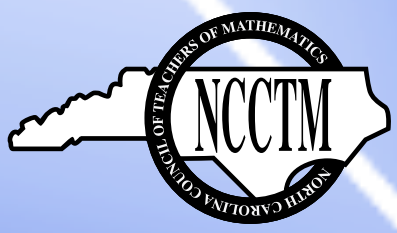
The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:



Volume 43, Issue 2 • Spring 2018

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

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Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:

  Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:

  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.

  Chapters or sections of books:

  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:

  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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9 am to 4 pm, March 23, 2018
Radisson Hotel High Point; High Point, NC

Speakers:
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Dr. Ted Coe (Achieve): What does "out of proportion" really mean?
Dr. Jane Wilburne (Penn State Harrisburg): Using tasks to promote rich and engaging mathematical experiences

Visit ncctm.org for more information and to Register!

NCCTM State Math Conference and Fall Leadership Seminar

October 31, and November 1-2, 2018
Koury Convention Center, Greensboro, NC

More information coming soon at ncctm.org

President's Message

State President Julie Kolb
Meredith College, Raleigh, NC
kolbjuli@meredith.edu

Happy New Year! Thanks to everyone for a very successful Fall Conference. I am always amazed by what can be accomplished by a committed group of volunteers. In my first year as President of NCCTM, I was never really certain what my responsibilities were and as I wondered who was responsible for planning and executing various aspects of the conference, I was constantly impressed by and grateful to everyone who helped. Please know that I am sincerely appreciative of everything each and every one of you continue to do for the benefit of NCCTM.

I am not a big believer in making New Year's resolutions—I bet that most of you know why that is. But I will share with you some of my hopes for NCCTM. I hope that those of you reading this article will resolve to help me make our organization more vital, visible, and relevant. For starters, perhaps you could share *The Centroid* with someone who has no idea that it even exists! Talk about the goals of NCCTM with your colleagues and encourage them to join and attend a meeting and to help us further the mission of the organization by encouraging excellence in mathematics education in North Carolina. Help us increase our presence and share the excitement of your professional endeavors with other members of the organization across the state through the use of Twitter and Facebook. Please consider volunteering in some way—help out at the conference, encourage your students to enter the logo contest or participate in a math contest or math fair; you might even consider running for office, making a presentation at the conference, or submitting an article to The Centroid. Finally, make plans now to attend the Spring Leadership Seminar in High Point on March 23. Presentations at the Spring Leadership Seminar will be made by Fran Arbaugh, Ted Coe, Jane Wilburne, and NC DPI mathematics leaders.

During his presentation, Ted Coe will "show how a single way of thinking should unify all understandings of proportional relationships." Dr. Coe compares the prior emphasis on "doing" mathematics to the current emphasis on "developing ways of thinking" about mathematics. While there are potentially many anecdotes that I could share from my experiences in teaching mathematics, I'd like to share a recent event that is related to the difficulty many individuals experience regarding proportional reasoning and the act of "doing" rather than "thinking about" mathematics. The story is about the recent distribution of luminaries in our neighborhood. (Now, whatever the organizers decide to do is fine with me as I prefer NOT to be the one in charge, but seriously!) It was decided that each household in our neighborhood (regardless of the size of their lot) would receive an equal amount of supplies to make luminaries – ten – they were "doing" mathematics – equally dividing the supplies they had obtained by the number of households in the neighborhood. You can imagine the display that evening as neighborhood lots with 300 foot frontages were decorated with one luminary every 30 feet while other neighbors decorated their frontage with one luminary every 10 feet! Needless to say, it looked "interesting." I'm still debating whether or not to suggest that the organizers "think" about a different apportionment strategy for next year. Perhaps using the perspective of Jane Wilburne, I could modify this task to "Promote Rich & Engaging Mathematical Experiences" for my students and a possible distribution plan for next year's luminary display.

Dr. Fran Arbaugh's presentation is titled "Reasoning-and-Proving: Do you Know It When You See It?" Dr. Arbaugh states that research shows that K-12 students do not participate in regular reasoning-and-proving activities. This reminds me of a second recent experience for which I am compelled to send a note of thanks and support to my granddaughter's kindergarten teacher! When asked at the conclusion of her first week of kindergarten what she had learned, my 5-year old granddaughter replied that she and her classmates learned about subitizing. The adults at the table were amazed and perplexed, so I asked her to explain what she meant. She very accurately informed us that she was able to look at a small group of objects and determine how many there were without actually counting them! Now that's impressive, especially since I did not learn what subitizing was until I was about 50 years old, and, apparently, Microsoft Word still doesn't know the word! It was exciting to learn that 5 year-olds are already engaging in reasoning activities.

I hope to see you at the Spring Leadership Seminar. Best wishes for 2018 – make it a great year!

Professional Learning Communities RULE

Bryant Ford, Montgomery County Schools (retired), Mt. Gilead, NC

The author presents the findings of a study that examines the implementation of PLCs and makes recommendations about the number and duration of PLCs, discussion topics, and involving parents.

Teachers who have taught for any length of time know about the trends that have come about in education over the years. Before the 1980s, the principal and the upcoming mathematics teacher stressed content. Every mathematics teacher had to cover the necessary material for students to do well in the upcoming mathematics course. In the 1980s, there was *Effective Teaching*. With *Effective Teaching* came the six-point lesson plan and observations by *Observer Evaluators* (OEs) from the central office. This blended into the *Career Ladder*, which labeled teachers as Level 1 or Level 2 from performance observations. Level 2 teachers were paid more. After the *Career Ladder*, teachers were using cooperative learning and in particular my school used *Paideia Seminars*. These proved to be effective teaching methods. This paragraph does not need citing because I lived through it.

The finer points of these teaching methods from the 1970s until now evolved into *Professional Learning Communities* (PLCs). Schools from across the state and the country adopted this method of sharing ideas about teaching (Dufour, 2004). Teachers get together once a week and strategize about their practice, student data, and student work. There has been a great deal of time and money spent on professional development for PLCs, and stakeholders want to know whether student learning and achievement are impacted by PLCs and whether PLCs are worth the time and money invested in the program. They want to know whether students show positive gains, whether teachers are adjusting their practices to assessments, and are those assessments predicting accurate scores on the End of Course (EOC) test. They discuss *Common Formative Assessments* (CFAs) and *Predictive Assessments* (PAs) as preparation for the EOC (Principal B. Brown, personal communication, 2016).

The school in this study previously had low test scores. The school is in a low wealth area, and stakeholders had to be convinced that money spent on PLCs would not be wasted. PLCs were introduced because teachers and administrators believed that this method of collaboration, and not teaching in isolation, was smarter teaching (American Institutes for Research, 2016). Professional development introduced CFAs and PAs. CFAs were given frequently and PAs, which are benchmarks, were given at the end of nine weeks. The conceptual framework was set up to focus on student learning, collaboration, and focusing on results (Datt, 2015).

At the beginning of this study, there were limitations. Teacher absences could be a problem if they are chronic, and they could miss important PLCs. Chronic student absenteeism is always a problem for the student and teacher. There is another high school in the county that the researcher should have included because there are only six mathematics teachers at this school. This group of six had to be both the mathematics teacher focus group and the mathematics teacher group.

There were benefits from this study of PLCs. According to the focus group, there has been positive change (Provini, 2012). Young teachers and new teachers had more confidence because the department chairpersons

and other mathematics teachers made them comfortable. Young teachers were encouraged to give suggestions and contribute to the discussion because no one will be negatively critical of them (American Institutes for Research, 2016). After observing several PLCs at the school, I saw that these mathematics teachers were inspired and excited. These teachers said that students and teachers were more interested and both were adhering to visions and mission statements.

In this study, the methodology used was "mixed methods," where both quantitative and qualitative data analysis are used (Creswell, 2015). I sat in on PLCs at the school in this study and observed their agenda and the manner in which it was covered so that items on the survey instrument could be created. Both quantitative items and qualitative items were on the same survey. The quantitative data are numbers that are easy to tally and apply statistical tests to verify the fact that PLCs impact student learning (Creswell, 2015). They also verify the fact that this program is worth the time and money spent. Surveys were created using a Likert scale of one through six: strongly disagree, disagree, and slightly disagree to slightly agree, agree, and strongly agree (Gross, 2017). There was no neutral category. There were four research questions used for the surveys. In the mathematics teachers' survey, the four research questions had five quantitative items to rate, except on research question four there were four items. There were 19 quantitative items in all. There were five qualitative items where the participants wrote their innermost feelings about the PLC program. The analysis identified themes that occurred most.

The parent survey was much shorter. There were eight quantitative items. Two items supported each research question, but the research questions were not headings as they were in the mathematics teacher survey to avoid confusion. There were four qualitative questions, which were actually the research questions. The parent focus group had 15 parents who received invitations, but only eight came to take the survey and discuss them. The administration would not allow me to obtain addresses or email addresses to send parents the survey, so I had to depend on children to take them. Four hundred surveys were sent home with students and only 56 returned, which was disappointing, but expected. The first period with the biggest percentage of return received a doughnut breakfast. At this point, items were compiled for each research question into one distribution. Now, there were only four sets of means and standard deviations for each survey group instead of 19 for the parents and eight for the parent focus group and parent group.

When applying statistical tests, I did not realize that three were necessary. I actually thought two tests would be enough, and I could have stopped with two, but I have a thirst for being thorough. I was very thorough in my 34 years of teaching, and have not changed. Descriptive statistics were used to start, then chi square, and Cronbach's Alpha (Trochim, 2006; University of Minnesota, 2017). Descriptive statistics were the beginning because finding basic values tells a great deal. Descriptive statistics is always a good place to start for any statistical test. The researcher can do different things to the mean and standard deviation. With each of the nineteen quantitative items and the eight items on the parent survey, I ran a descriptive statistics test. In the teacher focus group survey, the means were high and the standard deviations were mostly under one, meaning that there was close agreement among the teachers. The results of the parent survey were disturbing. The means were low and the standard deviations were above one and some were approaching two, meaning that had there been a neutral category, that is where the parent statistics would lie. The lowest mean of the teacher group was higher than the highest mean in the two parent groups. The highest standard deviation of the teacher group was lower than the lowest standard deviation in either parent group. On a scale of one through six, these standard deviations were far apart, meaning that the parents appeared to be apathetic. Subtracting the standard deviation from the mean and adding it to the mean would make for a large range. The parents seemed uncertain about their responses. I recalled an incident years ago of about three sets of parents that were trying to get a PTA started and they could never get it to come to fruition. Parents were apathetic about the PTA. The researcher became concerned, so, a chi square test was in order.

I decided to get more specific with the chi square test. This test would give an analysis by having a chi factor and a chi critical for rejecting, or not rejecting, the null hypothesis. The researcher entered the data for the four research questions in a chi square calculator for analysis (Turner, n.d.). The calculator was a four by six contingency table that could be adjusted for other dimensions. For the teachers, the chi factor was more than the chi critical, therefore the null hypothesis was rejected. This test of the teacher survey told that the PLCs were doing great and that teachers were excited about them and thought that PLCs were helpful. In both parent surveys, the null hypothesis was not rejected because the chi critical was larger than the chi factor. These parents, again, seemed

apathetic about their children's education with PLCs. What was so striking was the probability factor. In the teacher survey, there was a 0% p-value. In the focus group of parents, the p-value was 67%, and in the parent survey sent home, there was an 84% p-value. These probabilities showed that the parents were truly uncertain about this survey. The parent survey was much shorter than the teacher survey, but very similar. I thought about these results for a period of time and was somewhat upset about the parents. I wondered why the parents felt so apathetic, but felt confident in the analysis by this time and decided to run a Cronbach's Alpha test on the four research questions for reliability and internal consistency. The test showed that the research questions were very reliable.

The qualitative statistics solved the problem of parent apathy. It turned out that the parents were not apathetic at all. The school had been using PLCs for four years and the parents did not know about them. The researcher thought that the school had informed teachers about the way teachers were teaching. In the 1980s, the district informed parents about Effective Teaching and in 2002, Paideia seminars. The parental feeling was that the professional development taught the teachers and then the teachers taught their children. They also felt like teachers knew what to do, so everyone should let them teach and support them. They also felt that the central office did a great job of running interference for teachers, so that they could put their minds to teaching with no worries.

The teachers felt strong about the PLCs collaborating for their mathematics classes, but they felt that their students were not being held accountable. They told the researcher that a student could make level 1 on the EOC and get a passing grade for the exam on their report card. The parent and student both thought they were completing passing work, but actually had a false sense of security. Teachers felt that instead of the EOC score, students should be graded on growth. On the other hand, they felt that the PLCs pulled the mathematics teachers closer, and they listened to each other without judgment. There was an atmosphere of collaboration. "Plan, do, study, act" (Minnesota Department of Health, n.d.) was another theme that was prevalent in their qualitative perceptions. Another theme that came from these questions was that PLCs enable them to share their strengths and correct their weaknesses. As a result of the PLCs, scores did go in the positive direction. They were not dramatic, but they did increase. There was one year where the College Career Ready decreased slightly, but everything else increased from 2013 to 2016.

Regular summative testing in the classroom can be stressful for students, also. There was a teacher in the system who solved this problem. There was a problem with her students doing homework in ninth grade algebra I before common core. They would not do their homework. These students were not the academically gifted students. She told them that there would be no more homework. The students were happy, but there would be common formative assessment (CFA) every morning on what they had learned the day before. The CFAs were very short, but were given every day with one review problem. She gave a summative test on the week's material every Friday that lasted thirty minutes. This constant and structured reiteration of material was all a wonderful review for the EOC. In this small, low wealth, school, her scores were some of the best in the state.

In conclusion, there were limitations in accomplishing this study. As mentioned previously, teacher and student absences were a problem for any department at anytime. At the beginning of the school year, the principal needs to encourage maximum teacher attendance because there is no substitute for the actual teacher. The other high school in the county should have been included to have a bigger sample of mathematics teachers. A big limitation was not having addresses or email addresses for students. The teacher would have included a SASE for return of the survey. Could there have been special permission to have these addresses? I did not check on special permission.

Recommendations

The following recommendations emerged from this study.

1. Allow more than thirty minutes a week for PLCs. Twice a week would be great.
2. PLC participants need to read and research articles on PLCs. There are many.
3. The district should provide more professional development for more PLC strategies. This group is excited about them, and would like to learn more about them.
4. Get videos of PLCs in action. If there is no money, write for a grant. There is plenty waiting to be claimed.
5. Let parents know about PLCs on the school website or newsletter. Explain them to parents. If they are not a part of the website, make them a part of the website.
6. Focus on results and research how students achieved them, or how they can improve them.

Future Directions

Professional learning communities and plan, do, study, act, are great ideas that need more research. There also needs to be more research on students doing more assignments in collaboration and less testing. If the state and country would limit testing, there would be less stress among students and teachers, and less expense. Learning should not be stressful. The state standards need more research. Education researchers should consult with educators from countries that have high achievement scores in mathematics to see what they are doing in their classes.

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Trust Fund Scholarships: Now $1000

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the “grants an scholarships” link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (nctm.org). More information can be obtained from: Janice Richardson, richards@elon.edu.

Using R Markdown with the RStudio Integrated Development Editor

Alan T. Arnholt, Appalachian State University, Boone, NC

The author provides an overview of R Markdown, an add-on to the statistical computing language, R, for producing high quality documents in various formats that incorporate professionally typeset mathematics, graphics, and numerical calculations.

This article will focus on the benefits a high school mathematics teacher will realize from using R Markdown for the creation of documents to use in a high school mathematics course. R (R Foundation, 2017), RStudio (RStudio, Inc., 2016b), and LaTeX (LaTeX Project, n.d.) are all open source software projects which means there is no cost to download and install them on your personal machine. The R Markdown language is an extension of Markdown (Daring Fireball Company, 2004) that is used inside of R by loading the rmarkdown package written by Allaire et al. (2017). R Markdown supports many static and dynamic output formats including: HTML, PDF, MS Word, Beamer, HTML5 slides, Tufte-style handouts, books, dashboards, shiny applications, scientific articles, and websites (R Studio, Inc, 2016a).

Benefits of using R Markdown

An immediate benefit of using R Markdown is the easy creation of professional documents from simple text files. Since R Markdown uses text files, the user's workflow is recordable and can be easily maintained with a form of version control such as Git (git-scm.com/) on a platform such as GitHub (github.com/) or Bitbucket (bitbucket.org/product), both of which offer unlimited free public repositories. The number of R Markdown commands needed to control the appearance of a document (less than 20) and the ease of using the commands is a huge selling point. RStudio has a quick reference guide the user can view in the right pane of editor by clicking Help and then choosing Markdown Quick Reference (Figure 1).

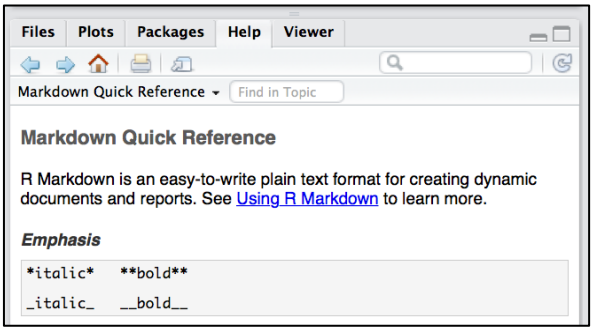


Figure 1. Markdown Quick Reference Help displayed in the right pane of the RStudio editor.

RStudio also maintains a two page R Markdown Cheat Sheet (www.rstudio.com/wp-content/uploads/2016/03/rmarkdown-cheatsheet-2.0.pdf) that discusses the YAML header, important code chunk options, and many other useful tidbits. For a mathematics teacher, one of the greatest advantages of using R Markdown is that mathematical symbols and equations can be inserted using standard LaTeX without having to spend an inordinate amount of time specifying a front matter as is typical when using LaTeX alone. R Markdown documents are dynamic, meaning program code and text narrative are in the same document so analysis/computing and

writing stay together. The user may choose whether to display or hide the computer code by using code chunks with appropriate options.

A Minimal Example

The source code in Figure 2 is used to create Figures 3 and 4. The source code illustrates a LaTeX environment (\begin{align}), several R code chunk options, R code to draw and compute the roots of a polynomial, and inline R code that extracts answers from R code and weaves the result into the text. The YAML written between lines of ---, has only one key:value pair, title: "A Minimal Example." Inline mathematics is written between single dollar signs ($) and display mathematics is written between double dollar signs ($$). More complex LaTeX environments for HTML and PDF documents are specified in the same fashion as one would use when writing with LaTeX. There are many free online references for using LaTeX (e.g., en.wikibooks.org/wiki/LaTeX/Mathematics). Equations, theorems, figures, and tables can all be cross-referenced using the bookdown package by Xie (2017). While the source code in Figure 2 does not use numbered equations or figures, the interested reader can refer to the documentation (bookdown.org/yihui/bookdown/) for examples to follow.

.

````
---
title: "A Minimal Example"
---

Solve $6x^2=3x$ by factoring.

$$\begin{align*}
 6x^2        &= 3x \\
 6x^2 - 3x   &= 0 \\
 3x(2x - 1) &=0 \\
 3x          & =0 \Longrightarrow x = 0 \\
 2x - 1      &= 0 \Longrightarrow x = \tfrac{1}{2} \\
\end{align*}$$

```{r, fig.align = "center", fig.width = 4, fig.height = 4, comment = NA}

f <- function(x){6*x^2 - 3*x}          # define function f
curve(f, from = - 0.25, to = 0.75)     # graph function f
abline(a = 0, b = 0, lty = "dashed")   # add dashed horizontal line
polyroot(c(0, -3, 6))                  # finding the roots with polyroot()
Re(polyroot(c(0, -3, 6)))              # Real roots
```

The real roots of $6x^2=3x$ are `r Re(polyroot(c(0, -3, 6))[1])` and `r
Re(polyroot(c(0, -3, 6))[2])`.
````

Figure 2. Source code for an example that draws and computes the roots of a polynomial.

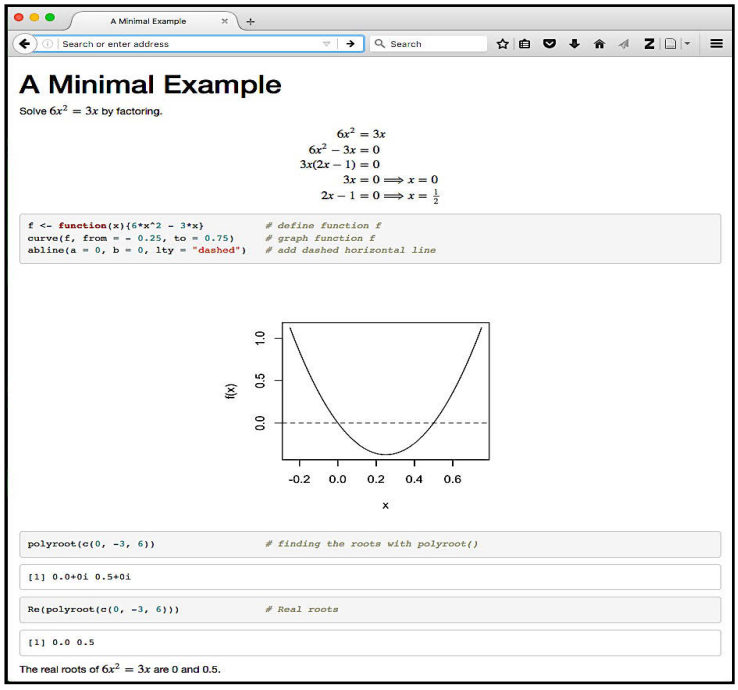
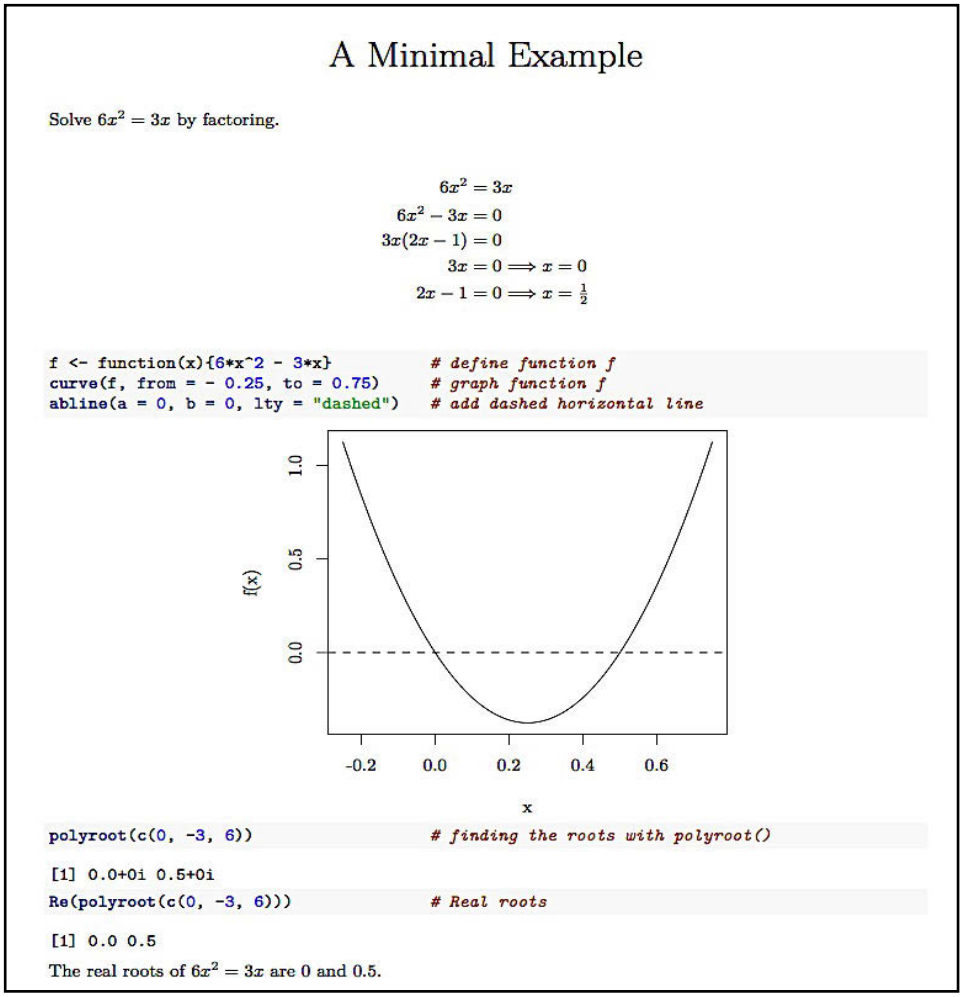


Figure 3. Webpage (HTML) output resulting from the code in Figure 2.



A Minimal Example

Solve $6x^2 = 3x$ by factoring.

$$6x^2 = 3x$$
$$6x^2 - 3x = 0$$
$$3x(2x - 1) = 0$$
$$3x = 0 \Longrightarrow x = 0$$
$$2x - 1 = 0 \Longrightarrow x = \tfrac{1}{2}$$

```
f <- function(x){6*x^2 - 3*x}          # define function f
curve(f, from = - 0.25, to = 0.75)     # graph function f
abline(a = 0, b = 0, lty = "dashed")   # add dashed horizontal line
```



```
polyroot(c(0, -3, 6))                  # finding the roots with polyroot()
```

```
[1] 0.0+0i 0.5+0i
```

```
Re(polyroot(c(0, -3, 6)))              # Real roots
```

```
[1] 0.0 0.5
```

The real roots of $6x^2 = 3x$ are 0 and 0.5.

Figure 4. Document (PDF file) output resulting from the code in Figure 2.

Source code is written the top left pane of the RStudio integrated development environment (IDE; Figure 5). To start a new R Markdown file, click File > New File > R Markdown…. To compile the document, click the Knit Button.

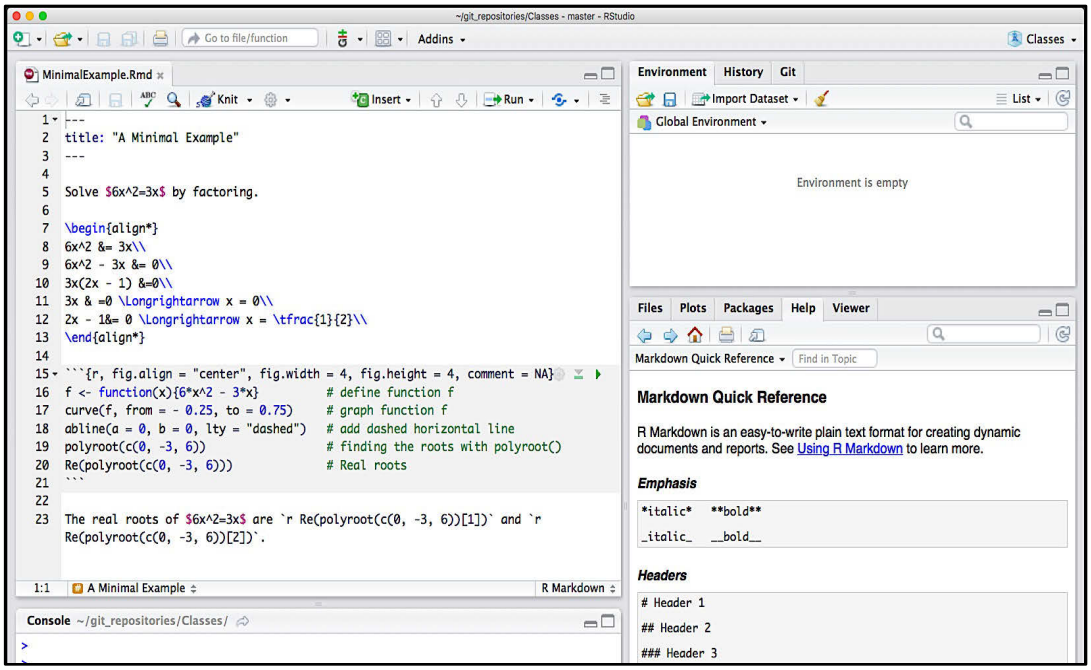


Figure 5. The RStudio Integrated Development Environment (IDE).

An electronic version of the code for this example can be copied from:
raw.githubusercontent.com/alanarnholt/GeneralStatistics/master/rmarkdown/UsingRStudioWeb.Rmd

Downloading and Installing R, RStudio, and LaTeX
R, RStudio, and LaTeX run on a wide variety of UNIX platforms, as well as Windows and MacOS. Download these tools as follows.

- **R:** go to cloud.r-project.org/ and select one of the precompiled binary distributions that matches your computer's operating system.
- **RStudio:** go to //www.rstudio.com/products/rstudio/download/, and select the installer that matches your computer's operating system. Install both the downloaded files. For the more adventurous user, it is possible to create your own RStudio server by following the directions Dean Attali has posted at deanattali.com/2015/05/09/setup-rstudio-shiny-server-digital-ocean/.
- **LaTex:** There are several options here. TeX live (www.tug.org/texlive/) is a TeX distribution for Linux, Windows, and MacOS; however, if you are using Windows, you may prefer MikTeX (miktex.org/) and if you are using MacOS, MacTeX (www.tug.org/mactex/). Follow the directions provided on the operating system specific web sites for installing LaTeX. The author recommends performing a complete LaTeX installation.

Additional Resources
If you are new to R, DataCamp (www.datacamp.com/) offers both free and premium content courses on a wide variety of R topics. Look for RStudio-maintained Cheat Sheets for many topics as well as user contributed sheets at www.rstudio.com/resources/cheatsheets/. The article *Using R with the RStudio Integrated Development Editor - Ideas, Pointers, and Teasers* (alanarnholt.github.io/GeneralStatistics/rmarkdown/IntroductionToUsingR.html) may provide the reader with additional resources and ideas for using R with RStudio.

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Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher? Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

Presidential Awards for Excellence in Teaching

Reported by Joseph Reaper, North Carolina Department of Public Instruction, Raleigh, NC

Every year teachers from each state can be nominated to receive the Presidential Award for Excellence in Mathematics and Science Teaching (PAEMST) – the highest honor bestowed by the US government for K-12 mathematics and science teaching. A selection committee, coordinated through the North Carolina DPI, selects up to five names from among the nominees to forward for national consideration.

Three North Carolina mathematics teachers were named 2017 finalists of the PAEMST: Wendy Bartlett, Reagan High School; Suzanne Gibbons, Wake STEM Early College High School; and Christina Pennington, Ash County High School. Congratulations to these outstanding teachers!

Pictured (l to r): Wendy Bartlett, Suzanne Gibbons, Christina Pennington

2017 Outstanding Mathematics Education Students

Reported by Rose Sinicrope, East Carolina University, Greenville, NC

Each Fall, NCCTM sponsors the selection of Outstanding Mathematics Education Students, one from each region of NCCTM. This year's recipients are Eastern Region winner **Ashley Lawson**, from North Carolina State University, Central Region winner **Josselyn Geyer** from Elon University, and Western Region winner **Tiffany Frady** from Western Carolina University.

Ashley Lawson an NC State Park Scholar, her passion is mathematics education in rural areas. She has been an active member of the student chapter of NCCTM since her sophomore year and has served as vice-president for one year and as president for two years. Ashley has served as a research fellow in a state-wide project to support the teaching of new high school mathematics standards. Ashley has been engaged in other research projects and has presented sessions to various professional groups including the UNC General Administration, the UNC Board of Governors, and the American Education Research Association.

Josselyn Geyer has spent over 100 hours in high school math classrooms observing and assisting high school math teachers. She serves as a math tutor and assists in the teaching of math content courses for preservice elementary school teachers. Seeing a need, Josselyn took the initiative to organize the mathematics materials at Elon's Classroom Resource Center. Josselyn has also developed mathematics curricula for childcare programs.

Tiffany Frady is committed to teaching high school mathematics. Tiffany is an active member of the student chapter of NCCTM. She serves as the Secondary Mathematics Ambassador for Western Carolina. Tiffany serves as a mentor to college students with intellectual disabilities. She tutors middle school students. She has presented her research on teaching financial literacy and life skills at professional meetings.

2017 Outstanding Elementary Mathematics Teachers

Reported by Denise Schulz, North Carolina Department of Public Instruction, Raleigh, NC

Each year, principals are encouraged to nominate the teacher who does the most effective job teaching mathematics in their school. From those nominated, each LEA selects one teacher to represent the best in mathematics teaching from the entire system. The teacher receives a membership in NCCTM, recognition at the State Conference, and a special memento of the occasion. The grade level cycles, and this year the teachers were chosen from among the best elementary mathematics teachers in North Carolina.

2017 Outstanding Teachers by School System

Tracy Bolyard, Alamance-Burlington
Monica Mitchell, Alexander
Cecelia Hampton, Alleghany
Allison Shoemake, Ashe
Valerie Runnfeldt, Asheboro City
Lea Skeate, Avery
Jamie Bradsher, Beaufort
Demetris Jacobs, Bladen
Shane Terzaken, Brunswick
Sara McGee, Buncombe
Amanda Marley, Caldwell
Elizabeth Gillikin, Carteret
Yvette Worsham, Caswell
Delaina Bryan, Catawba
Shalon Matthews, Chatham
Leslie Kale, Cleveland
Tyler Goodwin, Columbus
Courtney Merkel, Craven
Karlee Winicki, Cumberland
Kimberly Meidinger, Currituck
Jeanne Mouser, Dare
Dawn Courson, Davidson
Brandon Daniel, Durham
Pam Leary, Edenton-Chowan
Jenny Salsgiver, Edgecombe
Melanie Carter, Elizabeth City-Pasquotank
Vickie Willis, Franklin
Lori Lolies, Gates
Susan Smith, Granville
Heather Waddell, Greene
Marlene Creary, Haywood
Tara Hammond, Henderson
Anyea Gibson, Hickory City
Marcie Boutin, Hoke
Amanda Moore, Iredell-Statesville
Jessica Stewart, Jackson
Heidi Mills, Johnston
Mandy Futch, Jones
Rachel Leonard, Kannapolis City
Sonja LaBounty, Lee
Megan Lawson, Lenoir
Robin Bishop, Madison
Stephanie Roland, McDowell
Stephanie Shaw, Mooresville City
Amanda Robertson, Mount Airy
Karen Renee Barbour, Nash-Rocky Mount
Leah Southerland, New Hanover
Randy Abernethy, Newton-Conover
Kathleen Lester, Onslow
Michelle Lee, Pender
Gloria Ray, Perquimans
Kelly Pearce, Person
Kelli Peel, Pitt
Sarah Wood, Randolph
Erin Norris, Richmond
Amy Bullins, Rockingham
Hayley Dobbins, Rutherford
Misty Peed, Scotland
Karen Morton, Stanly
Marty Erskine, Stokes
Ashley Queen, Surry
Cheryl Saavedra, Swain
Erin Watson, Union
Amanda Hightower, Vance
Mary Catherine Gilbert, Wake
Martha Trimble, Watauga
Holly Medlin, Wayne
Jay Duncan, Wilkes
Paige Davis, Wilson
Katherine Boone, Winston-Salem/Forsyth
Jessica Duncan, Yancy

2017 Innovator Award Winner

Ellen Hilgoe, NC-EMPT

Reported by Rose Sinicrope, East Carolina University, Greenville, NC

The purpose of the NCCTM Innovator Award is to recognize and reward individuals and/or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM. The Recipients of this year's award is Ellen Hilgoe of the North Carolina Early Mathematics Placement Program.

Ellen Morrison Hilgoe, Associate Director of the North Carolina Early Mathematics Placement Testing Program (NC-EMPT) housed at East Carolina University, has received national attention for her work in preparing high school students for college-level mathematics courses.

Starting with a project funded by the State to assess high school students' readiness for college mathematics, this innovator developed a program to serve and support high school students, high school teachers, and university teacher educators in North Carolina. With her creativity, her passion for mathematics teaching, her commitment to the mathematics education of students in North Carolina, Ellen developed the NC Early Mathematics Placement Testing (NCEMPT) program into a vibrant service for thousands of high school mathematics teacher and three quarters of a million high school students and their parents across North Carolina

Accommodating and impressively efficient, Ellen has developed a stream-lined "processing machine" from test requests to test results. Students receive a personalized report and teachers receive summary data for their classes. With this innovator, NCEMPT has become much more than a testing service. With web-site and newsletters, secondary mathematics teachers and mathematics teacher educators can access games, problems, and activities for students. Whether teaching high school students or preservice teachers, supervising a high school mathematics intern, presenting at NCCTM or NCTM, or meeting with teachers, North Carolina Mathematics Education is fortunate to call Ellen Hilgoe one of our own.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website [ncctm.org]. More information can be obtained from Rose Sincrope [sinicroper@ecu.edu]

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Sandra Childrey [schildrey@wcpss.net], with questions.

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2017 Rankin Award Winners

Martha Ray and Kitty Rutherford

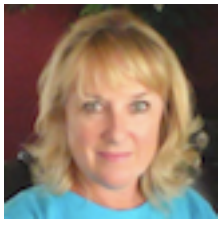
Reported by Lee Stiff, North Carolina State University, Raleigh, NC

At the 46th Annual State Mathematics Conference, NCCTM presented Martha Ray, Guilford County Schools, and Kitty Rutherford, formerly of the NC Department of Public Instruction, with the W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual.

For more than 30 years, **Martha Ray** has served the teachers of North Carolina with great distinction. She has been a champion of quality instruction for the teachers and students of North Carolina. In addition, she has influenced the opportunities for hundreds of teachers and students to learn math.

Ms. Ray has received the "Ron Hahn Outstanding Mathematics Teacher Award" and holds a National Board Certification in Adolescent/Young Adulthood Mathematics. She has been a curriculum writer and consultant, a high school mathematics department head, a mentor to countless teachers, a lead teacher for an Academic All-Star Camp, and an AP Calculus Audit Teacher Leader. She teaches courses via the North Carolina Virtual Public Schools and has assumed leadership roles in many local and statewide teacher-development projects and is a National Board District Coordinator. Moreover, she served NCCTM as a Regional Vice-President.

Ms. Ray is exemplary in helping teachers become better mathematics teachers. She has worked "with beginning teachers on lesson planning, content knowledge, deconstructing math standards, classroom management, and what it means to be an effective educator." Over her career, Ms. Ray is someone who represents the best that mathematics education has to offer in North Carolina.



Most recently as a mathematics consultant at the NC Department of Public Instruction, **Kitty Rutherford** has devoted her career to mathematics education. She provided strong leadership in the creation and implementation of the NCCTM-hosted Leadership Seminars. She has been active in the Association for Supervision and Curriculum Development, the Council of Presidential Awardees for Mathematics, National Council of Supervisors of Mathematics, and the National Science Teacher Association. She also served as a Member of the NCCTM Board of Directors as the State Elementary Vice President of NCCTM.

In support of this nomination to the Rankin Award Committee, one colleague wrote that "Kitty's only focus is 'what is best for the children' of North Carolina." He continues, "I have always been impressed with how Kitty has remained grounded…and kept the best interests of our state's children at the center of the work.…Great leaders raise everyone around them, and that is what Kitty has done over her career." Kitty will be missed in NC, but is now using her expertise to serve children of service members as an Instructional Support Specialist for the Department of Defense Educational Activity in Germany.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website [ncctm.org]. More information can be obtained from: Lee V. Stiff [lee.stiff@ncsu.edu].

Problems2Ponder

Holly Hirst, Appalachian State University, Boone, NC

Starting with this issue the Problems to Ponder column has received a face lift. The new Problems2Ponder will present problems similar to those students might encounter during elementary and middle school olympiad contests.

Student submissions are still welcome as are problem submissions from teachers. See the directions below the problem statements for more information. Enjoy!

Spring 2018 Problems

Problem A: Jerrod chose 5 numbers from the list 1, 2, 3, 4, 5, 6, 7, 8, 9. Two of the numbers he chose were 4 and 5. If these are the only two numbers chosen that differ by 1, what is the largest sum of the 5 chosen numbers?

Problem B: On a trip to the local amusement part, Gwyn agreed to drive the first 2/3 of the total distance if Shan drove the rest of the way. After changing drivers, Gwyn fell asleep and when she woke up Shan had 20% of her part of the drive left. For what percent of the whole trip was Gwyn asleep?

Solutions will be posted in the next edition of *The Centroid*.

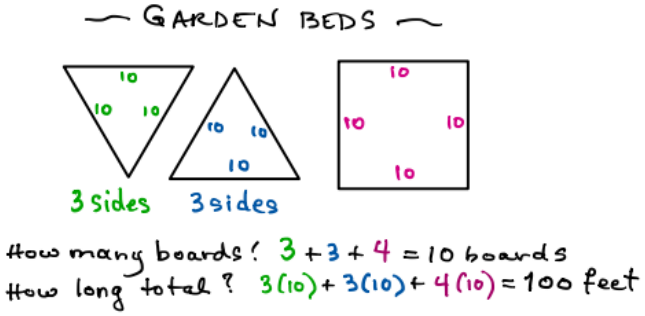
Problem submissions are welcome! If you have an idea for a problem to publish, please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of a typed or neatly written problem statement, along with a solution. Include your name and school affiliation so that we can credit you with the submission.

Solution submissions are welcome! In particular, if teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of *The Centroid*. Please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of the correct solution, with the name of the school, the grade level of the student, the name of the students (if permission is given to publish the students' names), and the name of the teacher.

Deadline for publication of problems or solutions in the Fall 2018 Centroid: July 1, 2018.

Fall 2017 Problem Solutions

Grades K–2: Henry wants to build three raised beds in his garden to plant vegetables. He wants to build two beds shaped like triangles and one shaped like a square. If he wants to make each side of the beds out of a board that is 10 inches long, how many boards does he need and how long are all the boards together?



Grades 3–5: According to wired magazine, when a squirrel hibernates in the winter, its heart only beats 10 times per minute, which is 5% of its normal heart rate. How many beats per minutes is the squirrel's normal heart rate?

hibernating heart rate:
10 beats per minute (bpm)
= 5% of normal heart rate
10 bpm = .05 x normal heart rate
$\frac{10 \text{ bpm}}{.05}$ = normal heart rate
200 bpm = normal heart rate

Another way to think about this problem: If 10 beats per minute is 5% of normal heart rate, then 20 is 10%, 40 is 20%, 80 is 40%, 160 is 80%. 160+40 must be 80%+20% or 100%.

Grades 6–8: Jody brags that his Dad's car can go 200 miles in 3 hours, and Steve says his Dad's is faster because it can go 300 miles in 4 ½ hours. Is Steve right?

Jody: 200 miles in 3 hours
gives $\frac{200}{3}$ miles per hour

Steve: 300 miles in $4\frac{1}{2}$ hours
or 300 miles in $\frac{9}{2}$ hours
gives $\frac{300}{\frac{9}{2}}$ miles per hour
or $\frac{600}{9}$ miles per hour
or $\frac{200}{3}$ miles per hour

They are the same! Steve is wrong.

2018 NCTM Annual Meeting and Exposition

25-28 April 2017
Washington, DC

Join thousands of your mathematics education peers at the premier math education event of the year! Network and exchange ideas, engage with innovation in the field, and discover new learning practices that will drive student success. Along with emerging issues and hot topics, the latest teaching trends and topics will include:

- Tools and Technology: Using Technology to Effectively Teach and Learn Mathematics
- Access, Equity, and Empowerment: Teaching Mathematics with an Equity Stance
- Purposeful Curriculum: Cultivating Coherence and Connections
- Teaching, Learning, and Curriculum: Best Practices for Engaging Students
- Assessment: A Tool for Purposeful Planning and Instruction
- Professionalism: Learning Together as Teachers
- Mathematical Modeling: Interpreting the World through Mathematics

Learn more at http://www.nctm.org/Conferences-and-Professional-Development/Annual-Meeting-and-Exposition/

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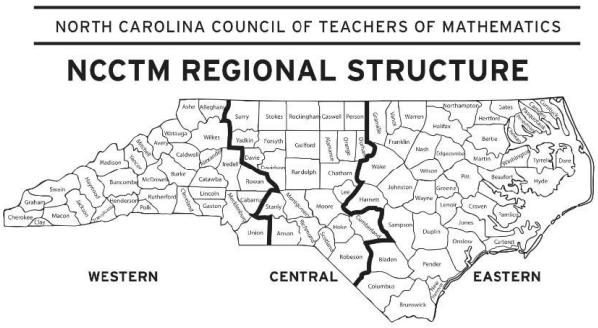

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