.

Discussion starter:

- Reiterate the idea that the experimental probability tends toward the theoretical probability as the number of trials gets larger.

Step 3: Combining Probabilities and Independent Events.

What happens in a two step race? Try the following three races using the race applet with a two-segment track, and report the experimental percent of the time blue wins with 100, 1000, 10000 trials, clearing the stats each time.

Two Roll Race	100	1000	10000
5: Blue wins on 4, 5, 6			
6: Blue wins on 5, 6			
7: Blue wins on 2, 3, 4, 5, 6			

Questions:

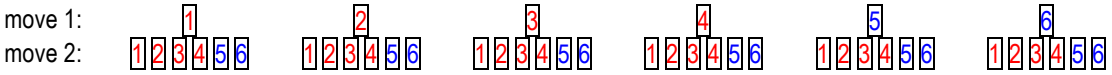
- What do the experimental probabilities approach as the number of trials gets bigger in each case?
- Was this what you were expecting?

Stop and share.

Discussion starters:

- A longer track changes things, at least for races 6 and 7. Consider race 6. In the one segment race, blue wins 2 out of 6 or about 33% of the time. In the two segment race, we seem to be getting an experimental probability of about 26% rather than 33%.
- Introduce building diagrams to show what the possible outcomes are – as shown below.

Let's investigate Race 6. One way that blue could win is B,B (blue moves twice in a row and red doesn't move). What is the probability that B, B happens? Let's color code the possible outcomes.

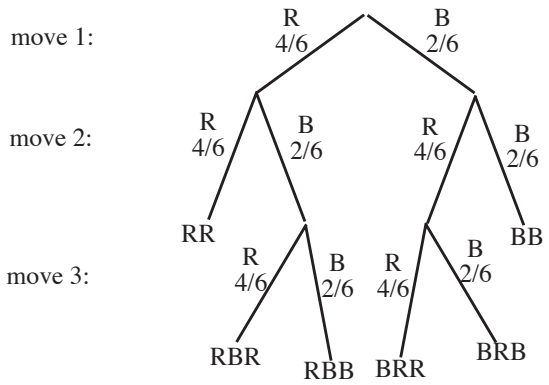


How many of the races have move 1 = B and move 2 = B? Four (5 5, 5 6, 6 5, 6 6). How many results are possible total? Thirty-six. So B,B has a 4/36 probability of happening. In one roll, blue has a 2/6 probability. The probability of two blues in two rolls is 2/6 x 2/6 = 4/36.

What other ways can blue win besides B, B? There are a number of possibilities. We'll organize this as a **tree** of possibilities, keeping track of what can happen at each move, along with probabilities at each move. We end each tree branch when one of the cars has moved twice, winning the race.

Each branch through the tree represents possible ways this race can happen. Let's write them all down, reading from the leftmost branch to the rightmost branch:

- RR: 4/6 x 4/6 = 16/36
- RBR: 4/6 x 2/6 x 4/6 = 32/216
- RBB: 4/6 x 2/6 x 2/6 = 16/216
- BRR: 2/6 x 4/6 x 4/6 = 32/216
- BRB: 2/6 x 4/6 x 2/6 = 16/216
- BB: 2/6 x 2/6 = 4/36



Did we find them all? All of the possibilities taken together should add up to 100%. Do they? Yes! Check it.

Add up the three parts that make up blue winning:

$$16/216 + 16/216 + 4/36 = 16/216 + 16/216 + 24/216 = 56/216 \approx .2593 \approx 26\%.$$

Capstone Exploration: Work on the tree diagram for race 5 and race 7 to see if you can find the theoretical probabilities for blue winning in those cases. Remember that in these races, the probabilities for moving are different!

Extra for Experts: Try the same analysis for races 5, 6, and 7 when there are three segments to the race.

2014 Outstanding Mathematics Education Students

Ashley Alston, Rebekah Currie, and Sara Martin

Reported by Bampia Bangura, North Carolina A&T State University, Greensboro, NC

Each Fall, NCCTM sponsors the selection of three Outstanding Mathematics Education Students, one form each region of NCCTM. The recipients of this year's awards are: REBEKAH CURRIE from East Carolina University in the Eastern Region, ASHLEY ALSTON from North Carolina A&T State University in the Central Region, and SARAH MARTIN from Western Carolina University in the Western Region.

Pictured left to right: Sara Martin, Rebekah Currie, and Ashley Alston

ASHLEY ALSTON is a Mathematics Education major at North Carolina A&T State University pursuing a BS Degree. A Teaching Fellow, Ashley has been very active in the mathematics and mathematics education program at A&T. She has been a Mathematics Department teaching assistant ; she has assisted with A&T's Annual High School Mathematics Contest; she has served as President of the NCA&T Student Affiliate of NCCTM. Ashley has attended the NCCTM Annual State Conference in Greensboro and is also a member of the MAA. Ashley is involved in other campus activities including: member of the Teaching Fellows Council, Welcome Week Ambassador, and member of the Student Union Activities Board.

REBEKAH CURRIE is a double major with a BS in Mathematics Education and a BA in Mathematics at East Carolina University. Rebekah came to ECU with great promise as a Maynard scholar and has more than lived up to that promise. She is part of the honors college at ECU, taking the rigorous honors courses. In addition to making the Dean's list every semester, Rebekah is a member of the Phi Eta Sigma National Honor Society, the Golden Key International Honor Society, and the National Society of Collegiate Scholars. In addition to her leading role in the ECU Student Affiliate of NCCTM, Rebekah is a well rounded person who is very active in the community.

SARA MARTIN completed a BS Degree in Mathematics with a concentration in Secondary Mathematics Education at Western Carolina University. A Teaching Fellow, Sara has attended NCCTM Annual State Conferences, served on the NCCTM Board of Directors as a student representative, and as President of the WCU Student Affiliate of NCCTM. She presented a paper at the Smoky Mountain Undergraduate Research Conference. She was also heavily involved in other campus activities as a member of the music fraternity, including acting as recording secretary and head of community service for four semesters. She marched with the Pride of the Mountains Marching Band for three years.

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Dr. Ronald Preston and Dr. Rose Sinicrope

Reported by Lee Stiff, North Carolina State University, Raleigh, NC

At its 44th Annual State Mathematics Conference held in Greensboro, NC in October 2014, the North Carolina Council of Teachers of Mathematics (NCCTM) presented Dr. Ronald V. Preston and Dr. Rose Sinicrope of East Carolina University with the W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual. Both Dr. Preston and Dr. Sinicrope were high school math teachers before joining the faculty at East Carolina University. Dr. Preston is originally from Indiana and Dr. Sinicrope from West Virginia.

Dr. Preston is universally regarded as an excellent mathematics teacher, a scholar of mathematics education, and as a person whose service to NCCTM and the teachers of NC is exemplary. Dr. Preston's love of mathematics and the desire to enhance the mathematical understanding of students and teachers alike have resulted in successful efforts to bring high quality teaching and learning to North Carolina via such programs as: the North Carolina Project in Improving Mathematics, the Noyce Scholars Program, and the North Carolina Middle Math Project.

Dr. Preston has provided outstanding leadership to NCCTM over many years, having served as the Program Chair of two Annual Conferences, a member of the Board of Directors and the Executive Board, and a Regional President. Moreover, he is the President-Elect of NCCTM. Dr. Preston has been consistent and tireless in supporting mathematics education in North Carolina and mathematics teachers across eastern North Carolina.

Dr. Sinicrope has been described as a person that has a "…passion for mathematics, mathematics teaching, and teaching preservice and inservice math teachers." This passion has resulted in her receiving two "Excellence in Teaching Awards" from ECU. As a scholar, Dr. Sinicrope has published in some of the finest practitioner's journals in mathematics education, including the *Arithmetic Teacher*, the *Mathematics Teacher*, and *School Science and Mathematics*. She was also recognized because of her service to such student-oriented pursuits as MathCounts, and other math contests and math fairs.

Dr. Sinicrope has also provided exceptional leadership to NCCTM over the years, having served as a member of its Special Awards Committee, Trust Fund Committee, and Program Committee for numerous Annual Conferences, as well as a Regional President, member of the Board of Directors and the Executive Board, and a Vice-President for Colleges at the regional and state levels of NCCTM. It is estimated that Dr. Sinicrope has taught thousands of students that have benefitted from her contributions to mathematics education and, as a result, have developed their own interest in and passion for the teaching and learning of mathematics.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Lee V. Stiff, lee_stiff@ncsu.edu.

2014 Innovator Award Winners

Individual: Dr. Drew Polly

Group: American Mathematics Competition (AMC), NC Component

Reported by Todd Abel, Appalachian State University, Boone, NC
and Janice Richardson, Elon University, Elon, NC

The purpose of the NCCTM Innovator Award is to recognize and reward individuals and/or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM.

DR. DREW POLLY has made significant contributions to elementary teachers in the Charlotte region as well as in the entire state. A prolific author and researcher, Drew has published in numerous professional journals, edited books, and made over 100 conference presentations in the past 10 years. Through his work on two Mathematics Science Partnership Grants, he co-coordinated the design, implementation, and management of 80 hours of professional development to over 250 teachers. According to nomination letters: Drew's "expertise and knowledge of mathematics have had a highly beneficial impact on the students, teachers, and leaders in our district. . . .He has worked with the district to create planning and pacing guides, formative assessment items, performance tasks, and units of study. He has co-directed a project to create curriculum units addressing the major ideas of the Common Core Math Standards from Kindergarten through grade 5. He has assisted with the writing of the Unpacking Documents describing the Common Core Math Standards for these same grade levels." "His innovative strategies for engaging both pre-service and in-service teachers in developing content knowledge and extending their repertoire of best practices has impacted countless classrooms across the state."

THE AMERICAN MATHEMATICS COMPETITIONS are the country's longest-running prestigious math contests. Started in 1950 in New York, they expanded into the *American High School Mathematics Examination*, which grew to include the *American Junior High School Mathematics Examination* in 1985. These exams would become known as the *American Mathematics Competitions*, and today there are three exams: AMC8 for students in grades 6-8; AMC10 for grades 9-10; and AMC12 grades 11-12. These exams are designed to emphasize problem-solving, expose students to a wide range of topics, and excite students about mathematics.

Pictured: Randy Harter - Accepting the Innovator Award on Behalf of AMC

A group of dedicated teachers and volunteers have provided this opportunity to thousands of students in North Carolina over the years, and this award honors their work. For many years, statewide efforts were coordinated by Harold and Betty Reiter, and in the last several years, Randy Harter has taken over coordination of AMC8. Over 10,000 students in 150 to 200 different schools across the state have engaged in exciting mathematical problem-solving and interesting mathematics by taking one of these three exams. Those opportunities, along with the several hundred students who have qualified for AIME, the dozens who have participated in the USA Mathematical Olympiad, and several International Mathematical Olympiad participants from North Carolina are the result of the efforts of the these coordinators, teachers, and volunteers who offer their time in service of improving mathematics opportunities for students in here in North Carolina.

Innovator Award Nominations

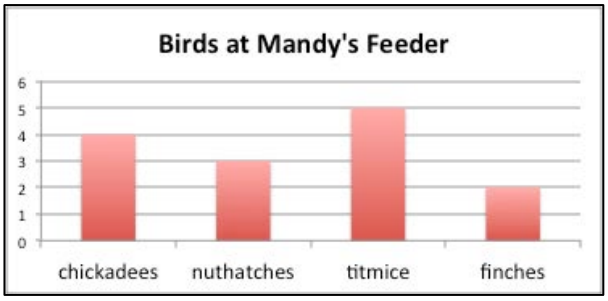
The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Todd Abel, AbelTA@appstate.edu.

Problems to Ponder

Holly Hirst, Appalachian State University, Boone, NC

Spring 2015 Problems

Grades K–2: Mandy has tallied the number of birds that come to her bird feeder at one time and created this bar graph. How many chickadees did she observe at her feeder? How many birds total?



Grades 3–5: Keshia is shopping for new clothes and finds some clothes she likes on sale. The sale is advertised as 30% off the retail price, and she has found the following garments with retail prices given. For which combinations of items does she have enough money if she has a total of $75.00 to spend?

$45 pair of jeans; $30 red blouse; $28 pink sweater; $39 sneakers; $25 sandals; $22 skirt.

Grades 6–8 Two middle schools have the same number of students. The ratio of boys in Lehman Middle School to boys in Dallas Middle School is 2:1 and the ratio of girls in Lehman Middle School to girls in Dallas Middle School is 4:5. Find the ratio of boys in Lehman Middle School to girls in Lehman Middle School.

Directions for submitting solutions:

1. Students: NEATLY print the following at the top of each solution page:
   - Your first name
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page. ***Students who submit correct solutions will be recognized by their first names only in the next issue of The Centroid.*** We will also publish one or two especially creative or well-written solutions from those submitted. If you would rather not have your solution published, please so indicate on your submission.

Proper acknowledgement is contingent on legible information and solutions. Send solutions by 30 June 2015 to:

Problems to Ponder, c/o Dr. Holly Hirst
Mathematical Sciences
BOX 32069 Appalachian State University
Boone, NC 28608

A submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

Fall 2014 Problem Solutions

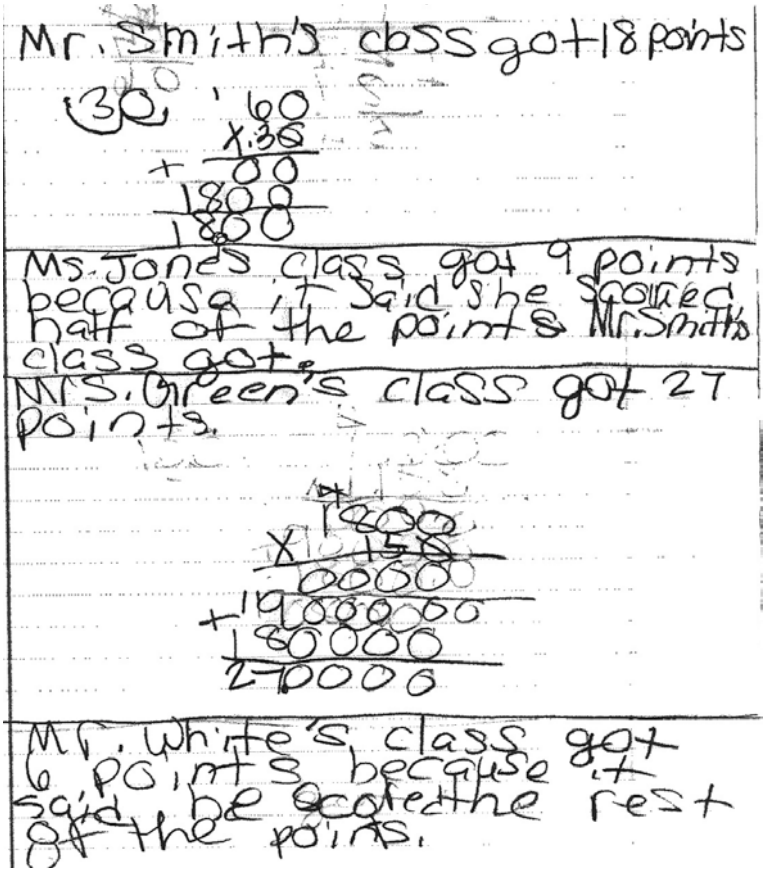
Grades K–2: Sonia, Megan, and Mike have collected sea shells on the beach. They put 5 shells in each pail. Sonia has 2 pails and 3 shells left over. Megan has 1 more pail than Sonia, but no shells left over. Mike has 2 more shells than Megan. How many shells does each person have?

No one submitted solutions to this problem. Here is a way to think about the solution: One pail holds 5 shells, so Sonia has 13 shells (5+5+3). Megan as 15 shells (5+5+5). Mile has 17 shells (15+2).

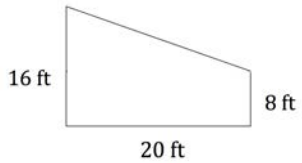
Grades 3–5: Four classes of students competed in field-day events in which it was possible to score 60 points total. Mr. Smith's class scored 30% of the points. Ms. Jones's class scored half as many points as Mr. Smith's class. Mrs. Green's class scored 150% of the points won by Mr. Smith's class. Mr. White's class scored the remaining points. How many did points did each class score?

Many students submitted solutions to this problem, but most of the students had trouble working with the 150% figure. They correctly calculated the number of points earned by Mr. Smith's and Ms. Jones' classes, but could not correctly calculate the number of points earned by Ms. Green's class.

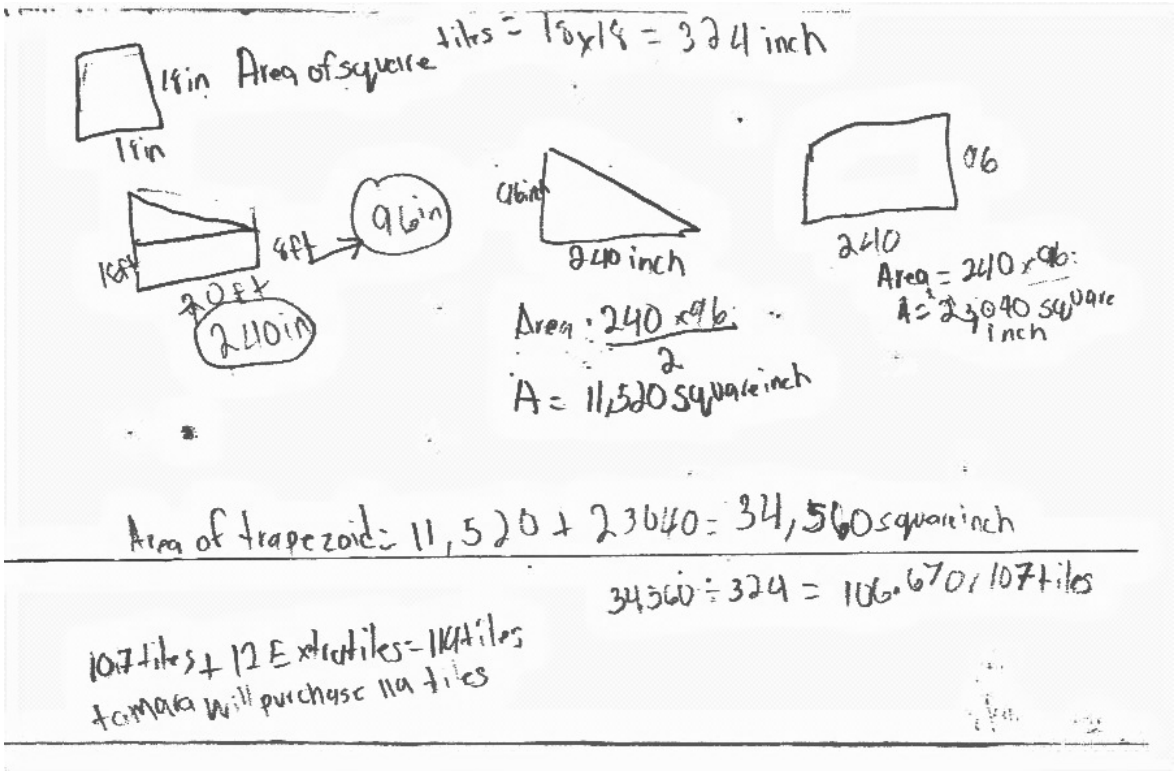
Below is a correct solution from a student in Ms. Brown's 5th grade class at Ahoskie Elementary School (good work Dianna!). In addition 3 students from Ms. Sitnic's 4th grade class got the correct answer (Altariq, Carlos, Mirocal).



Grades 6–8: Tamara wants to figure out how many 18-inch-square floor tiles her parents need to tile their new trapezoidal sun room the dimensions for which are given in the diagram. If purchasing one dozen extra tiles is recommended for replacements, how many tiles total should be ordered?



Many students submitted solutions, but only one student was successful at solving this area problem that mixes inches and feet units. The single correct solution came from Kent in Mrs. Bowser's 6th grad class. Way to go Kent!



There are at least three approaches to attacking this problem:

1. *Work with the areas in square inches, finding the area of the trapezoid and then dividing by the area of the tile – which is what Kent did.*
2. *Take the same approach, but work with square feet.*
3. *Think about the number of whole tiles needed to go across 20 feet (14 tiles) and up 8 feet (6 tiles) to fill the rectangular part of the sunroom (14 x 6 = 84). The triangular part of the sunroom is half as big as the rectangular part, so 48 more tiles would fill that area.*

In all three approaches, add 12 to the answer to account for the required extra tiles.

NCTM Mathematics Education Trust Award

Congratulations to Sabine Raquel Moses, a teacher at Clyda A. Erwin High School.

The Mathematics Education Trust Board of Trustees recently awarded Sabine a Future Leaders Initial NCTM Annual Conference Attendance Award. This award is supported by the Edwin I. Stein Fund and the National Council of Teachers of Mathematics. The purpose of this award is to provide financial assistance to a first-time attendee to the Boston NCTM Annual Meeting and Exposition in April 2015. The MET Board of Trustees congratulates Ms. Moses and Clyde A. Erwin High School for her successful proposal.

The next deadline for applications for MET Grants is May 4, 2015. Please check the website at http://ww.nctm.org/grants/ for information on all grants, awards and scholarships.

2014 Outstanding Secondary Mathematics Teachers

Reported by Kitty Rutherford, NC Department of Public Instruction, Raleigh, NC

Pictured: 2014 Outstanding Secondary Mathematics Teachers

Alamance Burlington: Steve Moore
Ashe: Josh Church
Asheboro City: John Phillips
Avery: Sherman Andrews
Beaufort: Heather Scott
Brunswick: Samuel Jennings
Buncombe: Kimberly Clark
Burke: Cheryl Weaver
Cabarrus: Jennifer McCarthy
Caldwell: Tamara Hines
Carteret: Anna Storms
Charlotte-Mecklenburg: Leslie Paytes
Chatham: Tracey Shaw
Cleveland: Shari Mansfield
Columbus: W.T. Edwards
Craven: Kelly Warren
Cumberland: Phyllis Cannon
Currituck: Diane Davenport
Dare: Wendy Lewis
Davidson: April Coggins
Davie: Yvette Shore
Duplin: Jeri Denise Humphries
Durham: Carla Joyner
East Wake Academy-Middle: Jodi Pearson
Guilford: Lydia Kirkman
Haywood: Deb Jones
Henderson: Tyler Honeycutt
Hoke: Tiffany Huggins-White
Iredell-Statesville: Melissa Calloway
Jackson: Renee Stillwell
Johnston: Karen Mills
Lenoir: Heidi Jones
McDowell: Miranda Ferguson
Moore: Kelly Frey
New Hanover: Taylor Spear
Newton-Conover City: Michael Allen
Onslow: Jennifer Simmons
Pamlico: Pamela Lefler
Perquimans: Kim Tyson
Person: Justin Sackett
Randolph: Donna Callahan
Robeson: Frances Sheats
Rockingham: Tracy Blackard
Rutherford: Lindsay Mays
Scotland: Theresa Houston
Transylvania: Adria Hardy
Wake: Suzanne Gibbons
Watauga: Jessica Smith
Wayne: Angela Lewis
Wilkes: Carol Canter
Winston-Salem/Forsyth: Franklin Wonsavage, Jr.
Yadkin: Kristy Fowler

Presidential Award for Excellence

Congratulations to Meredith Stanley, Kayonna Pitchford, and Heather Landreth, the 2014 state finalists for the Presidential Award for Excellence in Science and Mathematics Teaching.

Pictured left to right: Meredith Stanley, Kayonna Pitchford, and Heather Landreth

NCCTM Board

contact information can be found at ncctm.org

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Special Awards, Bampia Banguria
Student Affilliates, Lisa Carnell
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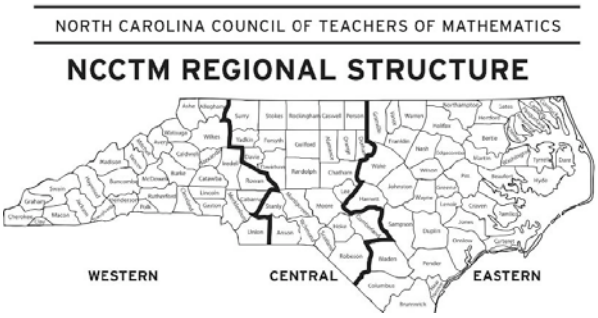
Becoming a Member

Follow the “Membership Information” link on the ncctm.org website, or go directly to:
http://www.ncctm.org/members/register.cfm



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