The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

- ✰ *Formative Assessment Utilizing the TI-Nspire Navigator System in 7th Grade Classrooms*
- ✰ *Forces in the Pairs Death Spiral: A Mathematics and Physics Modeling Activity*
- ✰ *Dr. Leibniz and His Amazing Triangle*
- ✰ *Awards: Outstanding Math Teachers, Outstanding Math Education Students, Outstanding Coach/Sponsor, Rankin and Innovator Winners*

Volume 42, Issue 2 • Spring 2017

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:
    Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.
    National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
  Journal articles:
    Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, 7, 196–199.
  Chapters or sections of books:
    Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.
  Websites:
    North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

Volume 42, Issue 2 • Spring 2017

Editorial Board

Editors
Deborah Crocker
Appalachian State University
Holly Hirst
Appalachian State University

Board Members
Betty Long
Appalachian State University
Jill Thomley
Appalachian State University
Solomon Willis
Cleveland Community College

About the Cover
The image on the cover depicts the centroid of a triangle.

Copyright
Educators are granted general permission to photo-copy material from *The Centroid* for noncommercial instructional and scholarly use. Contact the author(s) concerning other copying.

Contact Information
The Centroid
c/o Dr. Deborah Crocker, Editor
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
or send email to
CrockerDA@appstate.edu
Please include a return email address with all correspondence.

An advertisement in The Centroid does not constitute endorsement by NCCTM, and the opinions expressed or implied in this publication are not official positions of NCCTM unless explicitly noted.

TABLE OF CONTENTS

NCCTM Spring Leadership Seminar
March 24, 2017
Marriott Airport Greensboro in Greensboro, NC

NCCTM Leadership Seminar
November 1, 2017
Koury Convention Center in Greensboro, NC

NCCTM State Math Conference
November 2 and 3, 2017
Koury Convention Center in Greensboro, NC

Visit http://ncctm.org for more information!

President's Message

State President Ron Preston
East Carolina University, Greenville, NC
prestonr@ecu.edu

Members of the North Carolina Council of Teachers of Mathematics have much to celebrate as we look back over the past several months. The Fall Leadership Seminar was a success in terms of attendance and quality. There were 249 attendees for the seminar and they enjoyed talks by the mathematics consultants from the NC Department of Public Instruction, Jere Confrey, Dan Meyer, and Zach Champagne. The annual conference was held on 27-28 October 2016 and was attended by 1901 members and visitors. The Program Chairs (Jenna Regan, Kim Solomon, and Kwaku Adu-Gyamfi) and Conference Chairs (Stefanie Buckner and Drew Polly) lined up an exceptional array of keynote speakers, featured speakers, workshops, and sessions. The conference theme was "Leaping Forward: Teaching & Learning, Equity, Curriculum, and Assessment." Keynote and Featured Speakers included Dan Meyer, Jere Confrey, Zach Champagne, Jennifer Wilson, Mike Bossé, Susan Empson, Angela Gardner, Astrid Fossum, Maria Blanton, Robert Berry, Marni Greenstein, Jennifer Curtis, Kevin Dykema, and Ellen Whitesides.

Even as we celebrate past accomplishments, we always have an eye on the future and the new opportunities it holds. I hope you will make plans to attend the NCCTM Spring Leadership Seminar. It is scheduled for Friday, 24 March 2017 at the Marriott Greensboro Airport. Based on a recommendation from Julie Kolb (our NCCTM President-Elect), I have chosen the theme for this year's seminar to be "What's Your Message?" Speakers for the event are Gary Martin, individuals from the North Carolina Collaborative for Mathematics Learning (including Holt Wilson), and the mathematics consultants from the NC Department of Public Instruction. I feel that the "What's Your Message?" emphasis is very timely. The focus will be on providing leaders in math education with ways of communicating more effectively the intent and practices of math education in North Carolina to teachers, administrators, parents, elected officials, business leaders, stakeholders, concerned citizens, etc. Specifically, I would like for us to have strong messages of success to share with key stakeholders – in a proactive manner, instead of the reactive manner that many of us have found ourselves in recently. For example, at any given time, we may be asked about the following:

- Recent mathematics scores from the Programme for International Student Assessment (PISA), including the specific NC scores
- Parents who are unable to help their children with mathematics homework because the methods of instruction are unfamiliar to them
- Elected officials who hear more negative comments than positive about mathematics instruction
- Business leaders concerned with an underprepared workforce
- Higher education spending resources on remedial mathematics for some students

This is my last president's message for the Centroid. It has been a busy two years and I want to thank the NCCTM Board members, committee chairs and members, numerous volunteers, and all NCCTM members for your support over these two years. I also want to thank some people specifically, realizing as I do so that I will likely forget to mention some who made significant contributions. I could not have survived as president without the incredible assistance and information passed along by Debbie Crocker, Past President. Joette Midgett, Management Services, and Kay Swofford, Conference and Exhibit Services, were invaluable in their roles. Marilyn Preddy, Convention Services, went the extra mile to book facilities and then, as needed, re-book them when issues arose. Ray Jernigan did a tremendous job as chair of the Finance Committee and worked to keep us apprised of the financial status of the Council and to ensure we operate in the black. I have also greatly benefitted from the opportunity to work with the excellent mathematics consultants from the Department of Public Instruction. I am excited about turning over the leadership of NCCTM to our next president, Julie Kolb, and her team of regional presidents, Tim Hendrix, Julie Riggins, and Karen McPherson. I know they will do a great job and I hope you will support them and the organization in the same manner you supported me for the previous two years. NCCTM is a strong and effective organization and I remain committed to seeing it continue – and even advance – Its great work in the future.

Formative Assessment Utilizing the TI-Nspire Navigator System in 7th Grade Classrooms

Holly Henderson Pinter, Western Carolina University, Cullowhee, NC

The author discusses the findings of a study addressing middle school teachers' implementation of the TI-Nspire Navigator system in 7th grade mathematics classrooms.

Two main patterns emerged: ability to structure and organize lessons using the learning check feature; formative assessment in the form of adaptive instruction using the quick poll feature.

In the midst of a climate of standardized testing overload, there has been a parallel shift towards more intentional use of formative assessment in instruction (Black & William, 2009; National Council of Teachers of Mathematics [NCTM], 2013). *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) charges mathematics educators to not only monitor student progress by eliciting student thinking, but also adjust instruction in real time in order to offer students the necessary feedback to deepen student understanding. The NCTM position statement on formative assessment states, "Formative strategies embedded in instruction provide opportunities for students to make conjectures, incorporate multiple representations in their problem solving, and discuss their mathematical thinking with their peers" (2013). The influx of technology tools such as *Kahoot, Socrative*, and *Poll Everywhere* have become common tools for quickly assessing students within a lesson. For mathematics, the *TI-Nspire and Navigator* system is one technological tool that provides multi-faceted opportunities to facilitate instruction particularly regarding lesson structure and formative assessment.

Using local grant funding through the Sisters of Mercy, a school district in the southeastern United States was able to provide all six of the district's middle schools with TI-Nspire Navigator systems for 7th grade classrooms. Teachers were provided with three days of face-to-face training with a contracted Texas Instruments (TI) expert with an additional 16 days across the year for in-class coaching. To evaluate the use of the systems, we visited seven teachers at three schools and followed them through their year of implementation by conducting three formal observations throughout the year. In order to structure the observations, we used a modified version of the Mathematics Scan (M-Scan), an observational measure of standards based teaching practices (Berry et al., 2013) that allows the observer to take detailed qualitative notes regarding lesson structure, discourse practices in the classroom, and problem solving opportunities for students.

During our visits, two main themes emerged across the seven teachers we studied. First we noticed patterns in the use of the TI-Nspire Navigator System calculators in the ability to structure and organize lessons, particularly with novice teachers. There were also patterns in how the calculators were used for formative assessment in the form of adaptive instruction (Mosher, 2011). To explore these two themes, we present descriptions of two teachers and their varied experiences with the TI-Navigator systems.

TI Navigator Systems

The TI-Nspire Navigator System™ uses TI-Nspire calculators (also compatible with some earlier versions of TI calculators) and wireless access points to connect the teacher's computer to all of the students' devices. The system sets up a collaborative classroom where teachers can send, collect, analyze, and share student responses in ways to help build collective understanding. There are a few main features of the system that teachers can use for the purposes of formative assessment. The *Quick Poll* option allows teachers to pose questions to students for quick data collection. These questions may be in the format of multiple choice or open-ended questions. Quick polls can be questions planned in advance for formative assessment, or can be utilized more spontaneously by having students send in answers to a question asked on the spot. The use of quick polls was the function that was most widely used across the group of teachers in this study. All teachers used this function on a consistent basis, since it is a simple and effective function. The system quickly condenses the students' responses and provides a graph that can be shown to the class for whole group monitoring. A second component of the system used for formative assessment as well as efficiently structuring a lesson is the *Learning Check*. This feature allows the teacher to send a set of questions/tasks for students to work on. In our study we found that teachers (particularly first year teachers) were able to use this feature to help them in the planning and organization of their lessons as these pieces needed to be thought out in advance. Once again the technology collects and summarizes the data into a graph from which teachers can efficiently and effectively gauge student progress and adapt instruction as necessary. Teachers can also use the *Screen Capture* feature to quickly display student work in order to share and discuss student strategies.

Formative Assessment and Adaptive Instruction

The most prominent theme we observed in lessons was the variety of ways the calculator systems were utilized for formative assessment. Whether using the Quick poll feature, or simply displaying multiple student screens at once, teachers were able to use the calculators to help facilitate discussion and adjust instruction accordingly. Two teachers in our study were particularly skillful at this, Mr. Callahan and Mr. Fisher (pseudonyms). The following lesson vignettes from Mr. Callahan and Mr. Fisher's classrooms offer insight into how the use of the systems translates into practice for daily instruction.

Mr. Callahan. Mr. Callahan used a typical coherent routine for instruction. Students began their lesson with a short warm-up task used to formatively assess understanding from the previous day. Mr. Callahan had students explore patterns in linear relationships by having students create the graphs and equations of children with different walking rates. An example of the style of task is provided in Figure 1. Students used the TI-Nspire calculators to create their graphs and submit them wirelessly; Mr. Callahan could then display the graphs and have students discuss contrasts between graphs. In this moment, students were collectively contributing to the current understanding of the group based on the responses given. Mr. Callahan asked probing questions with leads such as "What would the walking rate need to be …." As students responded to this question Mr. Callahan was able to assess what parts of the linear relationships students understood (such as informal notions of slope and *y*-intercept). While, looking at all of the responses on the board at once and analyzing the students' verbal responses, Mr. Callahan said there were a few students he would check in with in a minute.

Sam challenges his little sister Katie to a race. To be fair, Sam gives Katie a 40 meter head start because he knows she will walk slower. Sam's walking rate is 2.5 meters per second while Katie's walking rate is 1 meter per second. How long should the race be so that Katie wins a close race?

Figure 1. Callahan example warmup task.

Meanwhile, Mr. Callahan presented them with a new challenge to build the connections between linear relationships and proportional reasoning. Students were given a set of rectangles, some of which were similar, and were asked to explore all aspects of the set. Some students created tables of lengths and widths of the rectangles and made conjectures about patterns among the rectangles; other students cut the rectangles out and compared them visually to each other. Students recorded their work and submitted responses using the calculators with which Mr. Callahan could easily pull up their work to compare different strategies. Regardless of the strategy students utilized, they were participating in problem solving strategies with multiple representations and using discourse practices to share their mathematical thinking with their peers. Similar to the beginning of the lesson, Mr. Callahan structured the closure of the lesson by utilizing the TI-Nspire calculators to gather multiple examples of student work to foster discussion that helps both him and the students know where they were in relation to the learning targets.

The NCTM position statement on formative assessment cites several important components necessary to productive formative assessment practices including giving feedback to students, having students involved in their own learning, and the adjustment of teaching based on the results of assessment (Hattie, 2012; NCTM, 2013). As evidenced by the vignette, Mr. Callahan's lessons captured these three components successfully.

Mr. Fisher. Like Mr. Callahan, Mr. Fisher had a typical routine structure for his lessons. Students consistently started the lesson by working on some sort of warm-up problem. In this lesson, Mr. Fisher had tasks like those in figure 2 as a warm-up for students. Students efficiently logged into the calculator system to report their answers. Once students had submitted their answers using the navigator system, Mr. Fisher was able to analyze the results quickly to make an instructional decision of how to use the next segment of class. He transitioned into a whole group discussion, sharing with the class, "Identifying the variables seems to be something that we are struggling with. So let's take a look at these." He then asks students a series of questions to help them make sense of the questions together. A few of the probing questions he used were, "What are the two variables? What depends on what? What do these numbers mean in this context?" By collecting all the student responses using the navigator system and analyzing the results, Mr. Fisher was able to give real-time feedback to students and adjust his lesson to focus on asking students scaffolded questions to help them make sense of the mathematics. In this instance it meant that more time was spent on the warm-up activity, but it allowed Mr. Fisher to know that students were ready to move on before starting something new. Mr. Fisher then adjusted the next tasks to appropriately work with the time left in the lesson. He used the learning check feature to send students a document with tasks to work on in partners to explore the new content. Before the lesson ended he was able to collect student responses once again and help students generalize their progress by asking targeted questions based on what the students had submitted. The way the system collects data and summarizes data allows students anonymity, while also getting the feedback they need. This process also helps them discuss and reflect on their thinking. Previous research has shown this feature of the navigator system to be beneficial for student engagement and effective formative assessment (Roschelle, Penuel, & Abrahamson, 2004).

Two rival taxicab companies offer the following deals:

Blue Ridge Taxi: $3 per mile
Red Rides: $5 plus $2 per mile

What are the variables? Make a table representing the cost of each company for the first 10 miles.

Match each equation to the appropriate scenario.

1. An amusement park charges school groups $50 for entry and an additional $6 per student in the group.
2. Henry's bank account had $100 at the beginning of the year. He deposits $20 per month to the account.
3. A ball is dropped from a 100-foot building and drops 20 feet per second.

a) $y = 2x + 50$
b) $y = 100 - 20x$
c) $y = 100 + 20x$

Figure 2. Fisher example warm-up task.

Teacher Voices

After the school year ended, we asked the teachers to reflect on their use of the TI-Nspire Navigator systems. Mr. Callahan and Mr. Fisher both spoke specifically about using the systems to support formative assessment in their teaching.

For Mr. Callahan the calculators seem to be the glue for fostering discussion in class. "I use my TI-Nspires daily for formative assessment by using the Quick poll feature, whether it is going over the warm-up, checking homework to see where misconceptions are occurring, or for exit tickets to see how the day's learning target went. They have improved class discussions, as students can be the presenter when they share graphs and tables, and can critique the thinking of others when we look at quick poll solutions, in addition to making it easy for me to hold students accountable when they are off task. I literally can't imagine going back to teaching a math class without them.

Similarly, Mr. Fisher stated, "The TI-Nspire calculators have been a great resource I use on a daily basis. I greatly value the immediate feedback of quick polls, which I use to gauge student understanding, as well as guide my future instruction. The students love using the calculators. They are more actively engaged in class and students that rarely volunteer get a chance to share their thinking and understanding in a way that is more comfortable to them."

Making Teaching More Adaptive

The TI-Nspire calculators with the Navigator system have offered this school district a tangible system for structuring lessons, as well as collecting and utilizing student data in real time that allows teachers to adapt and inform instructional practices. These teachers have been able to implement stronger standards-based instruction with coherent lessons giving students the opportunity to grapple with mathematics, discuss mathematical thinking, and use tools and representations that help them develop conceptual understanding. As evidenced in the vignettes, these teachers were able to efficiently distribute tasks, collect student data and analyze that data, provide real-time feedback to students, and then adapt their instruction by orchestrating discourse with and among students to reach collective understanding of mathematics.

While the tool certainly made this process more efficient and effective for this school district, it is important to note that the process is where the magic happens. Even teachers with fewer technological tools can use the same skills in providing successful classroom management strategies and strong formative assessment support. Students who complete work in the "old fashioned" pencil and paper way can still communicate their thinking. However, it is important that when quick informative feedback is provided, teachers collaborate with their students to discuss misconceptions, and for the teacher to then plan for the next steps based on the results of what the students provide.

References:

Berry, III, R. Q., Rimm-Kaufman, S. E., Ottmar, E. M., Walkowiak, T. A., Merritt, E., & Pinter, H. H. (2013). *The mathematics scan (M-Scan): A measure of mathematics instructional quality* [unpublished measure]. Charlottesville VA: University of Virginia.

Black, P., & Wiliam, D. (2009). Developing the theory of formative assessment. *Educational Assessment, Evaluation and Accountability* (formerly: *Journal of Personnel Evaluation in Education*), *21*(1), 5-31.

Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. New York, NY: Routledge.

Mosher, F. (2011). The role of learning progressions in standards-based education reform.

National Council of Teachers of Mathematics. (2013). *Formative assessment*. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Formative-Assessment/

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Roschelle, J., Penuel, W. R., & Abrahamson, L. (2004). The networked classroom. *Educational Leadership*, *61*(5), 50-54.

Forces in the Pairs Death Spiral: A Mathematics and Physics Modeling Activity

Diana S. Cheng and Asli Sezen-Barrie, Towson University, Towson, MD
Alexander C. Barrie, University of Colorado at Boulder, Boulder, CO
Timothy A. Akers and Kevin Peters, Morgan State University, Baltimore, MD

The authors discuss the mathematics and physics involved with the pairs death spiral figure skating move and provide an activity sheet to use with students at the high school level.

Pairs skating is a figure skating discipline that has been contested in the Winter Olympic Games since 1908. The International Skating Union (ISU), the governing body for international figure skating competitions, has made the death spiral a required element in pairs programs (ISU, 2015). In this article, we provide the background for a high-school-level exploration of the death spiral as a mathematical model based on physical relationships between the man and the lady. An activity sheet to use with students follows.

In the pairs death spiral, the male skater pivots on the ice while the female skater moves in a circular path around him (Kerrigan & Spencer, 2003). There is a connected pull between the skaters so that the woman does not fall as she is lowered toward the ice. Usually they hold hands or wrists to maintain this pull. Centrifugal force helps the woman move horizontally outwards from the center of the rotation. A video of death spirals that can be used to motivate the student actvity is available online: www.youtube.com/watch?v=H3T5A4WsWCQ

In Figure 1, on the left we show a picture of a side view of the pairs death spiral with the following key points marked:

- M: the man's head
- B: the center of the man's pivot
- L: the woman's blade on the ice

On the right, we show the tracings that the skaters' blades make on the ice, which are approximated by circles for simplicity.

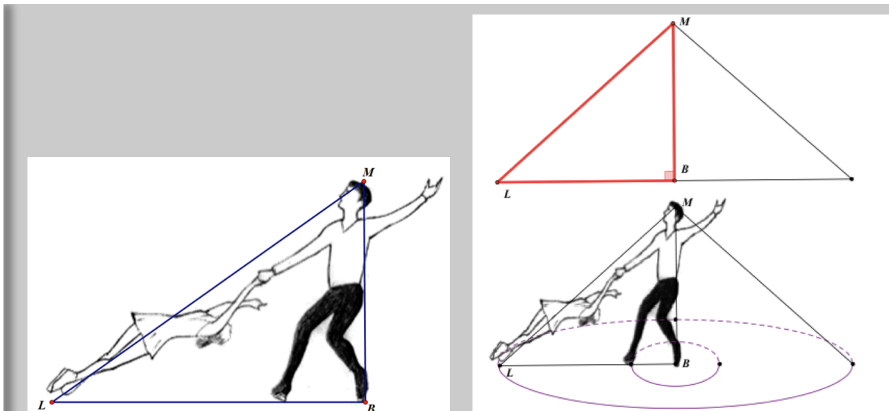


Figure 1. Performing a death spiral (adapted from Cheng & Berezovski, 2013).

The following quantities are relevant when examining the forces on the female skater during the pairs death spiral:

- m, her mass in kilograms
- r, the radius of the circular path that her blade traces, namely the distance BL in meters between her skating foot to the center of the rotation
- ω, the angular velocity as measured in radians per second
- $g = 9.8\, m/s^2$, gravity as vertical acceleration

While the female skater rotates around the male skater, the forces that she feels in the death spiral are the following:

1) Her net weight, represented by the downward vector $\vec{Y}$, which is the difference of the following two opposite forces:
    a. Her weight, the magnitude for which is calculated by $m \times g$, pulling downwards
    b. The weight she is supporting on her skating foot, supporting her upwards

    In this situation, we assume that the woman is supporting ¾ of her weight on her foot, so her net weight is ¼ of her body's weight and can be magnitude of the force can be written as ¼ $m \times g$.
2) Centrifugal force that pulls her direction outwards during circular motion, calculated from Newton's second law applied to circular motion as $m \times (\omega^2 \times r)$, represented by a horizontal vector $\vec{C}$ moving away from the center of rotation.
3) Tension in the hand that connects her to the man, represented by a vector $\vec{T}$ parallel to the straight-arms connection that she has with the man.

According to Newton's second law of motion, in order for the female skater to not fall to the ice the sum of these three forces must be zero. Thus $\vec{Y} + \vec{T} + \vec{C} = \vec{0}$. In order for the death spiral to be performed, the vector sum of $\vec{C}$ and $\vec{Y}$ must be equal in magnitude and oriented in the opposite direction of $\vec{T}$.

Figure 2 illustrates the vector diagram of these three forces: Shown on the left are $\vec{T}$, $\vec{Y}$, and $\vec{C}$; on the right are $\vec{C}$, $\vec{Y}$, and the magnitude and direction of the vector sum, $\vec{Y} + \vec{C}$.

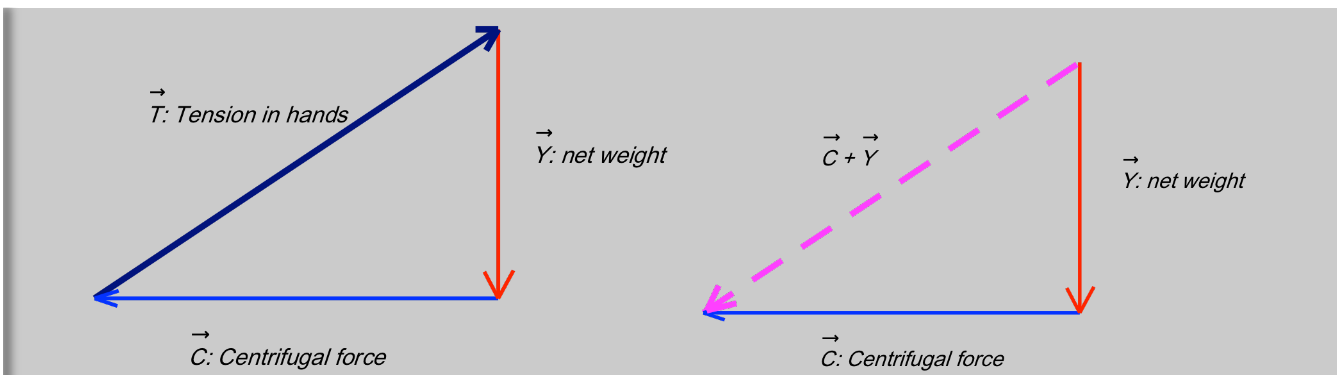


Figure 2. Vector sum of the three forces demonstrating a sum of 0.

Using the Pythagorean theorem relationship between the magnitudes of forces in the right triangle, $C = \sqrt{T^2 - Y^2}$. Substituting in the expressions for C and ω from above and simplifying yields:

$$\omega = \sqrt{\frac{\sqrt{T^2 - 0.25^2 m^2 g^2}}{r \times m}}$$

measured in radians per second. Note that the man's mass is not a variable in this equation, because we are only examining the forces on the lady during the death spiral.

Students completing the activity should be familiar with converting between measurement units. We suggest that students use technology such as graphing calculators or spreadsheets as aids in helping them solve the problems.

Common Core State Standards & Next Generation Science Standards

The activity based on this scenario addresses several Common Core State Standards for Mathematics (CCSSI, 2010) and Next Generation Science Standards (NGSS, 2013) at the secondary level. By completing the accompanying, students will have the opportunity to use the Standards listed in Tables 1, 2, and 3. The Common Core State Standards and the Next Generation Science Standards incorporated in this activity have a focus on modeling. In both mathematics and science, models help students represent real world phenomena such as the pairs death spiral. In this activity, students must conceptually understand both the physical relationships between the forces in the death spiral as well as apply the mathematical formulas explaining these relationships.

Table 1. Relevant Common Core State Standards: Standards for Mathematical Practice

Standard	Implementation
MP4: Model with mathematics.	The context of the death spiral and the relationships between the forces inherent in performing the death spiral serve as a cross-disciplinary model for using mathematics and physics.
MP5: Use appropriate tools strategically.	Students will make use of technology such as graphing calculators and spreadsheets to solve for the different variables in the formulae. They may also need to look up conversions between different measurement units.
MP7: Look for and make use of structure.	In order to respond to the qualitative questions, students need to make sense of the formulas conceptually and consider how certain quantities affect others by generalizing what will happen without exact values to substitute into the equations.

Table 2. Relevant Common Core State Standards: Content Standards

Standard	Implementation
6.RP.A.3.d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	Students are asked to convert between radians per minute and revolutions per minute, centimeters and meters, meters and feet, and kilograms and pounds.
HSN.VM.B.4.b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	The free body diagram representing forces on the lady uses vectors that need to be added. The tension between the hands of the man and the lady is equal in magnitude to the vector sum of the centrifugal and net weight, but is opposite in direction.
HSA.CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	In order to solve for mass, radius, angular velocity, and tension, it is necessary for students to rearrange a formula to highlight the quantity of interest, the unknown in each problem.
HSA.APR.D.6. Rewrite simple rational expressions in different forms	The formulas used involve rational expressions.

Table 3. Relevant Next Generation Science Standards

Standard	Implementation
MS-PS2-2. Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object. *[Clarification Statement: Emphasis is on balanced (Newton's First Law) and unbalanced forces in a system, qualitative comparisons of forces, mass and changes in motion (Newton's Second Law), frame of reference, and specification of units.]*	In the death spiral problem, Newton's Second Law is interpreted in the case of circular motion. The activity helps students relate the concept of Newton's Second Law to a real life situation, thus making it easier to understand why the lady's mass, the angular velocity, and the radius of the circular path relate to the forces in the system.

Systems and System Models. Models can be used to represent systems and their interactions—such as inputs, processes and outputs—and energy and matter flows within systems. (MS-PS2-1),(MS-PS2-4)	The death spiral is represented visually to support students' understanding of the movement. Students must understand the interactions between the lady and the man, as well as know the important points where the forces are being applied. A vector model is used to show how different forces (tension, weight and centrifugal force) interact with each other.

References

Cheng, D., & Berezovski, T. (2013) Ice math: Geometric software illustrating concentric circles in figure skating. *Journal on Mathematics, 2*(4), 14-20.

Common Core State Standards Initiative (CCSSI). (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from www.corestandards.org /wp content/uploads/Math_Standards.pdf

International Skating Union. (2015). *Communication no. 1944: Single & pair skating – Scale of values, levels of difficulty and guidelines for marking grade of execution, season 2015-2016*. Retrieved from static.isu.org/media/207718/1944-sptc-sov-communication-2015-2016.pdf

Kerrigan, N. & Spencer, M. (2003). *Artistry on ice*. Champaign, IL: Human Kinetics.

Next Generation Science Standards Lead States (NGSS). (2013). *Next generation science standards: For states, by states*. Washington, DC: The National Academy Press.

Student Activity: Forces in the Pairs Death Spiral

This activity sheet and solutions are available for download at
www.ncctm.org/resources-activities/the-centroid1/centroid-articles/

Part 1: Qualitative Questions

1. What force holds the man in place?
2. What prevents the man from tipping over towards the lady during the death spiral?
3. If the man squats down to lower his height, how will this motion affect the tension in the arms?
4. If the man suddenly lets go, in what direction / path will the woman go?

Part 2: Developing relationships between quantities

1. Draw a visual diagram representing the relationship between three forces that impact the woman as she performs the death spiral: 1) The straight-arms tension connecting her hand with the man's hand (T), 2) The lady's net weight (Y) going vertically downwards, and 3) Centrifugal force going horizontally outwards from the man.
2. What geometric shape is formed by the three forces, T, Y and C? Write an equation relating the magnitudes of these forces.
3. Use the following information to develop an equation for the lady's angular velocity (ω) as a function of the woman's mass (m), radius between her skating blade and the man's pivoting skate (r), gravity (g), and T.
   - The lady's net weight (Y) is the difference of two opposite forces: Her weight (calculated by $m \times g$), and the weight she supports on her skating foot (assume the lady supports $\frac{3}{4}$ of her weight on her foot).

- Newton's second law, Force = Mass × Acceleration, applies in this situation. In circular motion, the force involved is centrifugal force, C. The acceleration in circular motion can be represented as $\omega^2 \times r$.

4. Using your existing equation for ω, write equivalent equations to express the following:
    - m as a function of ω, T, r, and g.
    - T as a function of ω, m, r, and g.
    - r as a function of ω, T, m, and g.

Part 3: Quantitative Questions

Problem A: A female skater with mass 60 kg is performing a death spiral with her partner. The centrifugal force on the woman is 261.52 N. As she lowers her body, her skating boot travels from 1.5 meters away from his pivoting leg to 2.0 meters away. Will this increase in radius cause an increase or decrease in angular velocity, and why? By how many radians per second does her angular velocity change? What is the percentage change of her angular velocity (round to nearest tenth)?

Problem B: A female skater with mass 65 kg is performing a death spiral with her partner. When her skating foot is 4.92 ft away from his pivoting leg, the partners' hands maintain 400 N of tension. They then increase the amount of tension they have to 500 N but keep the same radius. How does this affect the angular velocity, and why? By how many revolutions per second does the angular velocity change?

Problem C: A female skater with mass 65 kg is performing a death spiral with her partner. When she is 150 centimeters away from the man's pivoting leg, they begin rotating at 2.078 radians per second. They stay the same distance apart but they decrease the angular velocity by 0.138 radians per second.

a) Will the tension in their hands need to increase or decrease in order to accomplish this change in angular velocity? By how much is the tension changed?
b) By how much does their centrifugal force change?

Problem D: A man is comfortable with providing 500 N of tension for his partner in a death spiral. His coach measured that he and his partner rotated at 0.374 revolutions per second. If he maintains the same radius and tension for a woman who has 8 kg less mass, how will his angular velocity be impacted? Will he rotate at a faster or slower angular velocity with the woman with less mass?

Problem E: If a man is able to use his hand to create tension between 300 N for a death spiral with radius 1.5 meters and 350 N for a death spiral with radius 2 meters, what is the range of masses of female skaters who he could support in a death spiral at an angular velocity of 1.629 radians per second?

Problem F: A man and a woman whose mass is 60 kg are rotating at 1.910 radians per second and they have straight-arms tension 400 N. The same man then performs the death spiral with the same angular velocity and tension, but with a different skater whose mass is 2.1 kg higher. Would the death spiral with the second skater be at a smaller or larger radius? What is the change in radius length in centimeters?

Problem G: If a man and a woman whose mass is 70 kg want to rotate in the death spiral at an angular velocity between 0.217 revolutions per second and 0.241 revolutions per second with radius 190 cm, what is the range of tensions they need to create?

Problem H: A female skater who weighs 132.28 pounds is performing a death spiral that has a maximum angular velocity of 2.1 radians per second.

a) Rounded to the nearest tenth, what is the maximum radius between the female skater's boot and the man's pivoting foot?
b) Rounded to the nearest hundredth, what is the centrifugal force this woman feels during this death spiral?

Trust Fund Scholarships: Now $1000

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

This year the Trust Fund Committee is pleased to announce that the amount that can be requested to help with the cost of graduate coursework is now $1000.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants an scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson, richards@elon.edu.

Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

Dr. Leibniz and His Amazing Triangle

William C. Bauldry, Appalachian State University, Boone, NC

The author discusses the Leibniz harmonic triangle, showing its relationship to Pascal's triangle and suggesting a number of investigations suitable for high school students.

Gottfried Leibniz created the Leibniz harmonic triangle in order to study sums, differences, and series. The harmonic triangle has fascinating properties that students can discover while reinforcing and extending their mathematical knowledge base. There are many connections to Pascal's triangle that students can also find. We'll look at several properties of the triangle, pointing to possible student explorations, and close with links to more information and resources. The exploration presented supports several Common Core State Standards for Mathematics (CCSSI, 2010): MP2, Reason abstractly and quantitatively; MP3, Construct viable arguments and critique the reasoning of others; MP7, Look for and make use of structure.

Development of the Triangle

Gottfried Wilhelm von Leibniz (1646 –1716) was a polymath who independently discovered calculus, invented the binary system and an improved mechanical calculator, and designed a system of logic (O'Connor & Robertson, 1998). He began his study of mathematics as a pupil of Huygens in Paris while there on a political mission; he quickly progressed to the frontiers of then current knowledge. The first version, shown in Figure 1, of what he would come to call the 'harmonic triangle' appeared in an unpublished paper in 1672 (Hofmann, 1974).

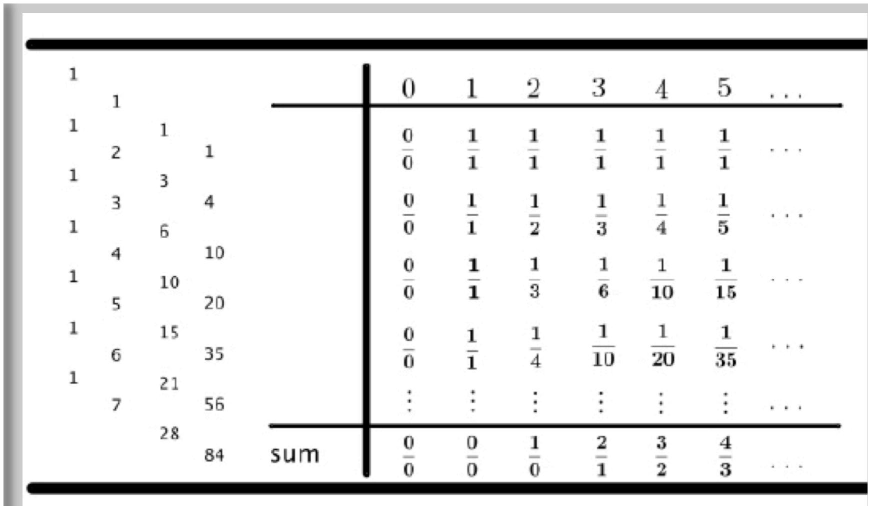


Figure 1. A reproduction of Leibniz's first harmonic triangle diagram.

Leibniz used Pascal's triangle to generate the harmonic triangle, developing the array to calculate infinite sums. Leibniz sent his new version minus the display of Pascal's triangle to Henry (Heinrich) Oldenburg, Secretary of the Royal Society of London, calling it the harmonic triangle in a letter (c.1676). Leibniz was fascinated by the analogues his triangle had to Pascal's; these observations helped steer him toward developing calculus (Boyer & Merzbach, 1991). Leading our students to follow Leibniz can enhance their pattern recognition and conjecture-making abilities (Stones, 1983).

Exploring Leibniz's Harmonic Triangle

Let's set Pascal's triangle and Leibniz's Harmonic Triangle side by side (Figure 2) in order to more easily see the relations. Number the rows and columns beginning with 0 ; i.e., the first row is Row 0 and the first column (first diagonal, going from upper right to lower left) is Column 0. Then the element in row n and column k of Pascal's triangle can be calculated as

$$P_{n,k} = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

for $n = 0, 1, 2, \ldots$ and $k = 0, 1, 2, \ldots, n$. This definition quickly leads students to ponder why 0! is equal to 1.

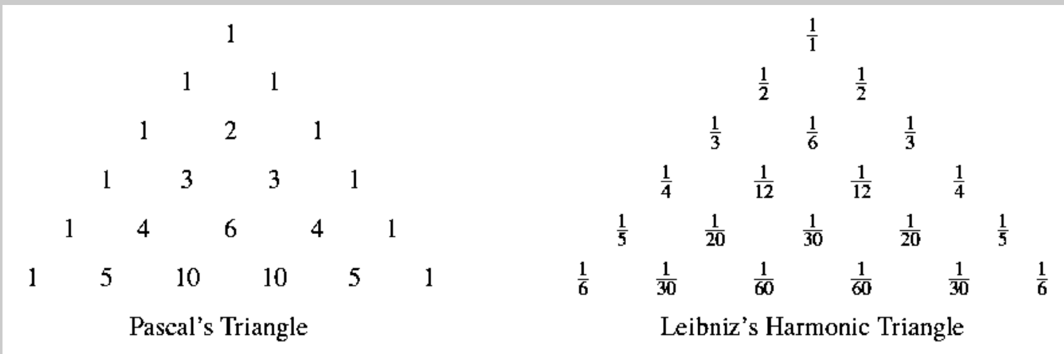


Figure 2. A comparison of the triangles of Pascal and Leibniz.

Some of the properties of Pascal's triangle that we typically study are:

1. $P_{n,k} = P_{n,n-k}$ for $n \geq 0$ and $k = 0, 1, 2, \ldots, n$.
2. $P_{n,k} = P_{n-1,k-1} + P_{n-1,k}$ for $n \geq 1$ and $k = 1, 2, \ldots, n$.
3. $\sum_{k=1}^{n} P_{n,k} = 2^n$ for $n \geq 0$.
4. $\sum_{j=0}^{n} P_{j,k} = P_{n+1,k+1}$ for $n \geq 0$ and $k = 0, 1, 2, \ldots, n$.

Proving Properties 1 and 2 is easy using the definition of $P_{n,k}$. Property 4 can be proved by iterating Property 2. Proving Property 3 can come directly from showing that

$$(x+y)^n = \sum_{k=0}^{n} P_{n,k} \cdot x^{n-k} y^k$$

and then setting both x and y equal to 1.

These properties lead us to ask, *Are there relations between Pascal's and Leibniz's triangles? Are there corresponding relations within Leibniz's harmonic triangle?*

Let $L_{n,k}$ be the number in row n and column k (diagonal) of Leibniz's harmonic triangle. Start by factoring the least common multiple from the denominators of Row 3 (remember that the first row is Row 0) thinking of the row as a vector. We see:

$$[4 \;\; 12 \;\; 12 \;\; 4] = 4 \cdot [1 \;\; 3 \;\; 3 \;\; 1]$$

Now try rows 4 and 5:

$$[5 \;\; 20 \;\; 30 \;\; 20 \;\; 5] = 5 \cdot [1 \;\; 4 \;\; 6 \;\; 4 \;\; 1]$$
$$[6 \;\; 30 \;\; 60 \;\; 60 \;\; 30 \;\; 6] = 6 \cdot [1 \;\; 5 \;\; 10 \;\; 10 \;\; 5 \;\; 1]$$

Aha! It appears that

$$L_{n,k} = \frac{1}{(n+1) \cdot P_{n,k}}.$$

Formalizing and proving this conjecture makes a good student project.

Another interesting aspect of Leibniz's harmonic triangle comes from looking at the elements in factored form. Consider the diagonals, shown in Figure 3. The pattern formed by these factored fractions is another straightforward conjecture that students can make and then derive using factorials.

Diagonal 0:	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\cdots$
Diagonal 1:	$\frac{1}{2}=\frac{1}{1\cdot2}$	$\frac{1}{6}=\frac{1}{2\cdot3}$	$\frac{1}{12}=\frac{1}{3\cdot4}$	$\frac{1}{20}=\frac{1}{4\cdot5}$	$\cdots$
Diagonal 2:	$\frac{1}{3}=\frac{1\cdot2}{1\cdot2\cdot3}$	$\frac{1}{12}=\frac{1\cdot2}{2\cdot3\cdot4}$	$\frac{1}{30}=\frac{1\cdot2}{3\cdot4\cdot5}$	$\frac{1}{60}=\frac{1\cdot2}{4\cdot5\cdot6}$	$\cdots$
Diagonal 3:	$\frac{1}{4}=\frac{1\cdot2\cdot3}{1\cdot2\cdot3\cdot4}$	$\frac{1}{20}=\frac{1\cdot2\cdot3}{2\cdot3\cdot4\cdot5}$	$\frac{1}{60}=\frac{1\cdot2\cdot3}{3\cdot4\cdot5\cdot6}$	$\frac{1}{140}=\frac{1\cdot2\cdot3}{4\cdot5\cdot6\cdot7}$	$\cdots$

Figure 3. The diagonals from Leibniz's triangle.

Recall that adding two elements in Pascal's triangle produced the element immediately below in the triangle. Let's try this for Leibniz's triangle. Does $L_{n,k} = L_{n-1,k-1} + L_{n-1,k}$? Checking with the first two entries in the fourth row (i.e., row 3, n = 3):

$$\frac{1}{20} \neq \frac{1}{12} + \frac{1}{4}$$

Oh, well. What about a different combination of entries, such as $\frac{1}{12}$, $\frac{1}{30}$, and $\frac{1}{20}$ (i.e., $L_{3,2}, L_{4,2}, L_{4,3}$)?

$$\frac{1}{20} = \frac{1}{12} - \frac{1}{30}$$

Hmmm. Does $L_{n,k} = L_{n-1,k-1} - L_{n,k-1}$? How would we test this idea? We can show that

$L_{n-1,k-1} = L_{n,k-1} + L_{n,k}$ for $n \geq 0$ and $K = 0, 1, 2, \ldots, n$.

Reversing Pascal's 'adding elements *above* to generate the element *below*,' in Leibniz's harmonic triangle we add the two elements *below* to generate the entry *above*!

With Pascal's triangle, for Property 3 above, we added down a column. Reversing the direction, in Leibniz's harmonic triangle, we would add going up the column, or in the triangular display, up the diagonal, but now the sums are infinite. Take the second diagonal. (Don't forget to start counting at 0.)

$$S = \frac{1}{3} + \frac{1}{12} + \frac{1}{30} + \frac{1}{60} + \frac{1}{105} + \cdots$$

Follow Leibniz's thinking by repeatedly applying our relation $L_{n,k} = L_{n-1,k-1} - L_{n,k-1}$. Therefore:

$$S = \frac{1}{3} + \left[\frac{1}{6} - \frac{1}{12}\right] + \left[\frac{1}{12} - \frac{1}{20}\right] + \left[\frac{1}{20} - \frac{1}{30}\right] \ldots$$

Re-associate the terms to see:

$$S = \frac{1}{3} + \frac{1}{6} + \left[-\frac{1}{12} + \frac{1}{12}\right] + \left[-\frac{1}{20} + \frac{1}{20}\right] + \left[-\frac{1}{30} + \frac{1}{30}\right] \ldots$$

This type of sum is referred to as *telescoping*. After making a convergence argument based on the terms cancelling, we arrive at:

$$S = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

Thus

$$\frac{1}{3}+\frac{1}{12}+\frac{1}{30}+\frac{1}{60}+\frac{1}{105}+\cdots=\frac{1}{2},$$

which is the element *above* $\frac{1}{3}$! In general, we can show that

$$L_{n-1,0}=\sum_{j=0}^{\infty}L_{n+j,j}.$$

Can you find other relationships? Here are a two more to think about: Use the telescope technique to demonstrate that:

$$L_{n-1,k}=\sum_{j=0}^{\infty}L_{n+j,k+j}$$

Look back at Leibniz's original diagram in Figure 1. How do you explain the bottom row labeled 'sum'?

Online Resources for Leibniz's Harmonic Triangle

There are many interesting web pages that provide sources for further study or student projects.

- A poster for the 'diagonal elements conjecture' at NRICH: nrich.maths.org/content/id/4745/NRICH-poster_HarmonicTriangles.pdf
- Toni Beardon's comparison of Pascal's and Leibniz's triangles at NRICH: nrich.maths.org/4781
- A 'calculator' to compute the first *n* rows of Leibniz's harmonic triangle: www.easycalculation.com/algebra/leibniz-harmonic-triangle.php?
- "Diagonal Sums in the Harmonic Triangle," by Marjorie Bicknell-Johnson, A. C. Wilcox High School, Santa Clara, CA [1]: www.fq.math.ca/Scanned/19-3/bicknell.pdf
- A short web page listing several properties: www.logicville.com/sel33.htm

Conclusion

Leibniz's harmonic triangle has quite a number of intriguing properties, some easy to discover, others very deep. He used the telescoping-differences procedure to sum the diagonals creating a number of interesting summation formulas. He also rediscovered the divergence of the harmonic series by analyzing Column 0. Investigating combinations of differences and sums was critical to setting Leibniz on the path to discovering the Fundamental Theorem of Calculus. Students following his lead can learn much about pattern recognition and conjectures, working with factorials, and make connections to the historical development of calculus.

References

Common Core State Standards Initiative (CCSSI). 2010. *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from www.corestandards.org /wp content/uploads/Math_Standards.pdf

Bicknell-Johnson, M. (1981). Diagonal sums in the harmonic triangle. *Fibonacci Quarterly, 19*, 196–199.

Boyer, C. B., & Merzbach, U. C. (1991). *A history of mathematics (2nd ed.).* New York: John Wiley & Sons, Inc.

Hofmann, J. E. (1974). *Leibniz in Paris, 1672-1676: His growth to mathematical maturity* [English translation of *Verlag, R. O. (1949). Die Entwicklungsgeshichte der Leibnizshen Mathematik warend des Aufenthalts in Paris (1672-1676)*]. New York: Cambridge University Press.

O'Connor, J., & Robertson, E. F. (1998). *MacTutor's Leibniz biography*. Retrieved from www-history.mcs.st-and.ac.uk/Biographies/Leibniz.html

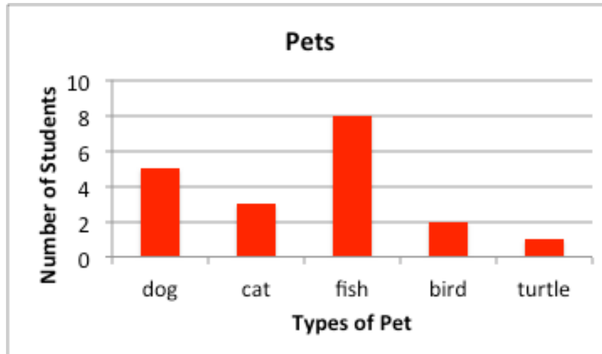
Stones, I. D. (1983). The harmonic triangle: Opportunities for pattern identification and generalization. *The Mathematics Teacher, 76*, 350–354.

Problems to Ponder

Holly Hirst, Appalachian State University, Boone, NC

Spring 2017 Problems

Grades K–2: Mrs. Burns' second grade class drew the chart to the right showing the pets belonging to the students in the class.



Joan has 2 dogs and Jayleen has 8 fish, but everyone else in the class has only one pet. How many pets are there total? How many students are in the class?

Grades 3–5: Kristin gave half of her jelly beans to her brother Ken and ate the rest. Ken ate half of those jelly beans and gave the rest to brother Kyle. Kyle ate 5 of those and gave the last 5 jelly beans to brother Kamal to eat. How many jelly beans did each child eat?

Grades 6–8: Mr. Gibson is creating a tulip garden and wants to plant a large area with 300 bulbs with equal amounts of each of four colors. He already has 32 red bulbs, 26 orange, and 9 yellow, but no white ones. How many more of each color does he need? If the bulbs cost 25 cents and you get 10% off if you purchase 50 bulbs of the same color, how much will Mr. Gibson spend?

Directions for submitting solutions:

1. Students: NEATLY print the following at the top of each solution page:
   - Your first name (we will not publish last names)
   - Your teacher's name
   - Your grade
   - Your school
2. Submit one problem per page. ***Students who submit correct solutions will be recognized by their first names only in the next issue of The Centroid.*** We will also publish one or two especially creative or well-written solutions from those submitted. If you would rather not have your name published, please so indicate on your submission.

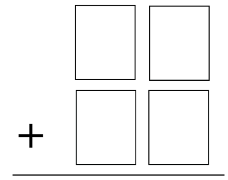
Proper acknowledgement is contingent on legible information and solutions. Send solutions by 10 June 2017 to:

Problems to Ponder, c/o Dr. Holly Hirst
Mathematical Sciences
BOX 32069 Appalachian State University
Boone, NC 28608

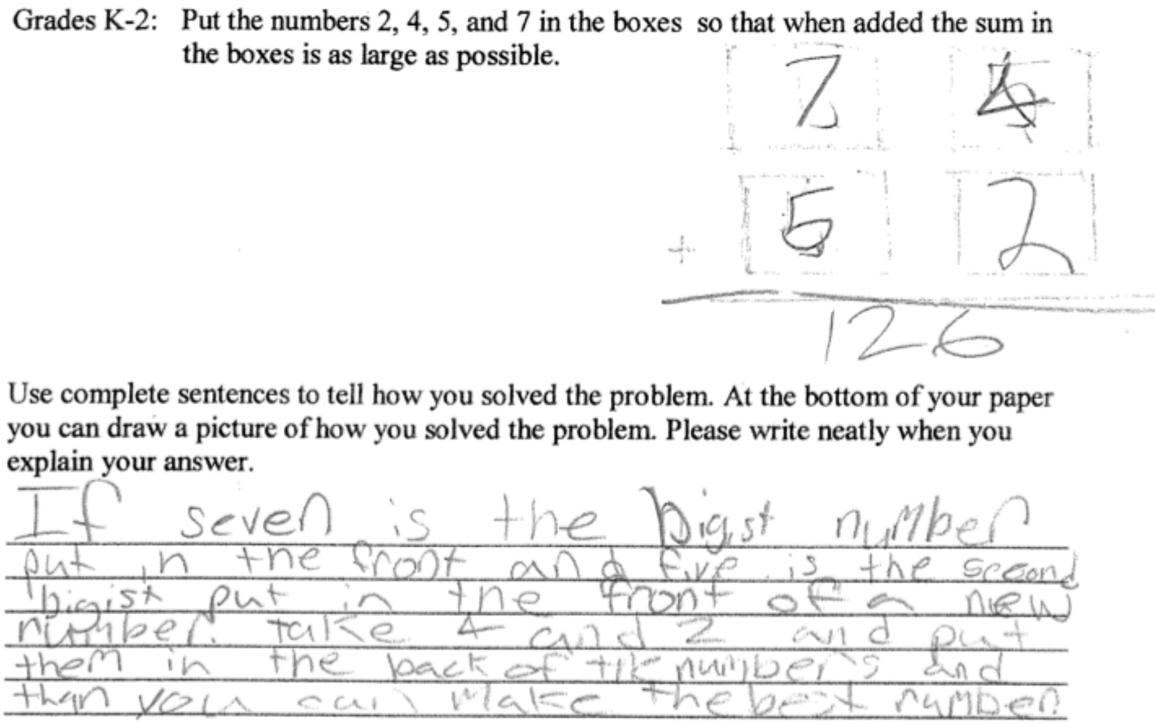
A submitted solution indicates the student completed a significant part of the work. Please try to have the students use complete sentences when they write up their solutions to promote effective communication of their ideas.

Fall 2016 Problem Solutions

Grades K–2: Put the numbers 2, 3, 5, and 7 in the boxes so that when added the sum is as large as possible.



Three Correct solutions were received from Mrs. Byrne's and Mrs. Lane's second grade classe at Ravenscroft School in Raleigh. Here is one of them:



Grades K-2: Put the numbers 2, 4, 5, and 7 in the boxes so that when added the sum in the boxes is as large as possible.

$$\begin{array}{r} 7\,4 \\ +\ 5\,2 \\ \hline 126 \end{array}$$

Use complete sentences to tell how you solved the problem. At the bottom of your paper you can draw a picture of how you solved the problem. Please write neatly when you explain your answer.

If seven is the bigst number put in the front and five is the second bigist put in the front of a new number. take 4 and 2 and put them in the back of the numbers and than you can make the best number

Grades 3–5: A triangle with a perimeter equal to 30 inches has sides in a ratio of 2:3:5. What is the length of the longest side?

Grades 6–8: A company produces fire crackers. During a recent test of the quality of the fire crackers, 90 firecrackers were lit and 3 failed to explode. If the company produces 1500 fire crackers each day, how many are expected to fail on average in each daily batch?

Editor's note: We did not receive any solutions to the 3-5 or 6-8 problems, both of which could be approached as ratio problems; perhaps these would be better as spring problems given the topic! Here are some hints to get students started, and please do send solutions for these problems by 10 June 2017!

Triangle Side Ratio: If the ratio of side lengths is 2:3:5, then a triangle that fits this ratio could have sides 2, 3, and 5. Of course, that triangle would not have perimeter 30. What if we double the shortest side? Then to preserve the ratio, we would need to double the lengths of the other sides as well: 4, 6, and 10. Still not a perimeter of 30, but…

Firecracker Testing Ratio: If the test produced 3 failures out of 90, then the part : whole ratio of failures is 3: 90. The question then becomes what equivalent ratio would result from 1500?

2016 Outstanding Secondary Mathematics Teachers

Reported by Kitty Rutherford, North Carolina Department of Public Instruction, Raleigh, NC

Each year, school principals are encouraged to nominate the teacher they believe does the most effective job teaching mathematics in their school. From those nominated, each LEA selected one teacher to represent the best in mathematics teaching from the entire system. The teacher selected from each LEA receives a one year membership in NCCTM, recognition at the State Conference, and a special memento of the occasion. The grade level cycles, and this year the outstanding teachers were chosen from among the best secondary mathematics teachers in North Carolina.

Alamance Burlington: Deanna Beckham
Alexander: Ann Bebber
Alleghany: Derrick Murphy
Anson: Kristin Park
Ashe: Jennifer Williams
Asheboro City: Sarah Trollinger
Avery: Claudette Reep
Beaufort: Lisa Johnson
Bladen: Jill Smith
Brunswick: Susan Quinn
Buncombe: Leigha Jordan
Cabarrus: Christy Bentley
Caldwell: Jill Pippen
Carteret: Lynn DeRosia
Caswell: Jonathan Barnes
Charlotte/Mecklenburg: Todd Rackowitz
Chatham: Patrick Tillett
Clinton City: Angela Corrine Harding
Columbus: Brittany Edwards
Craven: Kerri Bogue
Cumberland: Kenneth Williams
Currituck: Diane Davenport
Dare: Elizabeth Brown
Durham: Christy Simpson
Edgecombe: Tiffany Warren
Franklin: Linda Stephens
Gaston: Susan Adams
Gates: Jennifer Dail
Greene: Heather Davis
Guilford: Danielle Hendren
Hickory City: Amanda Gerken
Iredell/Statesville: Kelly Lewis
Johnston: Bill Worley
Kannapolis City: Felicia Shepard
Lee: Whitney Testa
Lenoir: Nicollette Morgan
Montgomery: April Daywalt
Moore: Karen Rhea
Mt. Airy City: Kelly Holder
Neuse Charter: Paige Bruner
New Hanover: Emily Myers
Newton-Conover: Alicia Rayfield
Onslow: Robert Bryant
Person: Jaclyn TenEyck
Pitt: Theresa Boggs
Randolph: Preston Todd Lomax
Richmond: Kim Floyd
Rockingham: Tara Beal
Rowan/Salisbury: Ashley Lanning
Rutherford: Ashley McComas
Sampson: Whitney Lamm
Scotland: Michelle Williams
Stanly: Christi Ritchie Edwards
Swain: Katrina Frizzell
Wake: Lindsay Rice
Watauga: Andy Eggers
Wayne: Rebecca Hare
Wilkes: Katrina Hurley
Wilson: Debbie Bass
Winston-Salem/Forsyth: Charita Martin Ward
Yadkin: Abby Salas

2016 Outstanding Mathematics Education Students

Reported by Todd Abel, Appalachian State University, Boone, NC

Each Fall, NCCTM sponsors the selection of Outstanding Mathematics Education Students, one from each region of NCCTM. The two recipients of this year's award are Taylor Cesarski from Elon University and Sarah Marsh from East Carolina University.

Outstanding students Sara Marsh (l) and Taylor Cesarski

The 2016 Outstanding Mathematics Education Student for the Central Region is **Taylor Cesarski** from Elon University. Ms. Cesarski is a senior secondary mathematics education major with an overall GPA of 3.959 and a mathematics GPA of 4.0. In addition to her outstanding work as part of her program, she engaged in mathematics research as part of the Summer Undergraduate Research Experience at Elon and has presented it at local, state, regional, and national undergraduate research conferences. As an Elon University Teaching Fellow, she has served as the Special Events chair, and has also been asked to serve as student representative on many advisory committees. She is a member of honor societies Phi Mu Epsilon, Phi Kappa Phi, and Phi Beta Kappa in addition to minoring in Spanish, coaching, serving with her sorority Alpha Chi Omega, and volunteering with InterVarsity. As a Noyce scholar, she has dedicated many hours to working with students and teachers in the Alamance-Burlington School System and is committed to working in a high-need school district after graduation. It seems certain that she will serve those student very well.

The 2016 Outstanding Mathematics Education Student for the Eastern Region is **Sarah Marsh** from East Carolina University. Ms. Marsh is a senior, and has an overall GPA of 3.903 while working toward a *triple* major in Mathematics Education, Mathematics, and Psychology. She has served as the President of the Gamma Chapter, ECU's NCCTM affiliate, for which she has planned meetings, invited speakers, and coordinated social and fund-raising activities. She has also served on the NCCTM board as student representative from the eastern region. Her outstanding academic achievement has earned her several ECU mathematics and mathematics education scholarships. In her spare time, she tutors students in mathematics, works in the Mathematics, Science, and Instructional Technology Education Department, works in the department computer lab, swims on the ECU Club Swim Team, and serves as an official for youth soccer. It should come as no surprise, then, that she is described as "proactive, organized", "enthusiastic", "energetic", and a "strong leader" by her professors, the embodiment of the type of future teacher we are excited to honor.

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Sandra Childrey [schildrey@wcpss.net], with questions.

2016 Innovator Award Winners

Eric and Kimberly Marland

Reported by Todd Abel, Appalachian State University, Boone, NC
and Janice Richardson, Elon University, Elon, NC

The purpose of the NCCTM Innovator Award is to recognize and reward individuals and/or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM. The Recipients of this year's award are Eric Marland of Appalachian State University and Kimberly Marland of Marland Architecture in Boone, NC.

Eric and Kim Marland have given extensively of their own time and resources to grow interest in and excitement about STEM fields by creating opportunities for students in western North Carolina and East Tennessee to engage in rich, meaningful, and enjoyable problem-solving in mathematics and robotics. Motivated in part by the interests of their own children – but more so by a desire to help kids excited by mathematics and engineering to appreciate the process of solving a problem and feel some success in doing so – our awardees have founded, organized, run, and expanded two very successful competitions for students.

Each year since 2012, they have organized and run the *Watauga SumoBot* competition, where teams of elementary, middle, and high school students compete with autonomous LEGO robots in a sumo-style competition. If you've never watched it, it's immensely entertaining. As numbers have increased, our awardees have recruited sponsors and moved into the Holmes Convocation Center at Appalachian State University in order to accommodate the number of participants and observers. Students at these competitions often work extensively at their schools to prepare for the competition, and are excited to demonstrate their designs. When they don't win, they huddle to make modifications that will make their robots more successful, modeling the type of in-the-moment engineering and problem-solving that we hope to encourage in students. These competitions were initially part of of NASA NC Space Grant, which also funded a robotics workshop for teachers and made robotics kits available in the Appalachian State University Math/Science Education Center for teachers to check out in use in their schools. The director of that center, Dr. Phillip Johnson, reports that they are in great demand. In addition, the Marlands have founded the *Elevating Mathematics* tournaments, which give opportunities for students in grades 4-6 to engage in mathematical problem-solving. Though winners are certainly crowned at these competitions, the emphasis is on providing many students with opportunities to participate and experience success in mathematical problem solving. Growing out of volunteer work with a math club at a local school, these half-day events have been founded, organized, and financially supported by our awardees simply because they believe such an opportunity is important to provide for students. Up to 30 teams of five students each participate annually in an event hosted at Appalachian State University, and events have also been held at Lenoir-Rhyne University and East Tennessee State University in order to expand the reach of this opportunity. These two program are but a part of the service that the Marlands have offered to mathematics and science education in their community, showing how they work to create opportunities for students to learn, grow, and participate when they see that such opportunities are not present.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website [ncctm.org]. More information can be obtained from Rose Sincrope [sinicroper@ecu.edu].

2016 Rankin Award Winners

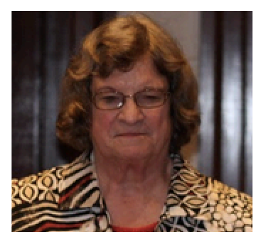
Bampia A. Bangura and Carol W. Midgett

Reported by Lee Stiff, North Carolina State University, Raleigh, NC

At the 46th Annual State Mathematics Conference, NCCTM presented Dr. Bampia A. Bangura of North Carolina A&T State University and Ms. Carol W. Midgett of Southport, NC with the W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual.

Dr. Bangura is widely regarded as a dedicated mathematics educator who is devoted to the teaching of mathematics and the advancement of the teaching profession. He has worked tirelessly to facilitate the success of students in classrooms as well as the teachers who serve these students, and is an individual who has demonstrated strong leadership in a variety of roles and situations, all of which promoted the enhancement of mathematics education in North Carolina and beyond.

Dr. Bangura has served as a department head of mathematics departments in secondary school and college, and as a department head of teacher education. He has provided leadership as coordinator of student teaching, student research, and mathematics education programs. Furthermore, he has championed the use of calculator technologies to serve the needs of students in the teaching and learning of mathematics over his long career. He has presented mathematics lessons on television for young math learners; served multiple times as a Member of the NCCTM Board of Directors; served the NCCTM Central Region as Chair for its Evaluation Subcommittee and as Vice President for Colleges; served as Chair of the NCCTM Special Awards Committee; received the NCCTM Innovator Award; and served as Chair of the Sub-committee for the Outstanding Coach/Sponsor Award of the State NCCTM Math Contest Committee. And, most importantly, Dr. Bangura has "enviable characteristics" for being able to work with all students, no matter what level of understanding of math they possess, in their efforts to learn high quality mathematics. Indeed, Dr. Bangura is recognized as a person whose service to NCCTM and the North Carolina mathematics education community, is exemplary.



For more than 45 years, **Ms. Midgett** has served the teachers of North Carolina with great distinction. She has been unparalleled as a champion of quality instruction for the teachers and students of North Carolina, and the Nation. And, she has influenced the opportunities for hundreds of students to learn math, especially beginning in the early grades.

Ms. Midgett is a National Board Certified Teacher who has successfully taught elementary, middle grades, university, and post-university students. She is a North Carolina Presidential Awardee in Elementary Mathematics, active in promoting the professional development of beginning and experienced teachers; providing insights, perspectives, and guidance through professional writings; and is described as a person who demonstrates the professionalism and commitment to quality mathematics teaching that all students in North Carolina should enjoy. In the State, Ms. Midgett served as a Leadership Coordinator for the "North Carolina Partnership for Improving Mathematics and Science," and a Leader in the implementation of the "TAP MATH Project" which develops school-based visions of quality mathematics instruction and creates K-8 Mathematics Instructional Leadership Teams in school districts across North Carolina. At the national level, she has served on NCTM Standards-based writing teams, and was a facilitator for EDC's "Lenses on Learning," a K-8 staff development program designed to help school leaders learn about standards-based mathematics education. She has conducted numerous workshops and sessions for the North Carolina Department of Public Instruction, school districts, and area colleges; has been a curriculum writer and math education consultant; helped develop North Carolina math standards and test specs for the End-of-Grade Assessments; and has served as a Regional Vice President and Board Member of NCCTM.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the “awards” area of the NCCTM Website [ncctm.org]. More information can be obtained from: Lee V. Stiff [lee.stiff@ncsu.edu]

Presidential Awards for Excellence in Teaching

Each year teachers from each state can be nominated to receive the Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) – the highest honor bestowed by the United States government specifically for K-12 mathematics and science teaching. A selection committee, coordinated through the North Carolina DPI, selects up to five names from among the nominees to forward for national consideration.

Two North Carolina teachers were recently named recipients of the PAEMST: The 2014 recipient is Kayonna Pitchford, previously with Stoney Point Elementary, Cumberland County Schools, and now with the University of North Carolina at Pembroke. The 2015 recipient is Lauren Baucom, Forest Hills High School, Union County Schools. The 2016 state finalists were Candace Crothers, Claudia Fann, and Heather Landreth.

Pictured (l to r): Lauren Baucom, Heather Landreth, Candace Crothers, Claudia Fann, Kayonna Pitchford

2017 NCTM Meeting

Creating Communities and Cultivating Change
5-8 April 2017
San Antonio, Texas

Network with thousands of your peers and fellow math education professionals to exchange ideas, engage with innovation in the field and discover new learning practices that will drive student success. Learn more at [nctm.org/annual].

Outstanding Coach/Sponsor Award

Celia Rowland

Reported by Philip Rash, NC School of Science and Math, Durham, NC

The purpose of this award is to recognize and reward an individual who has made an outstanding and noteworthy contribution to Mathematics Education and NCCTM by having formed, coached, and sponsored teams or groups of students in mathematics competitions. This year's awardee is Celia Rowland, a math teacher from William G. Enloe Magnet High School in Raleigh.

Celia teaches several math courses including AP Statistics, AP Calculus and Geometry. Celia has been involved with Math contests since moving to North Carolina in 2002. Celia is a very enthusiastic and helpful coach, and has encouraged hundreds of students over the past 14 years to participate in many contests. Some of those students have gone on to participate in the USAMO and been selected for the US IMO team.

Celia mentors teams each year participating in both the math contests and quiz bowl contests, while still keeping up with her teaching load and all she does with contests back at her school. To quote her nominator for this award, "in short, she is incredible!" Enloe is lucky to have Celia, and we are grateful for all she does for her students!

Puzzles

Here is a class of games called Shikaku (四角に切れ), a logic puzzle published by Nikoli. The rules are simple: Section off the grid into rectangular or square areas such that each cell in that area contains the same number AND the area is the size of the number given. Here is a 5 by 5 puzzle with a solution given. Can you solve the 8 by 8 puzzle? Hint: Areas the size of primes can only be 1 wide!

2				4
	2	5		
6				2
			4	

2	2	5	4	4
2	2	5	4	4
6	6	5	2	2
6	6	5	4	4
6	6	5	4	4

4			9	6		4	
							8
	2			3			
7							
2			5				
	6		2		4	2	

For more puzzles, see www.mathinenglish.com/Shikaku.php

NCCTM Board

contact information can be found at ncctm.org

Officers

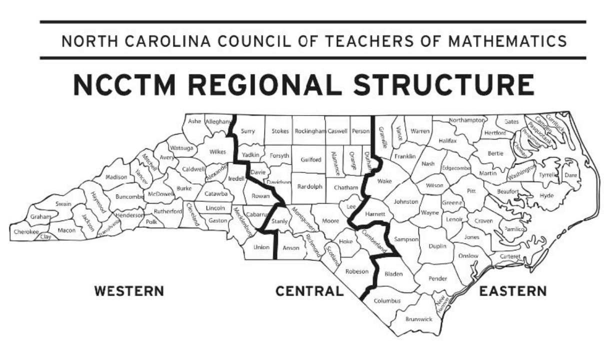
	State	Eastern Region	Central Region	Western Region
President	Ron Preston	Lynnly Martin	Maria Hernandez	Marta Garcia
President Elect	Julie Kolb	Tim Hendrix	Julie Riggins	Karen McPherson
Elementary Vice President	Carol Midgett	Michael Elder	Elisabeth Bernhardt	Jade Evaul
Middle Grades Vice President	Sheila Brookshire	Carla Sorrell	Jennifer Arberg	Angela Chappell
Secondary Vice President	Beth Lyton	Michelle Powell	Martha Ray	Christina Pennington
College Vice President	Shelby Morge	Ginger Rhodes	Denise Johnson	Emily Elrod
Other State Officers	Secretary Melanie Burgess	Parliamentarian Tim Hendrix		

Committee Chairs

Centroid Editors, Holly Hirst and Debbie Crocker
Webpages, Holly Hirst
Conference and Exhibit Services, Kay Swofford
Convention Services, Marilyn Preddy
Financial Chair, Ray Jernigan
Historian, Kathryn Hill
Leadership Conference, Ron Preston
Management Services, Joette Midgett
Math Celebrations, TBA
Math Contest, James Beuerle and Phillip Rash
Math Counts ,Harold Reiter
Math Fair, Betty Long
Minigrants, Sandra Childrey
NCDPI Representative, Kitty Rutherford
NCSSM Representative, Ryan Pietropaolo
NCTM Representative, Betty Long
NCSM Representative, Debbie Crocker
Nominations, Debbie Crocker
NC MATYC Representative, Luke Walsh
Rankin Award, Lee Stiff
Special Awards, Todd Abel
Student Affilliates, Lisa Carnell
Trust Fund, Janice Richardson

Becoming a Member

Follow the “Membership Information” link on the ncctm.org website, or go directly to:
http://www.ncctm.org/members/register.cfm



North Carolina Council of
Teachers of Mathematics
PO Box 33313
Raleigh, NC 27636