The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

Volume 44, Issue 2 • Spring 2019

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures, and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:

  Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.

  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:

  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.

  Chapters or sections of books:

  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:

  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

Volume 45, Issue 2 • Spring 2019

About the Cover
The image on the cover depicts the centroid of a triangle.

Contact Information
The Centroid
c/o Dr. Deborah Crocker, Editor
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
or send email to
CrockerDA@appstate.edu
Please include a return email address with all correspondence.

TABLE OF CONTENTS

NCCTM Spring Leadership Seminar

9 am to 4 pm, March 22, 2019
Red Lion Hotel High Point; High Point, NC

Speakers:

- Lee Stiff, Keynote
- Jennifer Wilson, Featured Speaker: "Cultivating Agency, Mathematical Understanding, and Language through Math Language Routines"

Visit ncctm.org for more information and to Register!

NCCTM State Math Conference and Fall Leadership Seminar

November 6, 7, 8, 2019
Koury Convention Center, Greensboro, NC

More information coming soon at ncctm.org

President's Message

State President Julie Kolb
Meredith College, Raleigh, NC
kolbjuli@meredith.edu

What a crazy school year this has been! So many different weather events have resulted in numerous starts and stops to the school year. Hopefully this spring will be less eventful and you and your students will have a productive and enjoyable semester. Please continue your support of our colleagues across the state – especially many in the east who have lost much. FAST NC is still accepting donations to provide assistance to schools, children, teachers and families affected by the devastation of Hurricane Florence; a link to their website can be found at NCCTM.org.

Thanks to everyone for a very successful Fall Conference. I am always amazed by what can be accomplished by a committed group of volunteers. I continue to be impressed by and grateful to everyone who helped. Please know that I am sincerely appreciative of everything each and every one of you continue to do for the benefit of NCCTM. In this, my last letter as President I would like to reiterate some of my hopes for NCCTM. I hope that those of you reading this article will resolve to make our organization more vital, visible, and relevant. For starters, perhaps you could share The Centroid with someone who has no idea that it even exists! Talk about the goals of NCCTM with your colleagues and encourage them to join and attend a meeting and to help us further the mission of the organization by encouraging excellence in mathematics education in North Carolina. Help us increase our presence and share the excitement of your professional endeavors with other members of the organization across the state through the use of Twitter and Facebook. Please consider volunteering in some way—help out at the conference, encourage your students to enter the logo contest or participate in a math contest or math fair; you might even consider running for office, making a presentation at the conference, or submitting an article to The Centroid. Finally, make plans now to attend the Spring Leadership Seminar in High Point on March 22. The theme of the day is *Targeted Support for Mathematics Learning.* In the morning, we will feature presentations by Lee Stiff and Jennifer Wilson. In the afternoon you will have the opportunity to attend two of five different break-out sessions. These sessions will include updates from NC DPI staff, information about Math Teacher Circles, Elementary Instructional Frameworks, TOOLS for elementary teachers, and TOOLS for secondary teachers.

We are looking forward to the Fall Conference in Greensboro; the theme for the conference is *Developing Mathematical Mindsets for All.* Fall 2019 will be our last meeting in Greensboro for a while. Future Fall Conferences will take place at Benton Convention Center in Winston-Salem. The first of these will be the Fall Leadership Seminar on November 11, 2020 followed by the Fall Conference on November 12-13, 2020. We have many new and innovative activities including the opportunity to sign up to visit to Old Salem. There are also a variety of great restaurants within walking distance of the convention center. Remember that this is the conference at which we will go digital!

You might recall that last spring I shared the story about the distribution of luminaries in our neighborhood – each household received supplies for ten luminaries regardless of the length of their lot's street frontage. You'll be pleased to know that the organizers this year (without any interference from me) distributed the luminaries proportionally – each household received enough luminaries to place one every ten feet! My new story is a probability question. Following Hurricane Florence we noticed that we had a very large, very dead tree on our lot and contracted with a tree service to have the tree removed. As you may know, the process involves a worker climbing the tree to remove limbs before cutting down the tree. Our lot is about ½ an acre; the area in which the tree was located is about 1/8 of an acre or 5,445 square feet or 784,080 square inches. So the question is this … what is the probability that a sawed limb about four inches in diameter would break apart, strike the ground, impale and cut a cable line ¼ in diameter buried six inches under the ground in two places a foot apart? You can do the math; it's not very likely, but it happened. So just because the probability of an event is almost zero, it doesn't mean that it can't happen.

I hope to see you at the Spring Leadership Seminar. Best wishes for 2019 – make it a great year!

Discovering Students' Mathematical World Through Mathematical Selfies

Axelle Faughn, Western Carolina University, Cullowhee, NC

The author presents a strategy for gaining insight into students' mathematical thinking through the use of student-generated selfies illustrating mathematical concepts.

In this paper we explore the visualization of mathematical concepts through photography. We show how mathematics can be popularized and student motivation increased by the use of visual mathematics embedded in a real-world context as a path to greater mathematical mindfulness. A "selfie" is commonly defined as a self-portrait photograph. We expand this definition to "mathematical selfies" as external representations of one's mathematical perception of the world; in other words, a self-portrait of one's mathematical world. Mathematical mindfulness is in turn defined as knowledge or perception of the presence of mathematics all around us. It takes a stand against the occasional mindlessness of school mathematics. Mathematics is often used as a gateway to, if not a gatekeeper from achievement in other fields of study (such as the medical field or engineering). It is therefore crucial to be concerned with increased equity and access for all in the teaching and learning of this social and professional determinant. We argue that mathematical mindfulness can provide all students with greater access to a desire to learn mathematics by providing a cultural perspective approach to mathematics teaching (Stinson, 2004).

In her foreword to the book *50 Visions of Mathematics*, Dara O'Briain explains that "Math is intense. But you stick with it because it is beautiful. And this beauty is often not clear to outsiders" (2014, p. iv). In her contribution to ICME 13 Topic Study Group 7 on popularization of mathematics, "Tapping into adolescents' main Characteristics to ignite Mathematics Interest", Duduzile Mkhize emphasized that popularization of mathematics is serious business for ethical and equity reasons. It helps students who may not otherwise be motivated to find a purpose in the study of mathematics, therefore developing their intrinsic motivation and providing them with further opportunity for success. Furthermore, in his plenary address at ICME 13, Bill Barton also insisted that "to the extent that we free mathematics (education) from culture, we are freeing ourselves of social and cultural responsibility. That makes us amoral." In a recent blog published at the Scientific American website, Barany (2016) argues:

> Recognizing that math has elitism built into its historical core, rather than pretending it is hidden all around us, furnishes a more realistic understanding of how math fits into society and can help the public demand a more responsible and inclusive discipline.

However, more often than not the representations used by teachers are catered to one specific and somewhat restricted type of student, leaving many behind in their appreciation and ability to get a grasp on the subject. Research on promoting classroom equity targets the use of students' mathematical representations in order to build community, and promote participation (Imm, Stylianou, & Chae, 2008). The work presented here is a study on student-produced representations of visual mathematical concepts through photography that focuses on mathematical visualizations and representations through pictures and quotes.

We often hear about real world mathematics, but there are different meanings for that phrase. First, there is the idea of how mathematics is used in the world of business or industry. In this view, the goal of mathematics education is to prepare students, at some point in the future, to take roles in society that require specific knowledge of mathematics such as finance, engineering, or advanced scientific research. Second there is a view of mathematics that primarily uses experiences or ideas of the teacher to create a real world in the classroom in which mathematics can be used. In essence, this is mathematics in the real world of the teacher that the student may enter for a short time. There is a third view of mathematics in the real world: mathematics that is in the real world that the student currently inhabits. It is this view of real-world mathematics that we seek to explore. The "Mathematical Selfies" project, while exploring ways to engage students in constructing mathematical knowledge and understanding, meets a recent push to integrate arts into the STEM curriculum, acknowledging the role of the arts in 21st century learning. The NCTM Research Committee recently wrote in *Grand Challenges and Opportunities in Mathematics Education Research* that there was a "need to see mathematics as something that human beings normally do and that has relevance and beauty" (Stephan et al., 2015, p. 139). The ideas behind the project were inspired by new modes of communications prevalent in social media through "pictures & quotes" that attempt to convey big ideas in a snapshot. The generation of upcoming students has mostly known a world where such modes of communication are prevalent in their daily interactions, which may be another way to engage in relevant mathematics beyond lectures and textbooks. In particular, we hoped that the effect generated by personal imagery could be harnessed to increase the enjoyment of learning and doing mathematics.

Initially, this project originated from a mathematics content class for grade school in-service teachers I was teaching in Jamaica while using a textbook from the United States. As a French national teaching for an American university among a group of Jamaican teachers, I found myself stepping out of their cultural realm on numerous occasions, and became increasingly faced with the issue of re-contextualization (referring to the idea that texts, images, situations and practices are transformed as they are moved between contexts) in the mathematics classroom. The obvious cultural distance in which we had been positioning ourselves encouraged me to think of ways to tap into the students' world of reference more extensively and more systematically, to make their context the basis for mathematical explorations. Meanwhile we had been thinking of activities we could do with college students to exploit the growing emphasis on the types of visual communication presented through social media. Finally, like most of us living and working in privileged academic realms, we were very moved by recent footages of refugee crisis situations in Europe, concerned with the reality of whole generations of children with no access to basic education while fleeing countries at war, and incredibly touched by the widely publicized drawings of some of these children depicting their daily lives. We thought giving others a chance to find something else in their world could lead to some interesting productions. All these circumstances merged into a "Mathematical Selfies" project we decided to conduct in college classes where students were asked to illustrate the mathematical concepts we discussed in class using pictures taken with their phones, with the addition of a quote for the purpose of explaining how the picture illustrated the selected concept. This project was well received by student and also gathered interest among colleagues in mathematics as well as other disciplines. Through mathematical discourse, students had an empowering voice to share and we as teachers came to know them on a different level. We then realized that this could be turned into an access door to the world of mathematics for a greater part of the community and a more general audience through photography exhibits. As teachers of mathematics we are all too aware of students' perennial question: "Where am I going to ever use this?" This project also attempts to provide a possible answer to this question through awareness of the relevance of mathematical objects that represent the world around us. We hope to raise interest in mathematics, its complexity, its relevance to our lives and its beauty. We provide ideas for using photography in the mathematics classroom as a tool to engage students and improve student success in a difficult subject that is too often thought of as irrelevant to students' lives. The images included in this paper are the results of extensive data collection on people's perception of and rendering of mathematical concepts through photographs taken in their daily world.

Analysing The Roles of Language and Visualization

> *"… mathematics [conveys the] deepest beauty of nature.… it is necessary to understand the language that she speaks in"* (Feynman, 1967; as quoted in Klinger, 2002).

A first step in analysing the wealth of data collected through the "Mathematical Selfies" project involves a combination of strikingly elegant (and at times ill-conceived) renderings by our students of their perceptions of mathematics, and the search for emerging trends in students' representations. Case studies will therefore support our discussion here, while pointing to identified redundancies in students' apprehension of mathematics. Our approach will be both empirical and theoretical. Simultaneously, the Standards for Mathematical Practices (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) will be used to highlight some of the pedagogical benefits gained by making Selfies a component of the mathematics classroom.

Research on visualization in mathematics tends to focus on mathematical objects/tools for visualization such as graph, diagrams, possibly with the purpose of supporting a proof or revealing properties and patterns (Arcavi, 2003). Speaking of visual knowledge and mathematics, Klinger (2002) notes:

> There are many mathematical topics that first arose in connection with something that people see. Hence both static and dynamic (animated) images have a major place in any program to ease anxieties and promote mathematical achievement….This material can be helpful as brief presentations that become the basis for writing and interchange among people needing to gain confidence in their ability to learn mathematics.

One of the components of the Mathematical Selfies project is to encourage students to write and discuss their thought processes when connecting selected photographs and the mathematics they are attempting to illustrate. Indeed, visualization and verbalization complement each other in meaning-making for the student of mathematics and encouraging discourse has been identified as a major practice that needs implementation in the mathematics classroom.

Image 1: Parallel lines

Mathematical Practice #6: Attend to precision

According to Presmeg (1986, 1997), concrete imagery needs to be coupled with rigorous analytical thought processes to be effectively used in mathematics. Through students' explanations of their picture selections, we are able to tap into their conceptual understanding of mathematical objects. Similarly, we can check on the accuracy of their definitions and whether they are able to use precise mathematical language that is consistent with the concepts they are trying to describe (see Image 1).

Mathematical Practice #3: Construct viable arguments and critique the reasoning of others

At times the use of language and written description also allows us to uncover misconceptions or discrepancies between common everyday language and mathematical language that can then be further addressed by inviting students to construct viable arguments and critiquing the reasoning of others during class discussions. For Image 2, the student's description highlights a misconception:

> *"The tree will only grow, it cannot shrink even though the rate of growth can differ from time to time. Therefore, if you graphed the growth of the trees the graph would produce a linear growth function."*

Image 2: Linear growth, misconception

The selfies then become a formative assessment tool and the student production becomes a resource in the classroom to explore the occasional breakdown between common language and mathematical conventions.

Mathematical Practice #4 and #5: Model with mathematics; Use appropriate tools strategically

The in-class discussion component of this project helps validate students' representations when asked questions such as "why is this mathematical idea/concept necessary/present in this particular situation/physical world?" (Think for instance of the equilateral triangles used to build playground features, or the congruent triangles on each side of an awning as in Image 3.) It becomes then possible to move from a singular representation to more generalized notions of where certain mathematical ideas are needed around us, training the mind to extract important information from a single situation, and tapping into problem solving skills of generalizing, streaming and filtering extraneous information. Image 3 also exemplifies appropriate use of the symbolic (here the added coloured strokes on the sides of the triangle to show SSS congruency), which is often necessary to illustrate a mathematical concept, in the same way that a legend on a map or diagram helps navigate it.

Image 3: Congruent triangles

We can therefore find a balance between the creativity of a representation (divergent behaviour of a new idea brought forward by the student), and ownership of new established knowledge once the representation is validated by the group (convergent behaviour through validation by peers, teachers). Other types of mathematical validations that we've used involve the use of graphing software when working with functions or geometric shapes, which allows us to check the actual veracity of what the student brings forward ("Is this really a quadratic function? Can we determine its equation?"). A common approach to mathematics is to go from real situation to its mathematization. However, reversing the process can be used as a communication exchange device: Students' visualizations can serve as a lens into their thinking and may be an easier media for them to use and express their understanding than through abstract mathematical language. This builds on documented equitable learning practices and could be particularly useful in multilingual classrooms to shift from a pictorial language common to all, to an informal language stage that may involve multi-language use (as in code-switching, Setati, 2002), to the more formal language of the topic taught. Indeed, Moschkovich (2013) recommends that teachers support students' engagement in the complexity of language in the mathematics classroom by focusing on mathematical practices first, not single words of vocabulary, and by making the most of home languages.

Mathematical Practice #7: Look for and make use of structure

Finally, another interesting idea that emerges from this project is to discover what students seem to find important in certain mathematical objects. For instance, when asked to illustrate the idea of a periodic function, some will provide a visual of a sinusoidal graph, while others prefer to photograph a periodic phenomenon such as a rotating fan. Mathematical objects also appear to have various functions such as the notion of volume which most students illustrate as capacity rather than 3-dimensional objects. We identified three themes as follows: what the mathematical idea is used for, what it looks like, and how it is created. This gives insight on what students find relevant, what they are attentive to, which could have significant impact on how they learn. The higher


Image 4: Sequence

the level of mathematics involved, the more students tend to create models that they then photograph to illustrate their thoughts. (See image 4.)

In this section we focused on one aspect of the project that promotes classroom communication and language development which were illustrated through individual cases. Overall, we find that students are able to develop mathematical mindfulness, finding relevance and beauty in the mathematical representation, along with a sense of community engagement which can turn this project into a community mathematical project for the classroom and beyond.

Discussion

People's reactions to mathematics vary from an innate fear acquired through unfortunate experiences with school mathematics, to expressed scepticism about the relevance of learning such difficult topics, to a fascination for a world mostly inaccessible but to a select few. By providing students and faculty an opportunity to share their mathematical visualizations through photographs, we are able to identify areas in the teaching and learning of mathematics where visualization of concepts by students, mathematicians, and mathematics education faculty are not always consistent. Mathematics teachers often provide students with representations illustrating mathematical concepts, however teachers less often experiment with student constructions of their own meanings and interpretations. The interdisciplinary and cross-cultural nature of picture taking positively motivates students to learn, build self-confidence and self-esteem, and focuses student attention while fostering valuable connections between the mathematics classroom and the community. By tapping into visual learning processes, we seek to develop powerful and effective teaching strategies that help students remember information better and longer. Breen (1997) identified two types of mathematical thinking: the one was the tendency towards abstraction, the other was the tendency towards intuitive understanding which stresses processes of visualization and imagery. As social media and visual culture transcend every aspect of our society, it is of utmost importance for millennials to be presented with the latter in their educational experience. By engaging in mathematical treasure hunts aimed at searching for and visually capturing mathematical ideas with a camera, one becomes more aware of the creative nature of the mathematics that surrounds us. These mathematical selfies become a tool for sustaining mathematical mindfulness, and finding beauty in mathematical constructs. As one student said after a few weeks of developing such awareness: "I cannot not see it anymore, mathematics is everywhere!" Another one soon exclaimed: "I never realized there were triangles in all these objects!", a prompt that can easily lead to further discussions on the structural qualities of triangles and their importance in studying shapes. In the previous section, we illustrated some of the initial findings through particular case studies of students' productions.

Although under scrutiny nowadays, research on learning styles was applied to the study of mathematics by Silver, Thomas, and Perini (2003), identifying four distinct mathematical learning styles: Mastery, Understanding, Self-expressive, and Interpersonal. The Mastery and Understanding students are more often those catered to in the traditional mathematics classroom. In our view, the mathematical selfies project can help the Interpersonal learners who tend to learn better through dialogue, collaboration, and cooperative learning and "approach problem solving as an open discussion among a community of problem solvers." It also supports the Self-reflective learners who use their imagination to explore mathematical ideas and like math that is "project-like in nature, and that allow them to think outside the box." This project presents yet another way to reach out to these learners and vary mathematical instruction and representations, along with other means of teaching/exploring mathematics creatively, such as using popular culture, story-telling, movies, pictures, software, and games in the mathematics classroom.

Conclusion

Through mathematical selfies treasure hunts offered at conferences and in other mathematical venues, we have been inviting other actors in the field of mathematics to share their meaning-making of the world around us. It will be particularly meaningful to compare the different layers of mathematical instruction and identify possible differences that emerge from such comparisons. Some limiting factors that came up as we were conducting this project is how restrictive static imagery can be when attempting to convey ideas of change. Naturally some students when asked to illustrate the notion of a derivative opted for videos rather than pictures so as to provide a more dynamic representation. This taps once again into students' mathematical mindfulness in realizing the difficulty of capturing change in stillness, and can help them understand the idea of change on a deeper level.

Possible extensions to this project involve cross-cultural comparisons of students' mathematical representations between France and the United States, as well as a more in depth investigation of the role of promoting language use through classroom discussions, especially in multilingual/multicultural teaching environments where pictures might offer a way for student to reach greater understanding and validation and help bypass the politics of language in the mathematics classroom that have been well documented as factors that can impact further mathematical success. (Setati, 2002).

Several comparative studies based on a wide multi-tiered collection of photographic representations of mathematics will eventually paint a picture of the variety of possible ways to interpret mathematical ideas, and of the differences through which one may express such ideas for others to understand.

With virtual reality opening its doors to research mathematicians and revolutionizing how we perceive and organize knowledge nowadays, allowing the visualizing of more extensive mathematical concepts with further possibilities for grasping abstract constructs that were previously out of reach, research on the interactions of visualization with the teaching and learning of mathematics is all the more current and necessary in order to prepare our students for a world that may not be so Euclidean after all. More examples of our students' work can be found at: www.mathematicalselfies.org. Note: The pictures in this article are not originals but were reproduced to exemplify students work submissions.

References

Arcavi A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, *52*, 215–241.

Barany, M. (2016). Mathematicians are overselling the Idea that "math is everywhere" [weblog]. Retrieved from http://blogs.scientificamerican.com/guest-blog/mathematicians-are-overselling-the-idea-that-math-is-everywhere/?WT.mc_id=SA_WR_20160817

Barton, W. (2013, July 26). *Mathematics, education and culture* [Plenary Lecture]. ICME13, Hamburg,Germany. Retrieved from https://lecture2go.uni-hamburg.de/l2go/-/get/v/19757

Breen, C. (1997). Exploring imagery in P, M, and E. In E. Pehkonen (Ed.), *Proceedings of the 21st PME International Conference*, *2*, 97-104.

Healy, L., & Hoyles, C. (1996). Seeing, doing and expressing: An evaluation of task sequences for supporting algebraic thinking. In L. Puig & A. Gutierrez (Eds.), *Proceedings of the 20th PME International Conference, 3*, 67-74.

Imm, K. L., Stylianou, D. A., & Chae, N. (2008) Student representations at the center: Promoting classroom equity. *Mathematics Teaching in the Middle School*, *13*(8), 458-463.

Klinger, A. (2002) Visual communication for mathematical confidence [weblog]. Retrieved from http://web.cs.ucla.edu/~klinger/vis_com_12_3_02.htm

Moschkovich J. (2013). Principles and guidelines for ELLs. In D. W. Stinson & J. A. Spencer (Eds.), Privilege and oppression in the mathematics preparation of teacher educators [Special issue]. *Journal of Urban Mathematics Education*, *6*(1), 45-57.

National Governors Association Center for Best Practices and Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Author.

O'Briain, D. (2014). A bad mathematician's apology. In S. Parc (Ed.), *50 Visions of Mathematics* (foreword). Oxford University Press.

Presmeg, N. C. (1986). Visualization in high school mathematics. *For the Learning of Mathematics, 6*(3), 42-46.

Presmeg, N. C. (1997). Generalization using imagery in mathematics. In L. D. English (Ed.), *Mathematical Reasoning: Analogies, metaphors and images* (pp. 299-312). Mahwah, NJ, USA: Lawrence Erlbaum.

Setati M. (2002). Researching mathematics education and language in multilingual South Africa. *The Mathematics Educator, 2*(12), 6-20.

Silver H. F., Thomas E., & Perini M. (2003). *Math learning style inventory*. Ho-Ho-Kus, NJ: Thoughtful Education Press.

Smith L. K., Moriarty S., Kenney K., & Barbatsis G. (2004). *Handbook of visual communication: Theory, methods, and media*. Boca Raton, FL: CRC Press.

Stephan, M., Chval, K., Wanko, J., Civil, M., Fish, M., Herbel-Eisenmann, B., & Wilkerson, T. (2015). Grand challenges and opportunities in mathematics education research. *Journal for Research in Mathematics Education*, *46*(2), 134-145.

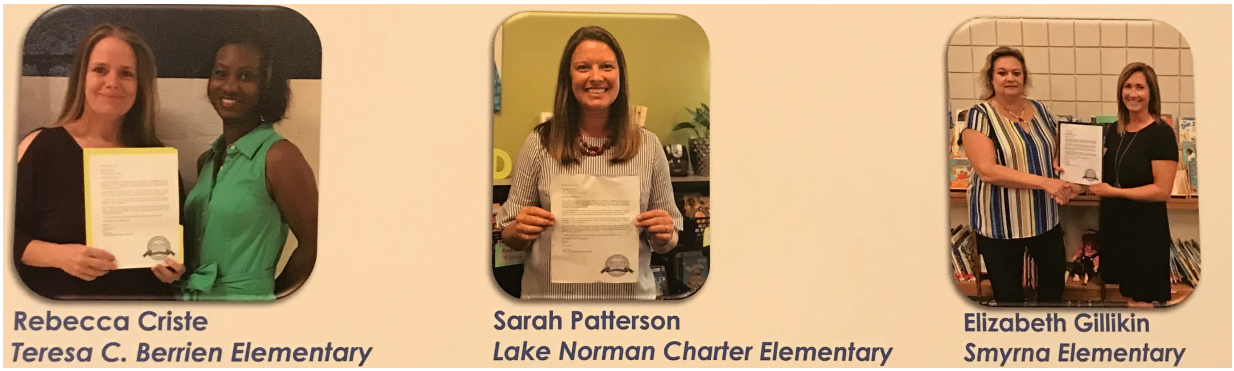
Stinson, D. W. (2004). Mathematics as "gate-keeper" (?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *The Mathematics Educator*, *14*(1), 8-18.

Presidential Awards for Excellence in Teaching

Reported by Joseph Reaper, North Carolina Department of Public Instruction, Raleigh, NC

Every year teachers from each state can be nominated to receive the Presidential Award for Excellence in Mathematics and Science Teaching (PAEMST) – the highest honor bestowed by the US government for K-12 mathematics and science teaching. A selection committee, coordinated through the North Carolina DPI, selects up to five names from among the nominees to forward for national consideration.

Three North Carolina mathematics teachers were named 2018 finalists of the PAEMST: **Rebecca Criste**, Teresa C. Berrien Elementary, Cumberland County; **Elizabeth Gillikin**, Smyrna Elementary, Carteret County; and **Sarah Patterson**, Lake Norman Charter Elementary, Lake Norman. Congratulations to these outstanding teachers!



Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [**jmccormick@rock.k12.nc.us**] with questions.

.

2018 Outstanding Mathematics Education Students

Reported by Ana Floyd, Randolph County School System, Asheboro, NC

Each Fall, NCCTM sponsors the selection of Outstanding Mathematics Education Students, one from each region of NCCTM. This year's recipients are Eastern Region winner **Renee Rhionon Fischer**, from East Carolina University, Central Region winner **Benjamin Rogers** from Elon University, and Western Region winner **Charlcy Carpenter** from Appalachian State University.

Picture (l to r): Ben Rogers, Charlcy Carpenter, and Renee Fischer

Renee Rhionon Fischer is a double major in mathematics education and mathematics at East Carolina University. Renee is a National Science Foundation Noyce Scholar. Renee is an exceptional student of mathematics education and mathematics, motivated not by grades but by learning and developing a deep conceptual understanding of mathematics. Renee has been an active member and officer of ECU's student chapter of NCCTM and has attended regional and state conferences. Renee is always eager to help. As a prospective teacher of mathematics, Renee has observed, served as an apprentice, and now as an intern in several different high schools. Renee has volunteered for one school year as a fifth grade mathematics tutor, for another year as a kindergarten mentor, and for a summer as an assistant in special program for high school students. One of Renee's strengths is her ability to listen to students and to engage students in productive classroom discourse.

Benjamin Rogers is mathematics education major with a statistics minor at Elon University. An Elon Teaching Fellow and a National Science Foundation Noyce Scholar, Ben has served as president of the Elon School of Education Ambassadors, President of the Elon Future Educators Club, and as a teaching assistant in a freshman orientation course. Always eager to help, Ben has provided assistance to many Elon University projects. This fall, Ben is mentoring high school students in statistics research through a Mathematical Association of America grant. Ben has completed multiple experiences in schools including a teaching experience in Oxford England and a Title One elementary school in Burlington.

Charlcy Carpenter is a middle grades major with a mathematics concentration at Appalachian State University. Charlcy has been an NCCTM member for two years and is also a member of NCTM. She attended the State Math Conference in 2017 and with her colleagues she led a workshop at the 2018 Conference entitled, "Classroom Ready Activities for Middle School Mathematics." In 2017, Charlcy served as the Western Region Student Representative to the NCCTM Board of Directors. Charlcy volunteered to work with student participants at NCCTM's 2017 Western Region Math Fair and served as a judge at the 2018 Western Region Math Fair. During 2017, Charlcy was invited and served on the NCTM panel for the webinar on "Start a Student Affiliate."

Fish Tanks and Beans: Related by Calculus!

Gail Kaplan, Department of Mathematics, Towson University, Towson, MD

The author presents a hands-on activity that can be used to help students understand the calculus definition of the average value of a function.

In the traditional calculus classroom of years past, the college professor or high school teacher stands at the front of the room and lectures as students diligently try to copy every word, symbol, and diagram written on the board. Deciphering the meaning of those notes is often a very laborious and demanding task. Technology now enables students to readily get a copy of the exact notes from the mere click of a mouse or a snapshot of the board. Yet mastering the concepts can remain a daunting task. How might teachers of Advanced Placement Calculus provide their students with a conceptual understanding of theoretical ideas? In this article we will explore how a simple kinesthetic experience can enhance comprehension. The hands-on activity provides a visual image that sparks student memory of the theory throughout the year.

The Curriculum Framework for Advanced Placement Calculus from the College Board (2016) states that part of the "*Essential Knowledge for Calculus"* includes the average value of a function:

The average value of a function $f(x)$ *over an interval* $[a, b]$ *is*

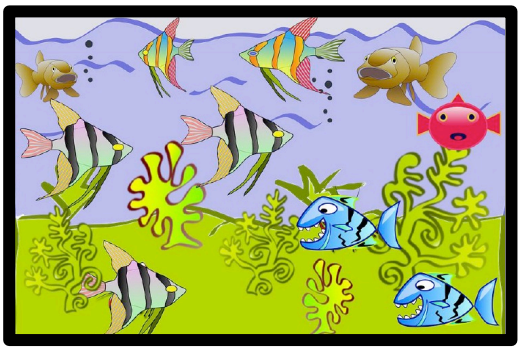
$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Far too often students memorize the definition and are able to find the average value of a given function, but lack an in-depth understanding of why the formula makes sense. This lesson is designed to creatively engage students in a simple experiential learning opportunity that leads to understanding the geometrical meaning of the formula. The approach in the following activity focuses on a sequence of challenges that requires students to use critical thinking skills and mathematical knowledge from courses prior to calculus as they work together to *discover* why it makes sense to call this integral the average value of a function on the interval $[a, b]$. We then connect the experience to a more theoretical approach. It is assumed that students have prior knowledge of how to represent the area under a curve on a particular interval using a definite integral.

The activity encourages and requires students to implement the following Common Core Mathematical Practices (Common Core State Standards Initiative, 2019).

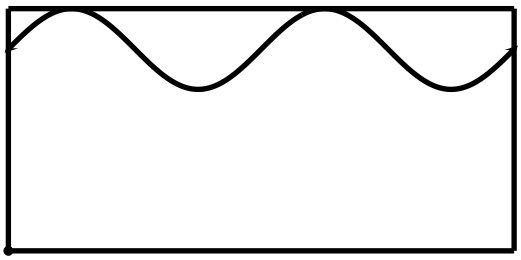
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

Part I: Fish Tanks



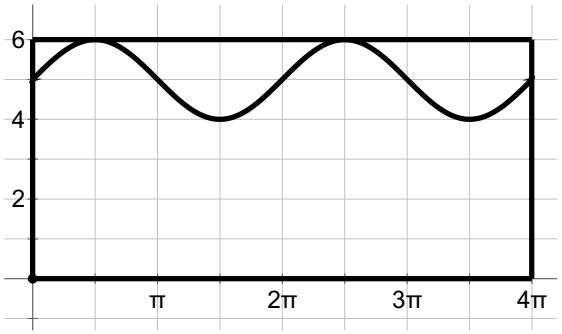
Fish Tank 1

i. Feeding fish in a calm tank often creates a frenzy of movement resulting in lots of waves. Imagine all the waves settle down and the surface becomes flat. Draw a line in Fish Tank 1 that represents the height of the water when the surface becomes flat. Compare the lines drawn by all members of your group, discuss your strategies, and justify which of the lines appears to provide the best solution.



Fish Tank 2

ii. The Fish Tank 2 diagram represents the waves in another fish tank. What type of function $f(x)$ might represent the height of the wave? Again, imagine that all the waves settle and the surface becomes flat. Draw a line in Fish Tank 2 that represents the height of the water in this tank when the surface becomes flat. Again, compare your line to the lines drawn by all members in your group, discuss your strategies, and justify mathematically which line appears to be the best solution.



Fish Tank 2 with Grid

iii. Let's now assume the lower left corner of Fish Tank II is the origin and consider the wave as a function $f(x)$ where x is the horizontal distance along the lower edge of the tank and $f(x)$ is the height of the wave at x. Reproduce the line drawn in the previous diagram that represents the height of the water when the waves settle. Find the coordinates of 3 points on your line, being sure that the x-coordinates are expressed in terms of π. List the points and label them on the graph.

iv. Write an equation for your line. Show that each point actually satisfies your equation. If any point does not satisfy your equation, revise.

v. Find the coordinates of 3 points **<u>on</u>** the actual waves **<u>but not on</u>** the line you drew, being sure that that the x-coordinates are expressed in terms of π. List the coordinates of the points and label them on the graph.

vi. Make an educated guess of the actual function $f(x)$ that represents the height of the wave. Show that all of the points from part v satisfy your function. If the points do not satisfy the function, revise, perhaps by modifying your function or the points. Repeat until all three points satisfy your function. Although the fish tank is 3-dimensional, the graph is 2-dimensional. We can think of the graph as representing a slice of the water. Since the amount of water in the tank does not change, the area of the region under the curve representing the height of the waves and the area of the water under the straight line must be equal.

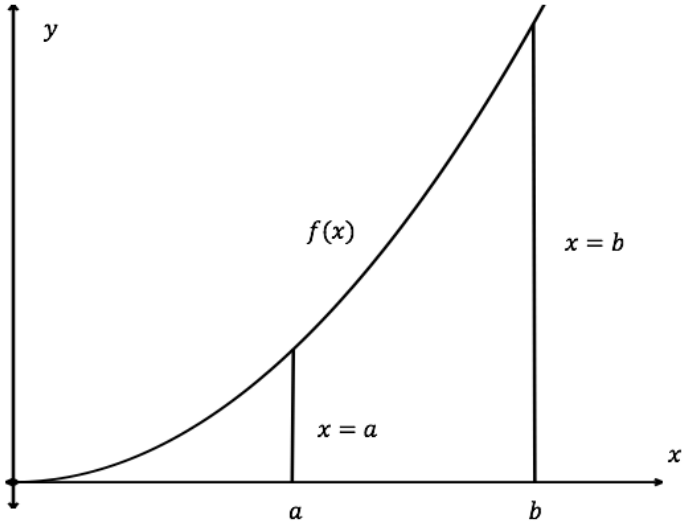
vii. Find the exact area between the x-axis and the straight line you drew for $0 \leq x \leq 4\pi$.

viii. Write and evaluate the integral that represents the area bounded above by your wave function and below by the x-axis for $0 \leq x \leq 4\pi$.

ix. Compare the values found in the last two computations. Explain why this relationship makes sense.

Part II: Playing with Beans!
As you proceed, be sure to discuss your responses with the other members of your group.

i. Using the graph below, fill the region bounded between $f(x)$ and the x-axis for $a \leq x \leq b$ with beans.



ii. Rearrange the beans to form a rectangle whose base is the line segment on the x-axis between the points $(a, 0)$ and $(b, 0)$. Draw the sides of your rectangle. Describe the width of your rectangle using only the constants a and b.

iii. Remove the beans.

iv. Find a point that is on the graph of $f(x)$ and on the top edge of your rectangle. Appropriately label c on the x-axis so that the coordinates of the point found are $(c, f(c))$.

v. Find the height, width, and area of your rectangle in terms of $a, b, c,$ and the function $f(x)$.

vi. Justify why the area bounded by the curve and the x-axis on the interval $[a, b]$ is the same as the area of the rectangle.

vii. Find an integral that represents the area the region bounded between $f(x)$ and the x-axis for $a \leq x \leq b$.

viii. Write an equation that sets your expression for the area of the rectangle equal to the integral that represents the area of the region bounded between $f(x)$ and the x-axis for $a \leq x \leq b$.

ix. Solve your equation for $f(c)$. Your solution is the formal definition of the **average value of a function $f(x)$ on the interval $[a, b]$**.

Part III: Putting it All Together!
To deepen understanding of why this makes sense, consider a brief teacher-led discussion using the following example. Suppose we want to find the average temperature on a particular day and have 10 temperature readings equally spaced throughout the day, $t_1, t_2, t_3, \ldots, t_{10}$. The average temperature is:

$$\frac{t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 + t_9 + t_{10}}{10}$$

Using sigma notation we write this as $\sum_{i=1}^{10} {}^{t_i}/_{10}$. For a more accurate average, we might use 20 or 100 temperature readings, $\sum_{i=1}^{20} {}^{t_i}/_{20}$ or $\sum_{i=1}^{100} {}^{t_i}/_{100}$. For any positive integer n, the average temperature of n readings is equal to:

$$\sum_{i=1}^{n} \frac{t_i}{n}$$

How might we get the best expression for the average temperature on that day? Students will generally respond that more readings will lead to better approximations. How can we get the *best* approximation? AP Calculus students will often consider using a limit. Mathematically, we want to let the number of readings get larger and larger. Theoretically, we need the number of readings to approach infinity. Using the concept of a limit, the average temperature for a given day is:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{t_i}{n}$$

Similarly, if we want the average value of a function on an interval $[a, b]$, we take the value of the function at more and more values throughout the interval $\big((x_1, f(x_1)), (x_2, f(x_2)), \ldots (x_n, f(x_n)\big)$, add the values, and divide by n, resulting in:

$$\sum_{i=1}^{n} \frac{f(x_i)}{n}$$

How does this connect to the beans? We used the beans to illustrate that the area of the region bounded by the curve and the x-axis is the same as the area of a rectangle where $f(c)$ represents the height of the rectangle. We know that the area of the region bounded by the curve $f(x)$ on the interval $[a, b]$ is:

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot \frac{b-a}{n}$$

Divide both sides of this equation by $(b - a)$ to obtain:

$$\frac{\int_a^b f(x)\, dx}{b-a} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{f(x_i)}{n}$$

Just as you did with the beans, the average value of the function on the interval $[a, b]$ is the value of the integral, $\int_a^b f(x)\, dx$, divided by the length of the interval, $b - a$.

It is truly a delight to observe the "aha!" experience as students connect the fish tank and beans to their understanding of the expanded meaning of average!

References

College Board. (2016). *Course and exam description: AP calculus AB and AP calculus BC.* Retrieved from https://apcentral.collegeboard.org/courses/ap-calculus-ab?course=ap-calculus-ab

Common Core State Standards Initiative. (2019). *Standards for mathematical practice.* Retrieved from http://www.corestandards.org/Math/Practice/

2018 Outstanding Secondary Mathematics Teachers

Reported by Denise Schulz, North Carolina Department of Public Instruction, Raleigh, NC

Each year, principals are encouraged to nominate the teacher who does the most effective job teaching mathematics in their school. From those nominated, each LEA selects one teacher to represent the best in mathematics teaching from the entire system. The teacher receives a membership in NCCTM, recognition at the State Conference, and a special memento of the occasion. The grade level cycles, and this year the teachers were chosen from among the best secondary mathematics teachers in North Carolina.

2018 Outstanding Teachers by School System

Kevin Scharen, Alamance Burlington	Shirley Lloyd, Edgecombe	Linda Lynch, Person
Lindsay Cox, Alexander	Melissa Sawyer, Elizabeth City/Pasquotank	Jon Ezell, Polk
Monica Davis, Anson	Linda Stephens, Franklin	Holly Grissom, Randolph
Heather Windish, Ashe	Deirdre House, Gates	Ismael Gomez, Richmond
Callie Everett, Asheboro City	Maegan Frederick, Granville	Tracy Snyder, Roanoke Rapids
Amber Benfield, Avery	Heather Davis, Greene	Lenise Stewart, Robeson
Cougar Caroon, Beaufort	Sara Vaughn, Guilford	Renee Gibbs, Rockingham
Sherry Autry, Bladen	Amy Carter, King Harnett	Jeniffer Bain, Rowan-Salisbury
Robin Kelly, Brunswick	Ryan Brumfield, Haywood	Amy Owens, Rutherford
Elliot Lunsford, Buncombe	Christian Gosnell, Henderson	Chrystal Pope, Sampson
Kelly Baker, Burke	Carie Kahn, Hickory City	Chesnee Palmer, Stanly
Julie Cline, Cabarrus	Brooke McCurdy, Iredell/Statesville	Maggie Mitchell, Surry
Joy Philyaw, Caldwell	Jessica Hatley, Kannapolis City	Amanda Chapman, Transylvania
Denise Thomas, Camden	LaKeisha Snipes, Lee	Marion Millican, Union
Kim Hanson, Carteret	Andrea Hardy, McDowell	Danilo Morales, Vance
Molly Caudill, Chapel Hill-Carrboro	Russell Barnett, Mitchell	Tonya Radakovich Wake
Daniel Wicks, Charlotte/Mecklenburg	Tabitha Richardson, Mooresville Graded	Jennifer Williams, Watauga
Karin Clamann, Chatham	Patricia Combs, Mount Airy	Andrew Bass, Wayne
Jennifer Collins, Columbus	Shauntel Palmer, New Hanover	Danielle Yount, Wilkes
Cindy Laird, Craven	Cheryl Rhea, Newton-Conover	Renee Pearson, Wilson
Makkeddah Gilchrist, Cumberland	Dawn Guzzi, Orange	Wendy Bartlett, Winston-Salem/Forsyth
Samuel DeWitt, Dare	Amanda Mehring, Pamlico	Angella Jordan, Yadkin
Dawn Lowery, Davie	Bethany Thomson, Pender	
Leah Perkins, Duplin	Kelsey Hagan, Perquimans	

2018 Innovator Award Winners

Individual: Dr. Denise Johnson

Group: North Carolina Collaborative for Mathematics Learning

Reported by Ana Floyd, Randolph County School System, Asheboro, NC

The purpose of the NCCTM Innovator Award is to recognize and reward individuals and/or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM. The Recipients of this year's award are, in the individual category, Dr. Denise Johnson and, in the group category, the North Carolina Collaborative for Mathematics Learning.

For 10 years, **Dr. Denise Johnson** has involved middle school girls in a program to empower them through the study of mathematics and science. The girls meet on Saturdays during the school year and participate in a week-long camp during the summer. Additionally, seven years ago she created a mentoring program across two NC universities to partner veteran teachers with beginning teachers. Using technology, mentoring is done virtually through Blogging, Vimeo, and FaceTime - all new technology at the time for mentors and mentees --- encouraging all to grow and develop professionally. Always reaching out, this Winston State University mathematics educator spread the work to after school-programs and community centers.

The **North Carolina Collaborative for Mathematics Learning** is a dedicated and selfless group of mathematics educators saw a critical need for resources and professional development as North Carolina transitioned to new mathematics standards. Starting at the high school level, Holt Wilson from UNC-Greensboro, Allison McCullough from UNC-Charlotte, Katie Mawhinney from Appalachian State, and Jared Webb from UNC-Greensboro developed educational briefs on content and pedagogy to support districts and math teachers across the state. They provided professional development modules and supporting documents to help teachers make sense of the mathematics content and put into practice research-based pedagogies to teach mathematics. They created virtual professional communities that enabled all teachers to have access to high-quality tasks and resources for students.

For elementary and middle school teachers, the work centered on the development, refinement, editing, and final creation of the Instructional Frameworks. This is the first time that there has been a state-wide group brought together, given ownership of, and produced state-wide recommendations for the unitizing of specific mathematics concepts and standards, as well as a recommended order in which to teach those units based on research. Katie Schwartz from East Carolina and Michelle Stephan from UNC Charlotte have led this work in elementary and middle grades, respectively. For the first time, all districts have access to common, research-based instructional frameworks to guide cohesive and coherent mathematics education. This is invaluable to small districts or districts with no curriculum team to do this work. The possibilities are endless as we think of how this common language and framework lends itself to collaboration, shared resources and research, and access to quality instructional materials across the state. Additionally, this work led to the creation of the Tools4NCTeachers website that includes elementary resources and materials that correlate with the new mathematics standards.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website [ncctm.org]. More information can be obtained from Ana Floyd [afloyd@randolph.k12.nc.us].

2018 Rankin Award Winner

Marta Garcia

Reported by Lee Stiff, North Carolina State University, Raleigh, NC

At the 2018 State Mathematics Conference, NCCTM presented **Marta Garcia**, with the W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual.

Marta Garcia is a former elementary mathematics teacher at Haw Creek Elementary School, former Elementary Mathematics Instructional Coach in Buncombe County, and now an independent Mathematics Education Consultant and on the mathematics education faculty of Mount Holyoke College. She has been an outstanding educator for more than 35 years and has contributed to the mathematics education of countless students and teachers during this time. First and foremost, Marta is unanimously regarded as a passionate teacher and learner of mathematics. In fact, "passion" is the one word that is repeatedly used to describe her. As a colleague wrote: "…to my knowledge, there has been no one in North Carolina who has been a more enthusiastic, articulate, and effective advocate … in support of the implementation of … Standards-based curriculum materials for elementary schools" than Marta Garcia.

Marta is a National Board Certified teacher and the recipient of the Presidential Award for Excellence in Mathematics and Science. She has certifications in elementary education, elementary mathematics education, early childhood education, academically gifted, and Spanish. As an elementary classroom teacher for more than 20 years, she has been consistently recognized for outstanding teaching and leadership: She has been named "Teacher of the Year" in her home county no less than three times! Marta is active in promoting the professional development of beginning and experienced teachers, serves as a role model in the classroom via coaching and supervising teachers, and is described as a person whose "…love of mathematics is pervasive; and [for] those who are fortunate enough to spend a few hours with [our honoree] in professional development, [they] walk away with a deeper understanding [of], and in most cases a more affectionate view, of mathematics and children…"

In North Carolina, Marta has participated on the NC Department of Public Instruction Standards Review Writing Team and numerous DPI projects, including being a Lead Writer on a Math/Science Partnership Grant. She has given numerous presentations at local and state NCCTM meetings, served as a program chair for NCCTM state math conferences, served on the NCCTM Board of Directors, and was Western Region President of NCCTM. At the national level, Marta has provided professional development in implementing the *Investigations in Number, Data, and Space* K-5 elementary math curricula, *Building Computational Fluency*, *Algebraic Reasoning*, and *Developing Mathematical Ideas* projects, and is a frequent speaker at NCTM and NCSM national conferences. Our recipient was a contributing author to such publications as: *Connecting Arithmetic to Algebra*, *My Kids Can*, and *Foundations of Algebra.*

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website [ncctm.org]. More information can be obtained from: Lee V. Stiff [lee.stiff@ncsu.edu].

Problems2Ponder

Holly Hirst, Appalachian State University, Boone, NC

Starting in 2018, the Problems to Ponder column received a face lift. The new Problems2Ponder will present problems similar to those students might encounter during elementary and middle school olympiad contests.

Student submissions are still welcome as are problem submissions from teachers. Please consider submitting a problem or a solution! Enjoy!

Problem submissions are welcome! If you have an idea for a problem to publish, please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of a typed or neatly written problem statement, along with a solution. Include your name and school affiliation so that we can credit you with the submission.

Solution submissions are welcome! In particular, if teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of *The Centroid*. Please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of the correct solution, with the name of the school, the grade level of the student, the name of the students (if permission is given to publish the students' names), and the name of the teacher.

Deadline for publication of problems or solutions in the Fall 2019 Centroid: June 30, 2019.

Spring 2019 P2P Problems

Problem A: It takes three people 1 hour to fill a storage room with boxes. How long would it take one person to fill three storage rooms, assuming all of the people work at the same rate and all of the rooms are the same size?

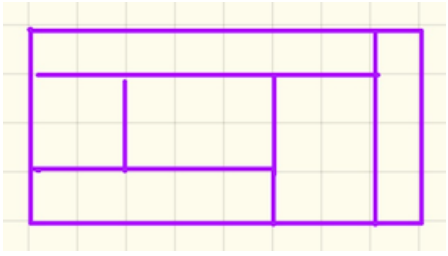
Problem B: How many different fractions (in lowest terms) between 1/4 and 3/8 are exactly equivalent to a decimal that is written using 3 decimal places?

Solutions will be posted in the next edition of *The Centroid*.

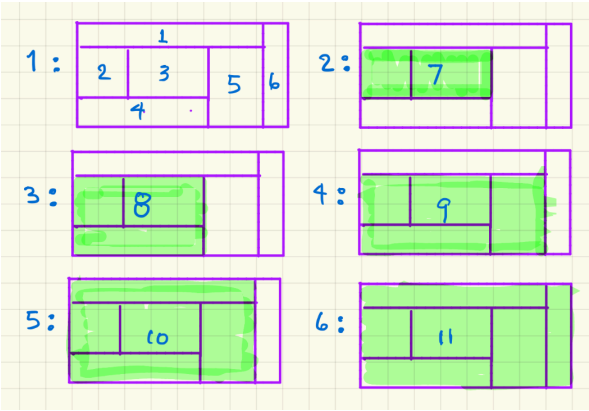
Fall 2018 P2P Problem Solutions

These two problems both involved counting carefully!

Problem A: A large rectangle is cut into smaller rectangles as pictured. How many rectangles of all sizes are in this diagram?

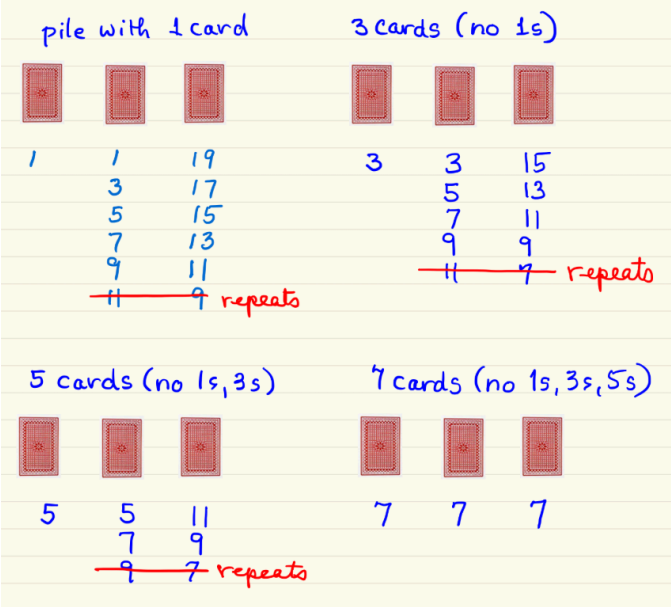


Solution: The answer is 11. The key to counting the rectangles is to be organized! Count the individual rectangles contained in the group (6), and then those that can be formed from two (1), three (1), etc., as pictured below. Don't forget that a square is also a rectangle!



Problem B: The answer is 12. If we have 21 cards, how many different ways can we organize them in 3 piles if the piles must all contain an odd number of cards?

Solution: The key to counting the possibilities is to be organized! Count all the possibilities when one of the piles has only 1 card and stopping when there the possibilities begin to repeat, then count all the possibilities when one of the piles as 3 cards (excluding piles of 1 card), then 5 cards (no 1s or 3s), etc., as illustrated below. Notice that we stopped at 7, because a pile of 9 cards would require other piles of fewer cards, which we already counted!



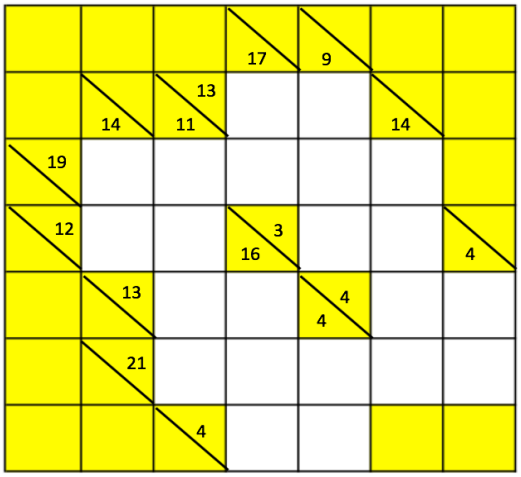
Puzzle

Here is a new puzzle challenge for you: a Kakuro puzzle, sometimes called a Cross Sums puzzle. According to Wikipedia, in Japan Kakuro puzzles are second in popularity only to Sudoku.

The directions: Place the digits 1 through 9 in the unshaded, empty squares so that the row or column adds to the number provided, and for each sum, no digit can be used twice.

In this puzzle, for example, the squares in the second row can be any two different numbers that add to 13. If we use 9 in the first square, then the 9 is also impacts the column sum of 17, requiring the square below the 9 to be 8, etc.…

Like this puzzle? Check out https://www.kakuros.com/ for more Kakuro puzzles! Another great puzzle site: http://www.nikoli.com/



Trust Fund Scholarships: Now $1000

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants & scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson Plumblee, richards@elon.edu.

Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

NCCTM Board

contact information can be found at ncctm.org

Officers

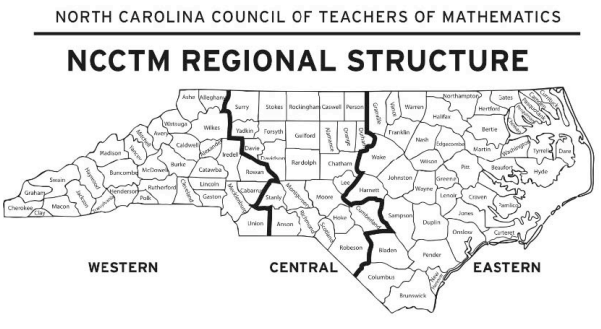
	State	Eastern Region	Central Region	Western Region
President	Julie Kolb	Tim Hendrix	Julie Riggins	Karen McPherson
President Elect	Kathy Jacqua	Ginger Rhodes	Denise Johnson	Sheila Brookshire
Elementary Vice President	Jade Evaul	Leah Southerland	Deanna Wiles	Brian Bettis
Middle Grades Vice President	Dawn Jenkins	Kim McCuiston	Sara Vaughn	Jessica Vernon
Secondary Vice President	Michelle Powell	Renea Baker	Allison Yokeley	Lauren Baucom
College Vice President	Katie Mawhinney	Charity Cayton	Tracy Howell	Holly Pinter
Other State Officers	Secretary Christina Pennington	Parliamentarian Wendy Rich		

Committee Chairs

Conference and Exhibit Services, Kay Swofford
Centroid, Holly Hirst and Debbie Crocker
Convention Services, Marilyn Preddy
Financial Chair, Ray Jernigan
Handbook, Ray Jernigan
Historian, Kathryn Hill
Leadership Conference, Julie Kolb
Management Services, Joette Midgett
Math Celebrations, Anthony Finlen and Pamela Parham
Math Contest, James Beuerle and Phillip Rash
Math Counts, Harold Reiter
Math Fair, Betty Long
Minigrants, Joy McCormick
NCDPI Representative, Lisa Ashe
NCSSM Representative, Ryan Pietropaolo
NCTM Representative, Debbie Crocker
NCSM Representative, Debbie Crocker
NCMATYC Representative, Luke Walsh
Nominations, Ron Preston
Rankin Award, Lee Stiff
Special Awards, Ana Floyd
Student Affilliates, Lisa Carnell
Trust Fund, Janice Plumblee
Webpages, Holly Hirst

Becoming a Member

Follow the “Membership Information” link on the ncctm.org website, or go directly to:
http://www.ncctm.org/members/register.cfm



NORTH CAROLINA COUNCIL OF
TEACHERS OF MATHEMATICS
PO BOX 33313
RALEIGH, NC 27636