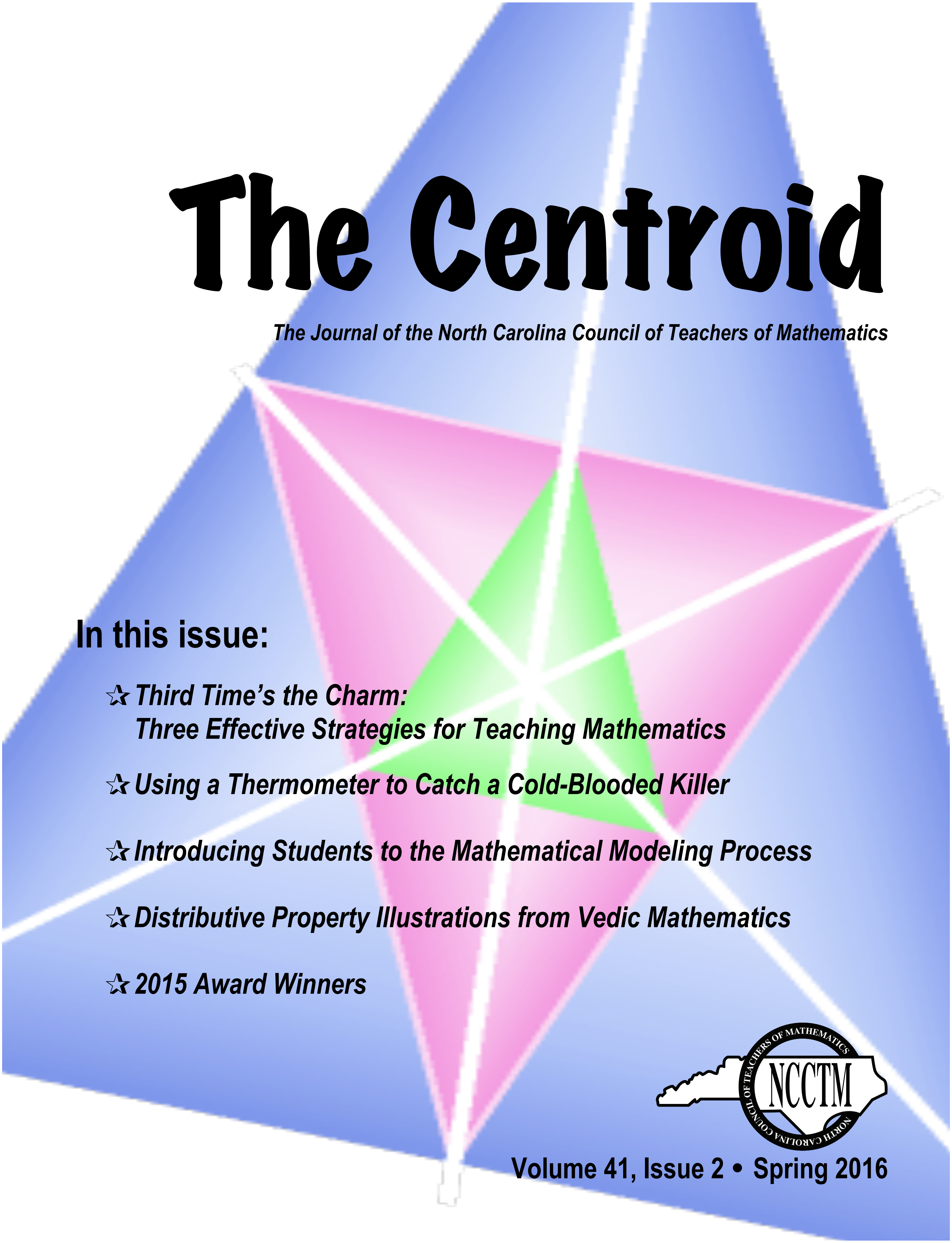

The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

Volume 41, Issue 2 • Spring 2016

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:
  - Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.
  - National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:
  - Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, 7, 196–199.

  Chapters or sections of books:
  - Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:
  - North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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NCCTM Regional Conferences
Western: February 27, 2016
Central: March 19, 2016
Eastern: March 19, 2016

NCCTM's Spring Leadership Seminar
Gates Wide Open in North Carolina: Working Toward Mathematics for All
9-3:30 on April 8, 2016
Marriott Greensboro Airport in Greensboro, NC

NCCTM's 46th Annual State Math Conference
October 27-28, 2016
Koury Convention Center in Greensboro, NC

Visit http://ncctm.org for more information!

President's Message

State President Ron Preston
East Carolina University, Greenville, NC
prestonr@ecu.edu

The State of the Council

Early each new calendar year, the president of the United States delivers the State of the Union Address. It is an opportunity to celebrate successes of the past year, to recognize current challenges, and to look to the future with an agenda for tackling those challenges. So what is the State of the North Carolina Council of Teachers of Mathematics? What have we accomplished recently? What challenges do we face? What should the officers, the board of directors, and the members do in light of those challenges?

When looking back at 2015, we celebrate the fact that noted mathematics educators like Jo Boaler, Doug Clements, Diane Briars, Dan Brahier, Jon Wray, and others visited our state and shared their expertise at the Spring Leadership Seminar, the Fall Leadership Seminar, or the Annual State Conference. We rejoice with Wendy Rich, Rankin Award winner, and The Burroughs-Wellcome Fund, Innovator Award winner. Speaking of winning, over 50 teachers were named elementary mathematics teacher of the year for their county and three preservice teachers made us proud with the accomplishments that won them their region's Outstanding Mathematics Education Student. Throughout the year, we were inspired by our Logo Contest winner, amazed by the top students at the State Math Contest, and impressed by Math Fair winners at the regional and state levels. Teachers won mini grants and some received scholarships for graduate work. Regions hosted their spring conference and several universities had active student chapters of NCCTM. A new and improved version of the website (ncctm.org) was launched, providing more information, a better look, and user-friendly interface for conference registration. I could go on, but our organization is better known for taking on the next challenge than resting on its laurels.

Mathematics teachers in North Carolina encounter a number of challenges on a regular basis. Mathematics textbooks are in short supply. Teacher morale has been higher. Many districts are dealing with high teacher turnover. You may be asked to teach an additional class because your school is short a mathematics teacher. Not all of our students, or groups of students, are succeeding mathematically. We are still transitioning to Common Core State Standards for Mathematics. On top of all these issues, I suspect that most who read this message will think, "he didn't even list some of the most important issues."

The North Carolina Council of Teachers of Mathematics has the ability to make a difference for most, if not all of the above issues. How? NCCTM can make a difference by utilizing the energy, expertise, and enthusiasm of its members, both individually and collectively. The Spring Leadership Seminar brings together national leaders to inspire and equip us to take on the challenge of providing access to high quality mathematics for all students, including students of color, English language learners, and those receiving free and reduced lunch. I highly encourage you to attend the 8 April 2016 leadership event at the Marriott Greensboro Airport. Still transitioning to the CCSSM? Come to our fall conference and see what your fellow teachers are doing to make mathematics come alive for their students. Feeling the pinch of the teacher shortage in your district? Encourage the students in your classes who you believe would make great teachers to, in fact, prepare to be a teacher. When they go off to school, make sure they are aware of the NCCTM student affiliate there. Have colleagues who are great teachers, but losing enthusiasm? Bring them to your regional conference or the fall conference. The energy of our state conference is contagious and is just the type enthusiasm we hope infects all of our fellow mathematics teachers.

Finally, the contests, conferences, seminars, grants, scholarships, and awards are great, but you may be asking, "What can I do on a day-to-day basis to improve mathematics education in NC?" Write a good CCSSM lesson plan and share it. Mentor a young teacher. Help us figure out what the federal Every Child Succeeds Act means for NC. Host an intern. Make a difference for the student everyone else forgot. Recruit somebody to be a mathematics teacher. Share your love of mathematics. In short, be a great NCCTM member!

Third Time's the Charm: Three Effective Strategies for Teaching Mathematics

Courtney Glavich, University of North Carolina – Charlotte, Charlotte, NC

The author describes three teaching strategies that are thoroughly grounded in education research:

- *Probing for student understanding*
- *Using accurate representational forms*
- *Encouraging curiosity and questioning*

She includes a review of research that supports these strategies.

Teaching mathematics to children can sometimes be a daunting and frustrating task. The article, "Understanding, Questioning, and Representing Mathematics: What Makes a Difference in Middle School Classrooms" (Capraro, Capraro, Carter, & Harbaugh, 2010) presents three different teacher quality measures that are likely to improve student understanding in mathematics.

Probing for student understanding is defined in the article as classroom discourse that can be achieved through effective questioning (Capraro et al., 2010). The most common model of discourse in the classroom is the IRE sequence, where the teacher initiates the questioning, a student responds, and the teacher evaluates the response (Capraro et al., 2010).

Using accurate representational forms is described as using concrete representations to help foster abstract mathematical thinking and concepts (Capraro et al., 2010). Mathematical representations are essential to deeper mathematical understanding. In addition, accurate representations are needed to ensure misconceptions are not formed within student learning.

Encouraging curiosity and questioning is defined as asking higher-level questions to ignite students' exploratory behavior (Capraro et al., 2010). Educators can encourage curiosity in the classroom by inviting other ways to solve solutions, guiding the search for students' answers instead of giving them the answer, and modeling the types of conversations and questions that students should be engaging in.

These three components of teaching have been shown to be directly related to higher student achievement, and are discussed in the following articles related to effective mathematical teaching.

Examples of Usage in Classrooms: Review of the Literature

Capraro et al. (2010) describe ways that teachers can make a difference in middle school mathematics classrooms. A study was conducted focusing on the theoretical framework for teacher quality measures (TQM). The three quality measures that were focused on were: probing for student understanding; encouraging curiosity and questioning; and using accurate representational forms.

The TQM probing for student understanding was likely to increase student achievement when a different type of sequence was used. Instead of the initiate, respond, and evaluate sequence, a suggested alternative to evaluating the response (e.g., that is correct, that is incorrect) is to expand or have a conversation about response instead. "Asking more open-ended questions…can contribute to the construction of more sophisticated mathematical knowledge by students" (Capraro et al., 2010, p. 1).

Regarding the second TQM, encouraging curiosity and questioning, the authors explain "Teacher questioning strongly influences students' mathematical learning. A learner-centered classroom encourages students' curiosity and questioning; teachers listen to students' explanations, probe for justifications, and encourage students to share their solutions with peers when working together to refine, revise and extend their solutions" (Capraro et al., 2010, p. 4). Overall, when the two teachers in this study had their teaching coded and analyzed, it was evident that all three of these TQMs had a huge impact on student achievement when used frequently.

In the article "Evolution of a Teacher's Problem Solving Instruction: A Case Study of Aligning Teaching Practice with Reform in Middle School Mathematics" by David Hough (2005), student curiosity in mathematics, as well as probing for student understanding is touched upon. In this case study, focused on a teacher named Bob Fern, problem-solving methods are discussed. It is often debated that students are using problem solving when given a problem with a set of strategies to use to derive the correct answer; however, when students are given procedural methods to solve a problem, it becomes a computational effort and not a cognitive problem solving exercise (Hough, 2005).

In this case study, Bob focuses more on the actual problem solving of the question, and not the computational method. He began by distributing calculators to his students, and became more focused on their student-invented ways to derive the correct answer to a problem instead of focusing on the paper-and-pencil correct way to solve a problem (e.g., giving them a calculator to divide fractions instead of focusing a large amount of time on the procedures for long division). When Bob implemented a problem-solving-focused lesson, he would give his students constraints to meet. For example, Bob posed the question to his students, "If you have 9 coins that are together, worth 58 cents, what are the kinds of coins you can have" (Hough, 2005, p. 7). He gave his students the constraints of having 9 coins totaling 58 cents with no 50-cent pieces. When interacting with his students he probes questions, and uses an open-ended response instead of evaluating his students' responses when asking questions (Hough, 2005). He puts a heavy emphasis on reasoning and problem solving, and wants his students to be able to "devise a plan" instead of using a predetermined method to solve. This case study found that utilizing reflection and classroom dialogue (questioning, igniting curiosity, etc.) could result in higher-student achievement, especially when it came to word problems and problem solving (Hough, 2005).

In "Research Summary: Teaching Fractions in Middle Grades Mathematics" by Z. Ebrar Yetkiner and Mary Margaret Capraro (2009), the authors discuss how teaching fractions in middle school is often fragmented and not taught conceptually. Teachers often teach fractions procedurally and do not rely on visual representations to help promote such an abstract concept (Yetkiner & Capraro, 2009). "In a study of U.S. and Chinese teachers' knowledge of dividing fractions, Ma (1999) found that U.S. teachers relied heavily on the traditional algorithm and lacked conceptual understanding to generate appropriate representations" (Yetkiner & Capraro, 2009, p. 5). When teaching fractions, teachers tend to not encourage students to use their own algorithms to solve and focus on the traditional algorithm. Due to this, students repeatedly struggle with grasping the concept of fractions. "Middle school teachers should also be equipped with the necessary knowledge to help students develop conceptual understanding of fractional concepts such as accurate and appropriate representations for particular purposes, effective questioning techniques to promote multiplicative and proportional reasoning…" (Yetkiner & Capraro, 2009, p. 7).

In the article "Statistics in the Middle Grades: Understanding Center and Spread," by Gary Kader and Jim Mamer (2008), the understanding of statistical information is connected to clear representations of the statistical information. For example, when asked to find the five-number summary of a set of data, students understood the statistical information in greater depth when it was connected to a concrete representation, in this case a box-plot. The set of statistical tasks in this study were all coupled with representations to clearly display the statistical data. "These statistical tasks promote student understanding of various numerical summaries and illustrate connections between various representations" (Kader & Mamer, 2008, pp. 43).

A deeper understanding of mathematical concepts through representations can also be tied to the article, "The Role of Representations in Developing Mathematical Understanding," by Stephen J. Pape and Mourat A. Tchoshanov

(2001). The authors refer to representations as both internal and external. In their study of high school students learning trigonometry, Pap and Tchoshanov discovered that students learned better through multiple representations; the student groups that used purely analytical teaching and were focused on one representation to describe the problem had lower student achievement than those groups who were focused on more than one representation (Pape & Tchoshanov, 2001).

Increasing mathematical thinking through concrete representations is a focus in the article “A Model for Understanding, Using, and Connecting Representations,” by Lisa Clement (2004). Clement describes a structure for teaching mathematics efficiently to students, using pictures as the starting point. Teachers should encourage students to draw their own pictures to demonstrate their understanding (Clement, 2004). Pictures can be easily linked to manipulatives. Manipulatives are defined by base-ten blocks, fraction bars, fraction circles, etc., or any type of mathematical tool that can be physically held or manipulated by the student to engage them in mathematics understanding.

Spoken language is another way to represent mathematical teaching. Teachers should encourage spoken language and vocabulary as a way for students to express their understanding (Clement, 2004). Written symbols are the most abstract of the representations, and therefore should be displayed after students have had opportunities to make connections between the other representations (Clement, 2004). Displaying all of these representations in relevant situations (i.e., word problems, real world-context) can help cement these representational ideas. Once again, when teachers used all five of these representational models to in their mathematical instruction, student understanding increased.

In the paper, “Use of Student Mathematics Questioning to Promote Active Learning and Metacognition,” Khoon Yoong Wong describes how “asking questions is a critical step to advance one’s learning,” (Wong, 2012, p. 1). “Children are naturally curious about themselves and their environment. A natural way with which they try to satisfy their curiosity is to ask questions” (Wong, 2012, p.1). Instead of the traditional sequence of “teachers asking questions, and students answering them,” it is more beneficial for students to guide their learning with the sequence of “students asking questions, and teachers answering them.” In order to achieve this model teachers should pose their lessons as problem-solving questions (Wong, 2012). In order to engage student curiosity teachers should ask higher-level questions and expand wait time in order to encourage deeper responses from their students, as well as move away from the traditional sequence (initiate, respond, evaluate). Teachers generating curiosity within their students’ mathematical thinking results in deeper student understanding.

“Using Questioning to Stimulate Mathematical Thinking,” by Jenni Way (2011), describes how good questioning has been long thought of the most important teaching tool in mathematics; however, research shows that teachers typically use low-order questions to teach mathematical lessons, instead of using higher-order, open-ended mathematical questions (Way, 2011). When teachers enhance their level of questioning, it sparks students’ curiosity and allows them to think critically about math concepts.

In the articles, “Impact of Middle-Grades Mathematics Curricula and the Classroom Learning Environment on Student Achievement” (Tarr et al., 2008) and “Five Factors that Contribute to the Success of Middle Grades Math Teachers in North Carolina’s Most Challenging Schools” (Lemon & Fischetti, 2011), effective teaching is presented as one of the most defining factors in student achievement. It is important to train teachers to practice higher-level thinking questions when teaching lessons or explaining concepts. Tarr et al. (2008) discuss textbooks and how they affect the quality of teaching; however, they state in the article that the textbook curriculum is hindered or supplemented by how the educator chooses to use the textbook (i.e., the types of questions asked, activities that are used with it, etc.). In textbooks, often representations are used to demonstrate math concepts. Clearer representations help engage students and create a pallet of understanding.

Throughout all of these articles, engaging student curiosity, using representations and probing for student understanding are constantly referred to in the different case studies and topics discussed. It is important to use all three aspects in teaching mathematics to ensure higher student engagement and deeper understanding of mathematical concepts, therefore leading to higher student achievement in math.

Reflective Analysis

These three different characteristics of teaching – using correct mathematical representations, probing for student understanding, and encouraging curiosity and questioning for students – have proven effective for higher student achievement. I often use mathematical representations to help foster students' understanding of math concepts. However, since reading this literature, I have decided to incorporate at least one or two mathematical representations in my teaching for each new concept that is taught. For example, we are currently learning about rate of change in my 8th grade math class. When introducing rate of change, we did an activity called "Tap Your Pencil." The students tapped their pencils a certain amount of times each minute. This demonstrated a constant rate of change. We also did an activity where every minute, I would give them a certain number of M&Ms. This concrete representation allowed them to see rate of change, and coupled a representation to an abstract idea.

Probing for student understanding is a vital part of being an effective mathematics teacher. Not only do students need to understand the skill that is being taught, but they also need to know the mathematical reasoning behind why the skill actually works. With the new shift in curriculum to Common Core, probing for student understanding is becoming more vital in classroom instruction. This year, I have started to implement a variety of performance-based tasks and investigations in my teaching. With investigations it is vital that the questions that are being asked to the students are crafted to fit the expected outcomes of objectives. After reading the articles, I have realized that the types of questions that are asked to students are an important part of student understanding. For example, I was reviewing rate of change with my students, and how to determine whether a linear equation was a function or not. We were discussing the vertical line test, where you draw a straight line on the graph to determine whether it is a function or not. However, my students did not seem to understand that listing out a set of ordered pairs and determining if the input repeated, was essentially the same thing as drawing a vertical line through a graph. We had a discussion about why drawing a vertical line through a graph, and the graph passing through it more than once, was essentially the same thing as having repeating inputs in a relation. I realized that a lot of my students did not understand what the purpose of the vertical line test was. Even more so, they did not understand that a vertical lines pass through *x* coordinates. They were confused since a vertical line typically on a coordinate plane signifies the *y*-axis. This discussion revealed misconceptions as well as gaps in understanding.

Encouraging curiosity and questioning is one thing I feel as though I struggle with as a mathematics teacher. I teach at a Title I school, and I often find that there are gaps in students' learning. It is difficult to foster student curiosity and questioning when they feel defeated in math. I sometimes feel as though the curriculum is too abstract for my students in the 8th grade, and it sometimes leads to very frustrating moments. I have learned that I need to acknowledge their successes in math, no matter how small. When students feel that they are successful in math, they tend to be more motivated, thus seeking out answers to questions, and engaging in investigations more actively. Investigations, hands-on activities, centers, and technology driven instruction does an excellent job at encouraging curiosity and questioning. Due to this reason, I will try to incorporate more hands-on actives with my investigations and well as small group instruction to encourage collaborative, student-centered learning in my classroom.

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2015 Innovator Award Winner
The Burroughs-Wellcome Fund

Reported by Todd Abel, Appalachian State University, Boone, NC
and Janice Richardson, Elon University, Elon, NC

The purpose of the NCCTM Innovator Award is to recognize and reward individuals and/or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM. The Recipient of this year's award is the BURROUGHS-WELLCOME FUND, a private charitable foundation dedicated to advancing science through research and education.

The Burroughs-Wellcome Fund recognizes the important role that K-12 teachers play in the lives of students in stimulating a passion for science, technology, engineering, and mathematics, better known as STEM. This group strives to support teaching professionals who are leaders in their field and who provide quality hands-on, inquiry-based learning that engages students. The Fund has invested over $53 million since 1996, primarily in K-12 education in North Carolina, to help build systemic reform in the STEM disciplines. The Fund has offered funding for teachers to purchase science and mathematics supplies so as to prevent out of pocket spending and at the preservice level, and has partnered with the UNC system to produce quality teachers in math and science.

The Burroughs-Wellcome Fund was selected to receive the Innovator Award to honor their program, *Career Award for Science and Mathematics Teachers*, which has been offered for four cycles, every other year since 2009. The Career Award is a five-year grant that recognize teachers who have demonstrated solid knowledge of science and/or mathematics content and have outstanding performance records in educating children. Over the five years, recipients of this award have opportunities for professional development and collaboration with other master science and/or math teachers who will help to ensure their success as teachers and their satisfaction with the field of teaching.

The Career Award also offers schools and school districts the opportunity to fully develop teachers as leaders in the field. For example, one recipient of this award used funds to purchase technology for the classrooms at her high school and received professional development not only for herself and colleagues to implement the technology practices seamlessly in their teaching of mathematics, which was a true passion of hers. Another recipient, in a different region of the state, is very passionate about ensuring access to high levels of mathematics for all students and used funds to create meaningful professional development opportunities for herself and colleagues as they implement the Common Core at her high school. A goal of another recent recipient is to help all mathematics teachers in her department to implement meaningful collaborative hands-on activities with 21st Century technology and innovative materials that help students understand the integrated nature of all STEM fields. As yet another example, one middle school math teacher recipient is currently using her five-year award to work with in-service and pre-service teachers so as to create design thinking into students' mathematical experiences which are authentic, issue-driven, and project based.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org.

More information can be obtained from: Todd Abel, AbelTA@appstate.edu.

Using a Thermometer to Catch a Cold-Blooded Killer

David Thompson, Baltimore City Public Schools, Baltimore, MD

The author describes an activity to engage calculus students in solving a differential equation – Newton's law of cooling – in the context of a murder mystery.

Many times in math classes we are asked, "Where does this apply to the real world?" Students are interested in authentic applications of the mathematical concepts that are presented in class. In calculus, students begin to study the beginning of differential equations. For the beginning student, this can be a dull concept. A classic textbook example gives students the chance to solve the differential equation solving Newton's Law of Cooling:

$$\frac{dT}{dt} \text{ is proportional to } T(t) - T_a,$$

where $T(t)$ is the temperature of the object at time t and T_a is the ambient temperature (temperature of the surroundings). In 1701 Newton anonymously published his cooling law in a short article, "Scala graduum caloris: Calorum descriptions & signa." Ironically, the self-proclaimed father of calculus did not write any formula but stated his cooling law as:

> *[T]he excess of the degrees of the heat…were in geometrical progression when the times are in an arithmetic progression.* (Newton, 1701)

In traditional textbook problems, Newton's Law of Cooling can be used to determine when a hot cup of coffee will reach room temperature or when a cold soda will reach the temperature of the refrigerator. An example using differential equations is seen in Stewart (2008), where he gives an example of how Newton's Law of Cooling and differential equations can be used to solve the latter problem. Sheree LeVarge (2005) suggested using Newton's Law of Cooling to find the time of death. In the activity following this article, calculus students practice solving a simple differential equation and reasoning through a scenario to not only find the time of death of a victim, but also determine who the killer is based on evidence provided in a scenario.

According to the Standards of Mathematical Practice MP4 from the Common Core State Standards Initiative, "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (CCSSI, 2010). In the activity, students have the opportunity to solve the differential equation that leads to finding the time of death of the Hollywood Casino manager. Also by the Standards of Mathematical Practice MP1, students must be able to "make sense of problems and persevere in solving them" (CCSSI, 2010). To complete the activity, students must be able to make sense of where each variable is used in Newton's Law of Cooling in order to be able to solve the problems. Students must also be able to use the formula more than once in order to first find the cooling constant, and then to find the time of death.

Finally, students must be able to use appropriate tools strategically (CCSSI, 2010). Students may need the use of technology to solve the exponential equations resulting from solving Newton's Law of Cooling.

For STEM teachers, the *Next Generation Science Standards* are also addressed by the Newton's Law of Cooling activity. In particular, in grades 6-8, students can "develop a model that predicts and describes changes in particle motion, temperature, and state of a pure substance when thermal energy is added or removed." Also, in grades 9-12 students can "plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics)" (National Research Council, National Science Teachers Association, American Association for the Advancement of Science, & Achieve, 2013). A hands-on activity for Newton's Law of Cooling has been developed by Abaid, Eckhardt, and Abdelnour. In their activity, students use physics software and lab equipment to show that Newton's Law of Cooling is an exponential function (2015).

Following this article is an engaging activity for students learning to solve differential equations. To add more interest, the names of the people on the "Time Sheet" can be changed to students in your class. The time sheet should not be given out until you have verified your students' solutions; in fact, the teacher could play the role of "Police Chief." With the popularity of television shows such as Numb3rs and other crime television shows, students should enjoy the opportunity to play the role of "crime solvers" in class.

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Activity: Using a Thermometer to Catch a Cold Blooded Killer

Directions

You and your partner are students at the police academy and have been requested to help the State Police Crime Scene Investigation Unit. You are investigating the murder of a Hollywood Casino manager; however, there is little evidence available to solve this murder mystery as the murder weapon has disappeared. You must figure out the time of death to determine a time frame of the murder. Upon determining the time frame, you and your partner will request a warrant to view the time card report of the restaurant for the day the murder occurred. You will use this information to determine who is the prime suspect.

Known Facts

- There are four sets of finger prints on the manager's desk.
- The first set has been discovered to belong to the manager, Jeff.
- The second set has been discovered to belong to Sean, the cook.
- The third set has been discovered to belong to Victoria, the waitress.
- The fourth set has been discovered to belong to Heather, the host.

- The temperature of the restaurant remains at a constant 65°F.
- The temperature of the body at the time of death was 98.6°F. (Assuming the victim was not sick at the time of death.)
- The temperature of the body at 10:00 am was recorded as 73.3°F.
- The temperature of the body two hours later was 71.1°F.

Question
What important pieces of information are needed to begin zoning in on a suspect?

1. Set up the differential equation.

   $$\frac{dT}{dt} \text{ is proportional to } T(t) - T_a,$$

   where $T(t)$ is the temperature of the body at time t and T_a is the ambient temperature (temperature of the surroundings).

2. Solve the differential equation.

3. Solve for C.

4. Find the cooling constant and set up an equation.

5. Find the approximate time of death.

6. Ask for a warrant to view the time card report.

7. Based on the time card report and your calculations on the time of death who is the prime suspect?

Extension Question
Explain how the time of death can be thrown off if the body was in a freezer at 32°F for three hours after the time of death, prior to being placed at the desk. Would you expect the body temperature to decrease at the same rate in the freezer as it would in a room at 65°F?

Time Card Report

Name	Date	Scheduled Shift	Actual Hours	Total Hours Worked
Summer	10/31/09	10:30am-6:30pm	10:30am-7:00pm	8.5 hours
Mycaylah	10/31/09	10:30am-7:00pm	10:30am-7:15pm	8.25 hrs
Ryan	10/31/09	10:30am-6:30pm	10:30am-7:00pm	8.5 hrs
Nathan	10/31/09	11:30am-7:30pm	11:30am-8:00pm	8.5 hrs
Kayla	10/31/09	10:30am-8:00pm	10:00:00 AM-8:15pm	9.75 hrs
Jeff	10/31/09	**4:00pm-1:15am**	**4:00pm-**	**Not Clocked Out**
Nehemiah	10/31/09	5:00pm-11:00pm	5:00pm-11:30pm	6.5 hrs
Heather	10/31/09	4:00pm-9:00pm	4:00pm-11:00pm	7 hrs
Emily	10/31/09	4:00pm-10:00pm	4:00pm-10:30pm	6.5 hrs
Raven	10/31/09	3:00pm-10:00pm	3:00pm-12:00am	7.5 hrs
Molly	10/31/09	3:00pm-11:00pm	3:00pm-12:00am	9 hrs
Victoria	10/31/09	5:00pm-12:30am	5:00pm-12:30am	7.5 hrs
Sean	10/31/09	5:00pm-1:15am	5:00pm-1:15 pm	8.25 hrs

2015 W.W. Rankin Award Winner

Wendy Rich

Reported by Lee Stiff, North Carolina State University, Raleigh, NC

At the 45th Annual State Mathematics Conference, NCCTM presented WENDY DENEEN RICH, Director of Elementary Education for Asheboro City Schools, with the W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual.

First and foremost, Wendy is unanimously regarded as an excellent teacher. In fact, "excellent" is the one word that is repeatedly used to describe her. It has been said that she "…motivates and ignites sparks…" for learning mathematics; and that her "…greatest accomplishment is fostering in teachers [and students] a love for mathematics and a desire to grow in their mathematical understanding." Wendy has been consistently recognized for outstanding teaching and leadership. She has twice been named "Teacher of the Year" in her home county, and served as a Lead Teacher for nearly 10 years! Wendy is active in promoting professional development, serves as a role model in the classroom, and is described as "…exemplifying the professionalism and commitment to quality mathematics teaching we would hope that all students in North Carolina could enjoy."

At the national level, Wendy served on a Teacher Enhancement Review Panel for the National Science Foundation and on a National Council of Teachers of Mathematics Yearbook Editorial Panel. She helped to facilitate the implementation of the TAP MATH Project, which develops school-based visions of quality mathematics instruction and creates K-8 Mathematics Instructional Leadership Teams in school districts across the State; served as a state-wide trainer for the K-5 Math Program, Developing Mathematical Ideas; and played a pivotal role in the development of Add-On Mathematics Licensure for elementary teachers in the State.

Wendy has provided outstanding service to North Carolina and NCCTM. She has given numerous presentations at local and state NCCTM meetings as well as workshops and sessions for the Department of Public Instruction, school districts, and area colleges; has been a curriculum writer and consultant; helped develop North Carolina math standards and test specs for the End-of-Grade Assessments; and has assumed leadership roles in many local and statewide teacher-development projects, such as PARTNERS, STAMP, and TEAM. And, perhaps most importantly, has served NCCTM as a Regional Program Chair, an Executive Board member, and as NCCTM President.

In support of this nomination to the Rankin Award Committee, a colleague wrote that Ms. Rich is "…an exceptional educator and is well respected among…colleagues across the State." Another supporter observed that she "…is the kind of leader who models a growth mindset" and radiates positive energy that rubs off on everyone around... The Rankin Award Committee agreed. Ms. Rich is committed to both in-service and pre-service teachers and is someone who represents the best that mathematics education has to offer in North Carolina.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, http://www.ncctm.org. More information can be obtained from: Lee V. Stiff, lee_stiff@ncsu.edu.

Introducing Students to the Mathematical Modeling Process

Todd Abel, Alana Baird, Holly Hirst, & Tracie Salinas, Appalachian State University, Boone, NC

The authors describe an activity that can be used to illustrate the modeling cycle.

In addition a framework is presented that can be to used to assess both a modeling task and a student's level of mathematical engagement.

Introduction

Mathematical modeling is increasingly becoming a point of emphasis in school curriculum. There are plenty of good reasons for this: Modeling provides context for mathematical concepts, grounding them in applications; it offers strong motivation for mathematical ideas; and it's an important way people actually use mathematics.

In order for students to engage in mathematical modeling, they must recognize the process that's involved. Here, we'll describe a task that can be used to introduce the modeling process and then discuss teaching using the modeling process more generally. The lesson featured here was originally designed for use in a teacher professional development setting, but it is certainly appropriate for middle school or high school as well. The task may be used to motivate or teach a variety of content: estimation, proportional reasoning, arithmetic sequences, linear equations, or piecewise linear functions, among others. The main thrust of the activity as illustrated here, however, is experiencing the key elements involved in mathematical modeling. Implementation may differ given different learning goals.

The Modeling Cycle

Some illustrations of the modeling cycle emphasize the different phases of work (e.g., Bliss, Fowler, & Galluzzo, 2014), while others emphasize the movement between real-world and mathematical environments (e.g., Giordano, Fox, Horton, & Weir, 2009; Zbiek & Conner, 2008). The Common Core State Standards for Mathematics (CCSSM) emphasizes the actions involved in each phase of the process in a very simplified diagram (Fig. 1) in which the movement between real world and mathematical entities – the *mathematization* of the real-world situation - is implicit (National Governor's Association & Council of Chief State School Officers, 2010). Regardless of the visualization, the modeling process includes the following elements: problem identification and model formulation, identifying assumptions and variables; solving the mathematical model; making sense of the solution; and then assessing the model itself.

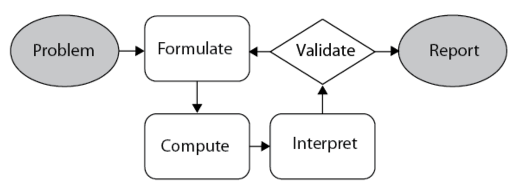

Figure 1: CCSSM Modeling Cycle
(National Governors' Association & Council of Chief State School Officers, 2010)

The modeling cycle also provides a framework for structuring lessons. It's relatively simple to provide students opportunities to compute, but authentic engagement in the other phases of mathematical modeling is more easily neglected. The activity described here focuses on helping students experience and understand the entirety of the modeling process, from problem and model formulation to validation and reporting.

The Cycle

Begin by giving the following prompt: *I have a birdfeeder outside of my house. Refilling it too often is a pain, but I want to make sure the birds keep coming back. Here are pictures of it at two different times.* (Fig. 2)

Ask students what questions occur to them. There may be a variety of responses, some more mathematical than others. It can be helpful to record all responses, since the list of initial questions can later inform the identification of assumptions and variables. However, the questions need to ultimately be refined to focus on specific questions that have quantifiable answers. Identifying that guideline for students can lead them to focus on a question such as "how long will it take the birdfeeder to empty?"

Figure 2: Birdfeeder pictures for motivating the problem

Formulate: Once the question is settled, ask students what information they need in order to begin answering the question. Too often, all pertinent information is provided to students within the question, removing the need to decide what is necessary and important. Students can either research to find the information they want, or the teacher may have the information at hand to provide when requested; it should be noted, though, that in some cases a bit of research can be a valuable experience for students. For instance, if students say they would like to know the dimensions of the birdfeeder, the image in Figure 3 could be provided.

Figure 3: Additional information for solving the problem

The class should also address assumptions of the model: Ask the class what assumptions might be reasonable or necessary. At this point, it can be helpful to refer to the original list of questions since many will point to issues that need addressed. For instance, students may choose to assume that birds feed on the seed at a constant rate, or that they only feed during daylight hours but do so at a constant rate (the number of daylight hours would have to be assumed in this case). Additional assumptions may become apparent as students formulate and solve their models, but it can be helpful to make simplifying assumptions initially then remove them as the model is improved.

At the end of this discussion, the teacher could prompt the class to reach consensus on assumptions or summarize the issues that need addressing and let individual groups choose how to do so for themselves. The latter option requires more independent thinking and reasoning but can lead to a wider variety of approaches.

Compute: In the computation phase, students do the mathematical work to find an answer, working within the framework of their assumptions. There are a variety of models that might be used to answer the question. A few examples include:

- Using approximations to estimate
- Using proportions to estimate the remaining time (could assume it needs to go all the way to the bottom or neglect the last 2 inches)
- Creating a linear function relating time to height of the seed
- Creating a piecewise linear function relating time to height, changing rules when the seed dips below the upper set of holes
- Modeling seed consumption with a periodic function based on daylight hours or feeding cycles

Other models are certainly possible. If the purpose of the activity is to illustrate the modeling process, gathering a variety of approaches can be useful. Note that assumptions may need to be revised as students work on their models.

Interpret: More than just adding units to an answer, interpretation means considering the answer in the context of the problem and examining its reasonableness. Teachers often ask students to consider whether an answer makes sense, and interpreting an answer in the context of a modeling problem provides a reason for students to care to do so and a way to determine what's reasonable. It also prompts examination of the model, which is the purpose of the next phase.

Validate: If the point of interpretation is to decide if the answer makes sense, the point of validation is to decide if the model itself makes sense. Students can consider whether the solution answers the original question, and question whether assumptions were reasonable. Prompt students to identify the strengths and weaknesses of their models and decide whether the assumptions were truly reasonable or not. Assumptions that are judged unreasonable can be removed as the model is improved. Students should have an opportunity to report their findings, either as a brief summary or a more fully-developed report. From a pedagogical perspective, this is an excellent opportunity to move from applying previously-learned material to motivating new material by moving around the cycle multiple times. For instance, if students use proportions to estimate when the seed will be depleted, they can be challenged to predict the amount of seed left at any time, thus motivating linear functions. Or the assumption that the seed is eaten at the same rate continuously could be challenged in order to motivate piecewise functions.

A Teaching Process

In addition to being an important way of applying mathematical knowledge, the modeling process is a teaching process. In validating a simple model, a class can find that their model leads to unsatisfactory conclusions, motivating new approaches. In this way, students can recognize the holes in their own knowledge. If an improved model requires mathematical content that students have not yet encountered, it provides an opportunity for the teacher to introduce that content.

Assessing Modeling Tasks and Student Engagement with the Modeling Process

Especially on a first experience with the modeling process, student engagement with the various phases will vary widely. It can be difficult to characterize high- versus low-level engagement with mathematical modeling. With that in mind, a group of teachers, mathematicians, mathematics educators, and college faculty worked together as part of the Implementing Modeling in the Secondary and Post-Secondary Classroom (IMSPC) project, funded by NC Ready for Success, to create a framework describing three levels of engagement with each phase of modeling (Fig. 4). Modeled after the "Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report" (Franklin et al., 2007) and drawing from "Standards for Student Practice in Mathematics" (Hull, Balka, & Harbin Miles, 2011), the framework describes characteristics of level A, B, and C modeling activities, and also describes characteristics of student work at each of these levels. It can be useful for assessing in what ways tasks might be useful for your classroom and for characterizing student work.

Level A tasks might be suitable for introducing the modeling process and require minimal engagement with each phase. For instance, a Level A formulation task might provide assumptions or direct the steps of creating a model. Computations might be procedural, and validation and interpretation are cursory, but each phase is addressed. Level B tasks require more independence from students, offering multiple solution paths and prompting consideration of alternative approaches. Level C tasks require full engagement in the modeling process. Tasks (or student work) may exhibit different levels for different phases.

The bird feeder task as presented would be Level B in most respects, but could be enacted in ways that increase or decrease the level. In general, the intended level may differ from the level of an enacted lesson. For instance, the formulation level would be lowered if students were directed toward a specific model, such as being told to "model the amount left at any time using a linear function." The validation level could be raised by requiring students to offer a full analysis of the strengths and weaknesses of their model, and, at minimum, propose changes that might improve its fidelity or lower its calculation cost. We challenge the reader to consult the framework and consider how the level of each phase could be raised or lowered.

Formulate

	Level A	Level B	Level C
Task	• Presents well-defined goals and questions • Provides Variables, constants, and assumptions • Provides or directs the process of mathematization	• Suggests questions and goals if not given OR provides questions and goals but requires explanation • Suggests mathematization if not given or requires explanation if given • Requires students to identify assumptions	• Provides situation but allows for clarification of goals and questions • Requires students to identify assumptions, variables, and resulting mathematization • Allows for a variety of formulations based on context
Student Work	• Explains rationale for assumptions and questions • Demonstrates understanding of why the particular mathematization was appropriate	• Demonstrates understanding of mathematizaation, either by adapting or explaining existing ones • Identifies all assumptions implicit in the mathematization	• Creates a precise and well-formed modeling question • Explains the assumptions made and identifies variables • Creates a correct mathematization within the context

Compute

	Level A	Level B	Level C
Task	• Directs computations that are procedural in nature • Prescribes and directs technology use	• Limits but does not prescribe computation methods • Provides multiple but limited solution paths • Guides but does not prescribe students use of technology	• Allows for multiple computation methods and technologies • Applies a broad range of math • Provides insight into mathematics concepts through computation methods
Student Work	• Evaluates or computes correctly following directed steps	• Chooses appropriate computation method and provides support for choice • Implements chosen computation method accurately	• Selects and explains computation method based upon comparison possible solution techniques • Produces procedurally and conceptually accurate computation

Validate

	Level A	Level B	Level C
Task	• Prompts minimally or not at all for reevaluating the components of the model (question, assumptions, variables, math models, computation, solution)	•Prompts students to consider other possible approaches and review the usefulness of their model	• Motivates students to consider whether the situation can be approached in other ways • Prompts analysis of solutions for sensitivity, efficiency, and sources of error
Student Work	• Demonstrates consideration of whether the solution answers the original question	• Shows evidence of reflection on correctness and challenges computations and answers • Demonstrates evidence of what is wrong, what is correct, and if other solutions are possible	• Includes assessment of assumptions and questions • Analyzes sources of error, precision, accuracy, sensitivity, efficiency, and generalizability

Interpret

	Level A	Level B	Level C
Task	• Requires straightforward or cursory interpretation of mathematical answers	• Prompts students to interpret answers in context and reflect on reasonableness of answers • Motivates students to assess computation	• Requires interpretation of answers in context and reflection on implications within the context • Prompts consideration of other solution methods
Student Work	• Provides solution with correct units or within the correct context	• Provides the solution with correct units in context and shows capacity to explain • Assesses the reasonableness of the answer	• Provides the solution in context and demonstrates consideration of whether question was answered, whether the answer makes sense, and whether computations need to be revisited

Figure 4: Assessing Modeling Tasks (A full sized version of this chart may be obtained at www.ncctm.org/membership/the-centroid/centroid-articles/)

Conclusion

Mathematical modeling is a process and an important way of thinking about and doing mathematics. When students leave school, many of those who encounter mathematics will do so in the context of modeling. The activity above provides an accessible introduction to the process of mathematical modeling, and illustrates how students can engage in each phase of it. It also emphasize how the modeling process can be used as a teaching process, with a first pass through the cycle reviewing and utilizing previously-learned material, and subsequent cycles motivating new material. By designing lessons that utilize such a structure, teachers may gradually incorporate higher-level modeling tasks.

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2015 Outstanding Elementary Teachers

Reported by Kitty Rutherford, North Carolina Department of Public Instruction, Raleigh, NC

Each year, school principals are encouraged to nominate the teacher they believe does the most effective job teaching mathematics in their school. From those nominated, each LEA selected one teacher to represent the best in mathematics teaching from the entire system. The teacher selected from each LEA receives a one year membership in NCCTM, recognition at the State Conference, and a special memento of the occasion. The grade level cycles, and this year the outstanding teachers were chosen from among the best elementary teachers in North Carolina.

Outstanding Elementary Mathematics Teachers

Ann "Renee" Boyd, Alamance Burlington
Melissa Bowman, Alexander
Tina Key, Ashe
Bailey Toomes, Asheboro City
Kara Rice, Asheville City
Kim Wiseman, Avery
Anita Rayburn, Beaufort
Carla Bethea, Bladen
Charlene Martin, Buncombe
Amy Shoe, Cabarrus
Melissa Wiant, Caldwell
Toni Luther, Carteret
Danya Diggs, Caswell
Lisa Schanilec, Catawba
Rebecca Tompkins, Chatham
Casey Avery, Clinton City
Amanda Greene, Columbus
Amber Mirise, Craven
Amber Everett, Currituck
Katherine McGee, Dare
Debbie Dawson, Davidson
Monica Rivenbark, Duplin
Britney Phillips, Edgecombe
Sarah Crisp, Gaston
Anita Winn, Gates
Marlene Creary, Haywood
Dana Wells, Hendersonville
Dorothy Dalton, Hickory City
Shequeta Fripp, Hoke
Leslie Buchanan, Jackson
Lauren Strickland, Johnston
Laura Baker, Kannapolis City
Brandon Morehouse, Lee
Monica Griffin, Lexington City
Jennifer Broome, Lincoln
Jessica Ross, McDowell
Rebekah Lonon, Charlotte/Mecklenburg
Amy Thomas, Mitchell
Melissa Cox, Moore
Kristin Mangano, Nash/Rocky Mount
Mischele Glover, Newton-Conover
Patricia Corbett, Onslow
Holly Winslow, Perquimans
Carrie Briggs, Person
Laura Skinner, Pitt
Rebecca Parks, Randolph
Kate Murray, Richmond
Tiffany Locklear, Robeson
Jonathan Pratt, Rockingham
Faith Bailey, Rutherford
Tricia Eury, Scotland
Pam Hyatt, Stanly
Kathy Wilson, Stokes
Tracy Cornett, Surry
Sandra English, Swain
Andrea Wisniewski, Union
Katie Melin, Wake
Chrissy Biggs, Washington
Jamie Dale Sherrill, Watauga
Donna Lewis, Wayne
Rebecca Kilpatrick, Wilkes
Robin Bass, Wilson
Jennifer Beach, Winston-Salem/Forsyth
Jill Owens, Yadkin
Kathleen Ellis, Voyager Academy

2015 Outstanding Mathematics Education Students

Reported by Todd Abel, Appalachian State University, Boone, NC

Each Fall, NCCTM sponsors the selection of three Outstanding Mathematics Education Students, one form each region of NCCTM. The recipients of this year's awards are: MARGARET LEAKE from NC State University in the Eastern Region, MIHOSHOTY YAMAGUCHI from North Carolina A&T State University in the Central Region, and KELSEY BROWN from Appalachian State University in the Western Region.

Pictured left to right: Margaret Leak, Kelsey Brown, and Mihoshoty Yamaguchi

MARGARET LEAK is a triple major in Mathematics Education, Mathematics, and Communication at North Carolina State University. Margaret is a Park and a Noyce Scholar. She has been involved in the NCCTM Affiliate of NCSU since her freshman year. She has been on the executive board for four years and is currently serving as president. Under her leadership, the Affiliate moved from a club with monthly meetings and some outreach to one with a more "official" membership requirement, extensive outreach to schools in the Raleigh area, meaningful professional development activities for members, and special recognition at graduation. In addition to her leadership with NCCTM, Margaret volunteers with many campus organizations and she is a College of Education Ambassador. She has studied abroad in Spain and in Costa Rica. Margaret is a natural leader who is constantly looking out for ways to grow as a mathematics teacher.

MIHOSHOTY YAMAGUCHI is a double major in Mathematics Secondary Education and Mathematics at North Carolina A&T State University. Miho is a very talented, hardworking, and goal-oriented student who always strives to do her best. She has been very involved in the Mathematics education activities at North Carolina A&T State University, and is currently serving as the Secretary of the NCCTM Affiliate at NC A&T. She has been a Mathematics Department teaching assistant. In addition Miho is involved in several other campus activities. She is a founding member of the Latino Culture Student Alliance. She has served as Secretary of the Tau Sigma National Honor Society, and she is currently the President of that honor society. She has volunteered with Habitat for Humanity, with Casa Azul of Greensboro, and has tutored numerous students in Mathematics and Spanish.

KELSEY SARAH-ELIZABETH BROWN is an elementary education major with a concentration in Mathematics at Appalachian State University. Her contributions and participation in mathematics education at ASU are closely tied to her involvement with the Prospective Teachers of Mathematics Association (PTMA), which is a student affiliate of NCCTM. Kelsey has been a member of this Association for four years and has held several positions including Secretary, Treasurer, and President. She has applied and received funds from ASU and NCCTM to send the PTMA officers to NCCTM State Math Conferences. She has represented the PTMA in the ASU Club Council. Kelsey occasionally covers a mathematics education class when the professor is away. Kelsey served as the western region Student Representative to the NCCTM Board Directors in 2014. She not only attended the NCCTM State Math Conferences, but she and two other students from ASU presented a workshop at the 2014 NCCTM conference.

Distributive Property Illustrations from Vedic Mathematics

Donald Hooley, Bluffton University, Bluffton, OH

How do you mentally calculate 3 x 17? I prefer using the distributive property as

$$3 \times (10 + 7) = 3 \times 10 + 3 \times 7 = 30 + 21 = 51.$$

This illustrates the number sense and reasoning that we would like to develop in all our students. To further concept development we might follow this question by asking for a good way to quickly compute 43 x 17. The first shortcut discussed in this article describes an alternative method to calculate this product.

The author describes four Vedic algorithms that can be used to strengthen students' understanding of the distributive property.

The algorithms are referred to as "Vedic mathematics" because they are based on sutras *in the writing style of ancient Indian Vedas. The Vedic mathematics* sutras *were developed by the Indian Jagadguru Swami Sri Bharati Krsna Tirthaji in the early 20th century.*

Use of the distributive property goes far beyond the ubiquitous FOIL method. This article presents four Vedic shortcut algorithms of particular algebraic interest for student enrichment and as possible examples for strengthening understanding of the distributive property. I first learned of these techniques while in a Delhi India bookstore and noticed a workbook for elementary students titled "Vedic Mathematics for Beginners (Level 3)" (Sinha, 2011).

Students preparing for the very competitive Indian university entrance exams commonly learn these one-line shortcuts since calculators are not allowed. The algorithms are referred to as Vedic mathematics because they are based on cryptic phrases called *sutras* in the writing style of ancient Indian Vedas. The Vedic mathematics *sutras* were developed by the Indian Jagadguru Swami Sri Bharati Krsna Tirthaji during the years 1911-1918 (Bathia, 2005). (For three arithmetic examples see Hooley, 2014.)

We start with three algorithms for which the distributive property is integral. These are short multiplication, squaring, and base method multiplication. In addition, an alternative method of factoring quadratics is illustrated.

Multiplication of Integers and Polynomials

Consider the problems 34×52 or $(3x + 4)(5x + 2)$. In both cases we can use:

$$(ax + b)(cx + d) = acx^2 + (ad + bc)x + bd$$

Placed vertically, these are examples of the *sutra* "vertical and crosswise" where ac and bd are found by multiplying vertically and $ac + bd$ is the cross multiplication term as in:

$$\begin{array}{c} (ax + b) \\ \underline{\times (cx + d)} \\ acx^2 + (ad + bc)x + bd \end{array}$$

When x = 10, our numerical example gives $3 \times 5 = 15$, $3 \times 2 + 5 \times 4 = 26$ and $4 \times 2 = 8$, so:

$$\begin{array}{r} 34 \\ \underline{\times 52} \\ {}^{1}5\ \ {}^{2}6\ \ 8 = 1768 \end{array}$$

Our familiar FOIL method works only when multiplying binomials. This shortcut can be extended to trinomial multiplication of $(ax^2 + bx + c)(dx^2 + ex + f)$ with a slightly more complicated cross-product term. Begin by placing the trinomials vertically as

$$\begin{array}{r} (ax^2 + bx + c) \\ \underline{\times\,(dx^2 + ex + f)} \end{array}$$

Now the coefficient of x^4 is found vertically as ad, that of x^3 is found crosswise as $ae + bd$, that of x^2 is found with crosswise including a vertical component as $af + be + cd$, that of x is found crosswise as $bf + ce$, and the constant term is found vertically as cf. So the solution can be written almost directly:

$$\begin{array}{c} (2x^2 + 3x + 1) \\ \underline{\times\,(3x^2 - 5x - 4)} \\ 6x^4 + (-10 + 9)x^3 + (-8 - 15 + 3)x^2 + (-12 - 5)x - 4 \\ = 6x^4 - 1x^3 - 20x^2 - 17x - 4 \end{array}$$

Squaring

Squaring integers can be considered a subcase of the multiplication method. If a is the tens value and b the units digit of a two-digit positive integer we can square the binomial $a + b$ and re-arrange to give an easier mental computation. Thus,

$$\begin{array}{c} (a + b) \\ \underline{\times\,(a + b)} \\ a^2 + 2ab + b^2 = a^2 + b^2 + 2ab. \end{array}$$

For example

$$\begin{aligned} 132 &= (10 + 3)2 \\ &= 102 + 32 + 2(10)(3) \\ &= 100 + 9 + 60 \\ &= 169. \end{aligned}$$

Note the computations are a little more difficult if the digits are larger, as in

$$\begin{aligned} 672 &= (60 + 7)2 \\ &= 602 + 72 + 2(60)(7) \\ &= 3600 + 49 + 840 \\ &= 4489. \end{aligned}$$

Base Method Multiplication

Consider multiplying two numbers that are slightly less than a particular value, called the "base." For example, if the base x is 100 then the product 88×91 can be represented using

$88 = 100 - 12 = x - a$ and

$91 = 100 - 9 = x - b$, with $x = 100$, $a = 12$ and $b = 9$.

Using the distributive property, then factoring, gives

$$\begin{aligned} (x - a)(x - b) &= x^2 - ax - bx + ab \\ &= x(x - a - b) + ab \\ &= 100(100 - 12 - 9) + 12\times 9 \\ &= 100(79) + 108 \\ &= 7900 + 108 \\ &= 8008. \end{aligned}$$

The Vedic mathematics *sutra* for this is "all from 9 and the last from 10." It reminds us of the subtraction needed to find the differences $x - a$ and $x - b$ between the base and the factors. This method is usually written

$$\begin{array}{rrl} & \text{(think)} & \\ & 88 \ \ (+12) & \\ \times & \underline{91 \ \ (+9)} & \\ & 79 \ \ {}^{1}08 & = 8008. \end{array}$$

So, we need only take the 12 and 9 away from the base 100 to find 79 hundreds and add $12 \times 9 = 108$ to find the solution 8008. Thus, to solve the problem 85×96 we write and think

$$\begin{array}{rrl} & \text{(think)} & \\ & 85 \ \ (+15) & \\ \times & \underline{96 \ \ (+4)} & \\ & 81 \ \ \ 60 & = 8160 \end{array}$$

with $100 - 15 - 4 = 81$ and $15 \times 4 = 60.$

Factoring Quadratics

We would like to use the distributive property to help find a way of reversing its multiplicative action. For one example, to factor quadratics we use the *sutras* "the first by the first and the last by the last" along with "proportionately". To illustrate, consider factoring beginning with distributing the factors for

$$\begin{aligned} 2x^2 + 7x + 5 &= (ax + b)(cx + d) \\ &= acx^2 + (ad + bc)x + bd \\ &= (2x + 5)(x + 1). \end{aligned}$$

First split the 7 into two integer values m and n "proportionately," so that $7 = m + n$ and $2/m = n/5 = c/d$, with c/d in lowest terms. Since

$$7 = m + n = ad + bc,$$

if we choose $m = ad$ and $n = bc$, then the proportion gives

$$\begin{gathered} ac/m = n/bd \\ ac/ad = bc/bd = c/d. \end{gathered}$$

This gives values for c and d and thus the factor $cx + d$.

In our example $m = 2, n = 5$ giving $c = 1$ and $d = 1$ so the factor is $1x + 1$. Now divide the "first (coefficient) by the first (m) and last by the last (n)" since

$$ac/c = a \text{ and } bd/d = b,$$

resulting in $2/1 = 2$ and $5/1 = 5$, so the remaining factor is $2x + 5$. Our final solution is

$$2x^2 + 7x + 5 = (2x + 5)\,(x + 1).$$

For another example, consider factoring $8x^2 + 22x + 15$. First find integers $m + n = 22$ so that

$$8/m = n/15.$$

This gives $m = 12, n = 10$ so

$$8/12 = 10/15 = 2/3.$$

One factor is then $2x + 3$. Now find the other factor with coefficients $8/2 = 4$ and $15/3 = 5$, leading to the final factorization

$$8x^2 + 22x + 15 = (2x + 3)(4x + 5).$$

Conclusion

The Vedic algorithms sometimes lack clear meaningfulness so I don't generally advise using them for whole-class normal instruction, but they are great for enrichment. As direct extensions of the examples in this article, can you modify the:

i) "Vertical and crosswise" multiplication method to find a binomial times a trinomial?
ii) Multiplication procedure for trinomials to square a three-digit number?
iii) Squaring procedure by subtracting when the units digit is large?
iv) Base method to multiply two numbers slightly larger than the base, or one larger and one smaller?
v) Base method to multiply two values slightly less than a base other than 100, say 60?

Other fascinating alternative algorithms have been developed as shortcuts for many algebraic, trigonometric and calculus computations. Excellent examples of these can be found in Tirthagi's (1992) classic book in English translation and on the internet.

References

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The Deadlines for applications are March 1 and October 1. The application can be downloaded from http://ncctm.org.

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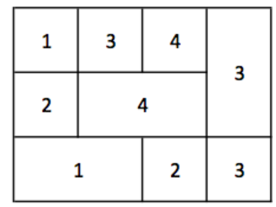
Problems to Ponder 

Holly Hirst, Appalachian State University, Boone, NC

Spring 2016 Problems

Grades K–2: Tom wants to purchase a really nice pair of ear buds for his iPod. Tom has already saved up $33, and he can earn $5 each week for doing his chores. If he needs $69.99 to get his ear buds, how many weeks of chores will he need to do to have the money?

Grades 3–5: Tena and Jai'Lisa are playing a game using the grid of squares and rectangles shown to the right. The winner is the girl who claims the most area. Each girl can choose a strategy from the list below. Which would be the best winning strategies to choose and which would be the worst? Explain why.



- Claim odd numbers
- Claim even numbers
- Claim all squares
- Claim all rectangles (and not squares)

Grades 6–8: A cube has a total surface area equal to 150 square feet when all faces of the cube are included in the measurement. What is the volume of this cube in cubic inches?

Directions for submitting solutions:

1. Students: NEATLY print the following at the top of each solution page:
   - Your first name (we will not publish last names)
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page. ***Students who submit correct solutions will be recognized by their first names only in the next issue of The Centroid.*** We will also publish one or two especially creative or well-written solutions from those submitted. If you would rather not have your name published, please so indicate on your submission.

Proper acknowledgement is contingent on legible information and solutions. Send solutions by 15 June 2016 to:

Problems to Ponder, c/o Dr. Holly Hirst
Mathematical Sciences
BOX 32069 Appalachian State University
Boone, NC 28608

A submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

Spring 2015 Problem Solutions

Grades K–2: Ms. Davenport has 24 students in her class. If she arranges 24 desks into 6 rows, how many students will be in each row of her classroom?

Editor's note: all the students tried the higher grade level problems this time! The solution to this problem is 4.

Grades 3–5: Tanisha wants to purchase a new iPad for $499. She will use a $50 gift certificate, and also $100 in money she had saved up. For her birthday, her dad gave her one-half of the remaining money she would need to get the iPad. How much more money does Tanisha need to save to make her purchase?

Here is one of the submissions with the clearest explanation (Rhyonna at Ahoskie Elementary School).

Correct solutions were submitted by:

- Ahoskie Elementary School (Ms. Harrison, Mrs. Liverman, Mrs. Mitchell, and Mrs. Ruffin): Akeyma, Ja'Bria, Kaitlyn, Preston, Rhyonna, Ruckira, Sophia, TyQuan

Editor's note: Many students submitted the amount Tanisha needed BEFORE her dad gave her half of the remaining money.

499.00
50.00 - gift certificate
449.00
100.00 - Savings
349.00
174.50 - dad ½
174.50

Tanisha needs to save $174.50 to purchase the ipad.

1. I subtracted the $50 gift certificate from the total price.

2. The difference was $449.00, I then subtracted her $100 savings.

3. $349.00 was left over and I divided that in half. Half of 349.00 is 174.50. I subtracted 174.50 and the difference was 174.50.

Grades 6–8: A picture that is 18 inches tall is to be hung on an 8 foot tall wall in Tanya's home. She wants the center of the picture to be 5 feet 2 inches off the floor. She needs to put the fastener on the wall at the top of the picture. How far from the ceiling should the top of the picture be?

Here is the neatest submission with the clearest explanation (Marc at Ahoski Elementary School).

Correct solutions were submitted by:

- Ahoskie Elementary School (Ms. Bowser, Ms. Canada, and Mrs. Moore): Asharie, Ashlyn, Colin, Dominique, Ethan, Ivin, Jane, Janiyah, Jerry, Khaniya, Kimore, Marc, Tristian
- Newport Middle School (Mrs. Corbett): Ben, Bradley, Julia, Lauren, Paige, Shawnta, Will

Editor's note: Many students provided a solution to the problem of how far from the FLOOR to put the fastener, rather than how far from the ceiling!

12/17/15

First I divided 18 by two because 9 inches is the center or middle of 18

$18 \div 2 = 9$

She wants the center to be 5 feet and 2 inches from the floor. So I added 9 inches from the 5 feet and 2 inches — this is the top of the picture.

$$5\tfrac{2}{12} + 0\tfrac{9}{12} = 5\tfrac{11}{12}$$

Then it asked how far the top of the picture was to the ceiling. Since the wall is 8 feet long I subtracted 8 from 5 feet and 11 inches.

$$8 - 5\tfrac{11}{12} = 2\tfrac{1}{12}$$

$\boxed{\text{2 feet and 1 inch away}}$

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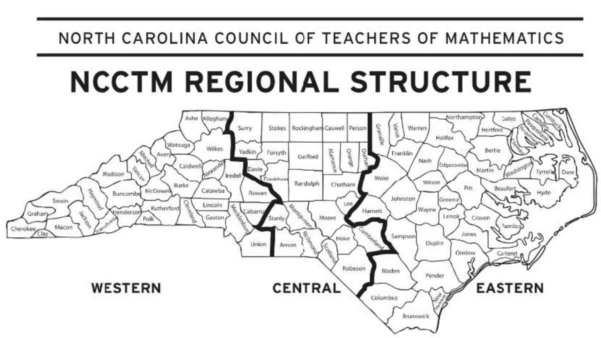

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