The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

Volume 45, Issue 2 • Spring 2020

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures, and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:
  - Bruner, J. S. (1977). *The process of education* (2nd ed.). Cambridge, MA: Harvard University Press.
  - National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

  Journal articles:
  - Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, 7, 196–199.

  Chapters or sections of books:
  - Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). Reston, VA: National Council of Teachers of Mathematics.

  Websites:
  - North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

Volume 45, Issue 2 – Spring 2020

About the Cover
The image on the cover depicts the centroid of a triangle.

Contact Information
The Centroid
c/o Dr. Deborah Crocker, Editor
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
or send email to
CrockerDA@appstate.edu
Please include a return email address with all correspondence.

TABLE OF CONTENTS

2020 NCCTM State Math Conference
November 12-13
Benton Convention Center
Winston-Salem

We hope you will join us for our Annual Fall Conference:

Rooted in the Past and Growing to the Future
50 Years and Counting!

We're celebrating 50 years of Math Education in North Carolina. Proposals are currently be accepted. Come and share your great ideas!

For more information go to

https://www.ncctm.org/

President's Message

State President Kathy Jaqua
Western Carolina University, Cullowhee NC
kjaqua@email.wcu.edu

Happy 50th Anniversary!

Did you know that NCCTM is celebrating its 50th Anniversary this year? In 1970, several leaders in Mathematics Education from across the state joined forces to share ideas and advocate for the students of North Carolina. That first conference marked the beginning of 50 years of sharing ideas and learning about our chosen profession—teaching children mathematics. You can find out much more about the early history of NCCTM and the leaders who founded our organization on the About NCCTM tab at NCCTM.org.

As we begin our next 50 years, we seek to continue our focus on helping all children learn mathematics and experience the joy and wonder that is mathematics. There are changes in store for the coming year, as we make our next 50 years ones of growth, development, and improvement in our work with teachers and through them every student in North Carolina.

We have already established a new Equity and Inclusion Committee, chaired by Dr. Denise Johnson at Winston-Salem State University, to help us improve our efforts to include all teachers and all students in the work of our organization. We want to plan our future with open minds and open hearts. As we move forward into our second 50 years, we want to become a model for how everyone can work together, learn from each other, and support all of our teachers

Also new this year is the move of our Annual Fall Conference and Leadership Seminar from Greensboro, its home for many years, to the Benton Convention Center in Winston-Salem. In Winston-Salem, we will be in the middle of a vibrant area of downtown. There are numerous restaurants within walking distance, and there are several hotels to choose from that offer different rates and amenities. We will have free wifi for everyone over the entire facility, and we are working on adding some new options for embedded field trips and evening excursions.

What's not changing? Our commitment to you and your students is not changing. The quality and scope of our Fall Conference, Leadership Seminar, and the CENTROID are not changing. The value of being an NCCTM member and joining your voice, your knowledge, and your passion for teaching children mathematics is not changing.

Plan now to attend one of the Regional NCCTM Meetings occurring in March, and mark your calendar for our two-day 50th Anniversary Party at the Fall meeting, November 12 and 13, 2020, at the Benton Convention Center in Winston-Salem.

Happy Anniversary!

Celebrate 100 Years of NCTM!

Did you know that the National Council of Teachers of Mathematics, NCCTM's parent organization, will turn 100 in 2020? The NCTM Centennial Meeting and Exposition will be held in Chicago from April 1-4, 2020. There is a conference strand for all interests!

Implement the Effective Teaching Practices
Experience the Depth and Excitement of Mathematics
Look Back and Move Forward: A Centennial View
Create Positive Change
Build Student Agency, Foster Student Identity, and Promote Social Change

For more information, check out: https://www.nctm.org/100/

Coordinates & Geometric Transformations: Four Activities

R. Michael Krach & Todd O. Moyer, Towson University, Towson, MD

The authors present four hands-on activities focusing on the relationship between geometric transformations and coordinate geometry through multiple representations. In addition, a web=based applet is provided for those who wish to provide a more dynamic experience for their students.

According to the Common Core State Standards (CCSS, 2010) and the National Council of Teachers of Mathematics (NCTM, 2000), the study of geometry is a vital component of the mathematical growth of all students at all levels of instruction (kindergarten through grade 12, inclusive). According to the NCTM (2000), the Standards of Mathematical Practice describe ways in which students in grades 1 through 12 should engage in learning concepts and skills as they mature mathematically. Therefore, a subset of these Standards should be an important consideration and component for all lessons/activities planned for and conducted with students. We present four activities that feature the following Standards for Mathematical Practice: M*ake Sense of Problems and Persevere in Solving Them, Construct Viable Arguments and Critique the Reasoning of Others, Use Appropriate Tools Strategically, Attend to Precision, and Look For and Make Use of Structure*. The four activities provided below are ones that have been successfully used when teaching middle and high school students (and preservice elementary and middle school teachers) the relationship between coordinate geometry and geometric transformations. When you use these activities, please let us know how well received and understood they were by the students!

Teacher Notes

For these paper-based activities, familiarity with the use of the Mira and tracing/patty paper is assumed. Materials needed: Mira, straightedge, and tracing or patty paper.

All four of the following hands-on activities have interactive applet analogs in our WebSketchpad (WSP) activity (Krach & Moyer, 2016). WSP is a form of The Geometer's Sketchpad (GSP) for which the user does not need GSP in order to complete the activities. Using appropriate educational technology (like WSP) provides multiple representations of these "hands-on" activities, which focus on the relationship between geometric transformations and coordinate geometry. According to the CCSS-M (2010), NCTM Principles and Standards (2000), and the van Hiele Model for Geometric Thought (Usiskin, 1982), students will better learn and understand geometric content by being exposed to multiple representations of relevant concepts and skills.

"Students can learn more mathematics more deeply with the appropriate use of technology….Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions" (NCTM, 2000, p. 25). In fact, NCTM recommends the use of dynamic geometry software. "Using dynamic geometry software, students can quickly generate and explore a range of geometric examples." (NCTM, 2000, p. 311)

Clements and Battista state that "… if a concept is tied too closely to a single image, its critical attributes might not be recognized or use of the concept in problem-solving situations might be limited because of over reliance on this image" (Clements & Battista, 1992, p. 444). One example would be that "… students' concept image of a right triangle was most likely to include a right triangle with a horizontal and a vertical side, less likely to include a similar triangle rotated slightly, and least likely to include a right isosceles triangle with a horizontal hypotenuse" (Clements & Battista, 1992, p. 447).

Based on the collective number of years of experience we have teaching geometry at the secondary school and college levels of instruction, we support and recommend that teachers use both forms of these activities to enhance their students' understanding of these important concepts and skills. Using a Mira and tracing/patty paper will yield a solid foundation of understanding; the WSP activity will allow students to see many more examples and also permit the asking of the "What if?" questions that should be asked and answered to deepen their understanding with ownership of the content as the goal of learning.

For solutions to the activities, contact us at rkrach@towson.edu and/or tmoyer@towson.edu.

References

Clements, D. & Battista, M. (1992). Geometry and spatial reasoning. In D. A. Growus (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 420-464). New York: Macmillan.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Krach, R. M., & Moyer, T. O. (2016). *Coordinate geometric transformations activity* [web-based applet]. Retrieved from https://tigerweb.towson.edu/tmoyer/coordinategeometrictransformationsactivity/

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry: CDASSG project*. Chicago, IL: The University of Chicago.

Activity One: Distance between points

1) Consider Graph Paper 1 (Fig. 1): P (3,4), Q (3,0), and O (0,0). Is ΔPQO a right triangle? Justify your answer.

Use ΔPQO and your understanding of the Pythagorean Theorem to compute the distance between P and O.

2) Graph the points (3,-4), (-3,4), and (-3,-4). Compute the distance:

between (3,-4) and O:_______________

between (-3,4) and O:_______________

between (-3,-4) and O:_______________

3) If (x, y) is a point in the coordinate plane, write an expression that can be used to compute the distance between (x, y) and O. Be sure to consider the Pythagorean Theorem as you respond to this question.

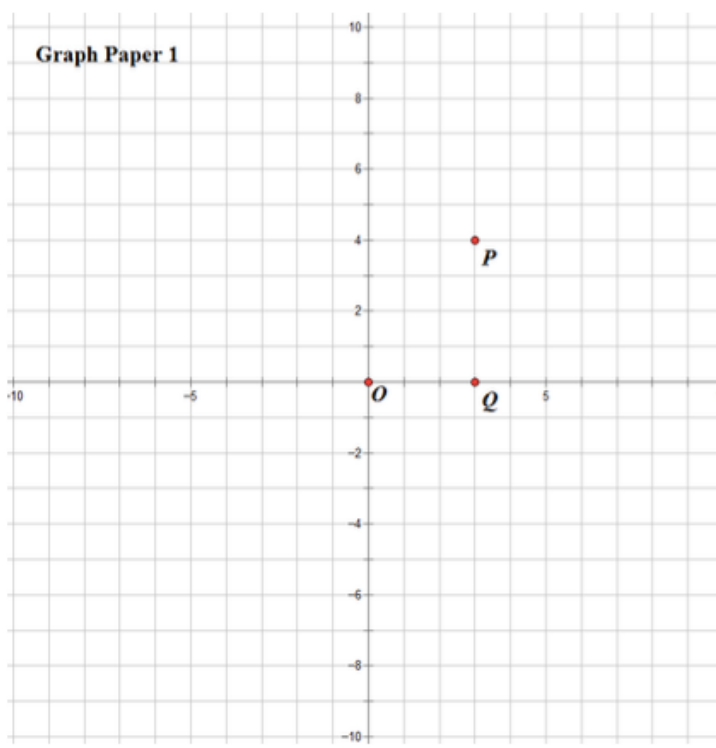


Figure 1. Graph for activity 1.

4) Graph the point (5,4). Now consider the points (5,7), (3,4), and (5,4). These three points are the vertices of a right triangle. Why?

5) Compute the distance between (5,7) and (5,4).

6) Compute the distance between (-3,4) and (3,4).

7) What do you notice about computing the distance between the points in Questions #5 and #6? Be specific in your discussion.

8) Compute the distance between the points (5,7) and (3,4).

9) Compute the distance between the points (5,7) and (-3,4).

10) What do you notice about computing the distance between the points in Questions #8 and #9? Be specific in your discussion. Would using the Pythagorean Theorem be helpful?

11) If (x_1, y_1) and (x_2, y_2) are points in the coordinate plane, write an expression that can be used to compute the distance between (x_1, y_1) and (x_2, y_2).

Activity Two: Reflections

1) The line containing the points (0,0) and (2,2) has been drawn Graph Paper 2A (Fig. 2). Graph the following points: (1,2), (7,5), (0,3), (3,-1), (-3,-6), and (5,5). Place your Mira on the line through (0,0) and (2,2) and provide the coordinates of the images of each of the points you labelled:

(1,2)_________	(3,-1) ________	(7,5)_________
(-3,-6)________	(0,3)_________	(5,5)_________

Make a conjecture about the coordinates of the image of (x, y).

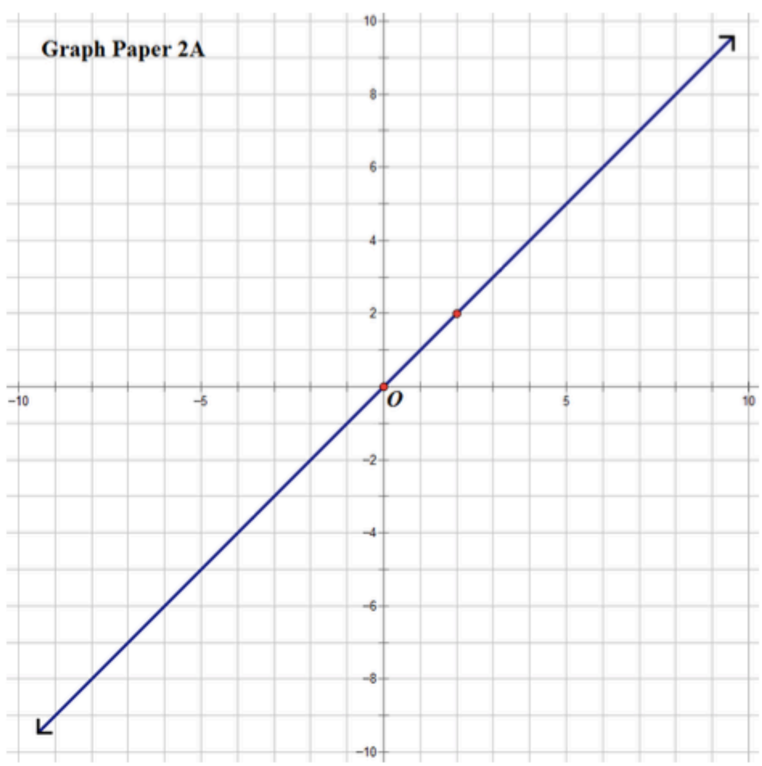


Figure 2. Graph for activity 2, problems 1 and 2.

2) Now test your conjecture using 3 different points. Was your conjecture verified? Explain your reasoning.

3) On Graph Paper 2B (Fig. 3), use the Mira to reflect ΔABC across the line containing (0,0) and (2,2). Label this triangle $\Delta A'B'C'$. What are the coordinates of each vertex point of $\Delta A'B'C'$? Is $\Delta A'B'C'$ congruent to ΔABC? Justify your answer.

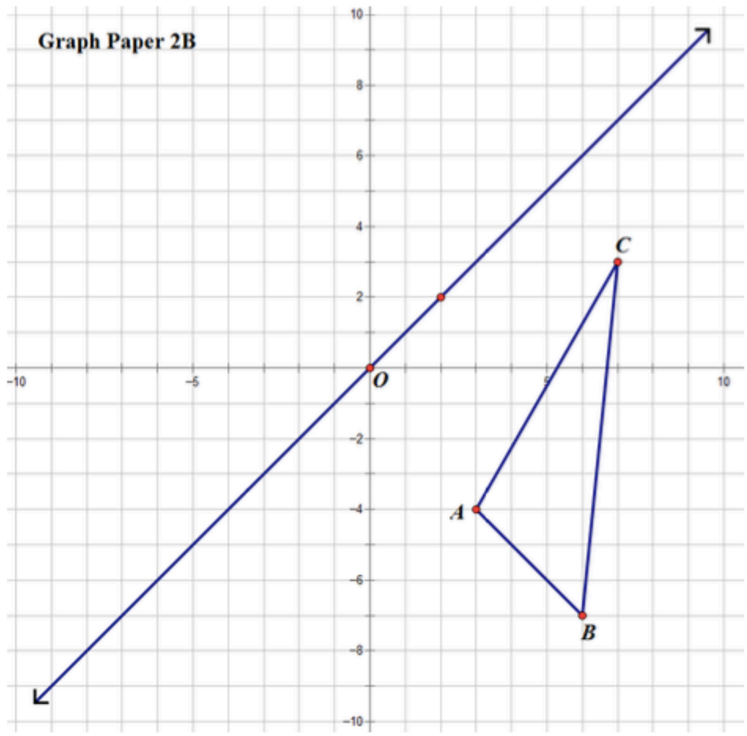


Figure 3. Graph for activity 2, problem 3.

Activity Three: Rotations

1) Using tracing paper and Graph Paper 3A (Fig. 4), graph the image of point A (4,1) after the plane is rotated counter clockwise about (0,0) through 90˚. Do the same for B (-4,1), C (-2,-3), D (0,0).

2) Using Graph Paper 3B (Fig. 5), draw the image of ΔEFG under the rotation above. Label this image $\Delta E'F'G'$. What are the coordinates of the vertices of $\Delta E'F'G'$? Is $\Delta E'F'G'$ congruent to ΔEFG? Justify your answer. Is there a relationship between the coordinates of the vertices of ΔEFG and the coordinates of the vertices of $\Delta E'F'G'$?

3) Using Graph Paper 3C (Fig. 6), reflect ΔEFG across the line containing points (0,0) and (2,1). Label this image $\Delta E'F'G'$.

4) Using Graph Paper 3D (Fig. 7), reflect $\Delta E'F'G'$ across the line through (0,0) and (1,5). Label this image $\Delta E''F''G''$. Carefully describe a rotation under which ΔEFG has image $\Delta E''F''G''$. Is there a relationship between the coordinates of the vertices of ΔEFG and the coordinates of the vertices $\Delta E''F''G''$? If yes, describe the relationship.

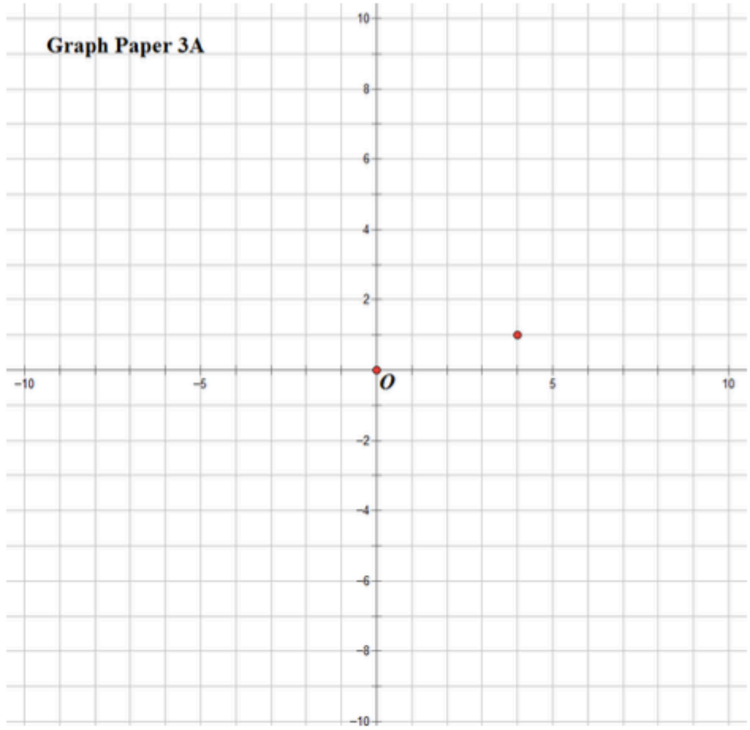


Figure 4. Graph for activity 3 problem 1.

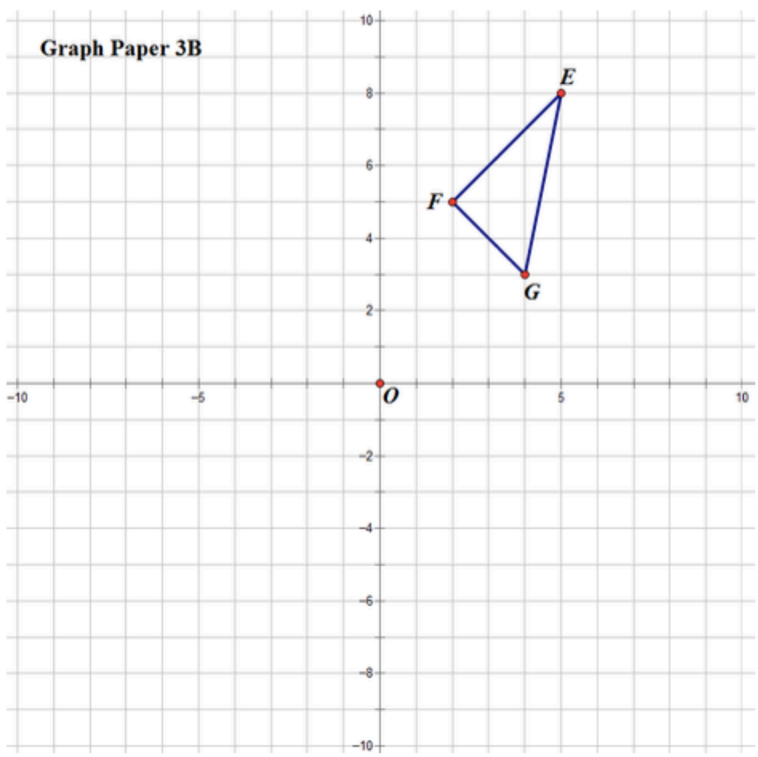


Figure 5. Graph for activity 3 problem 2.

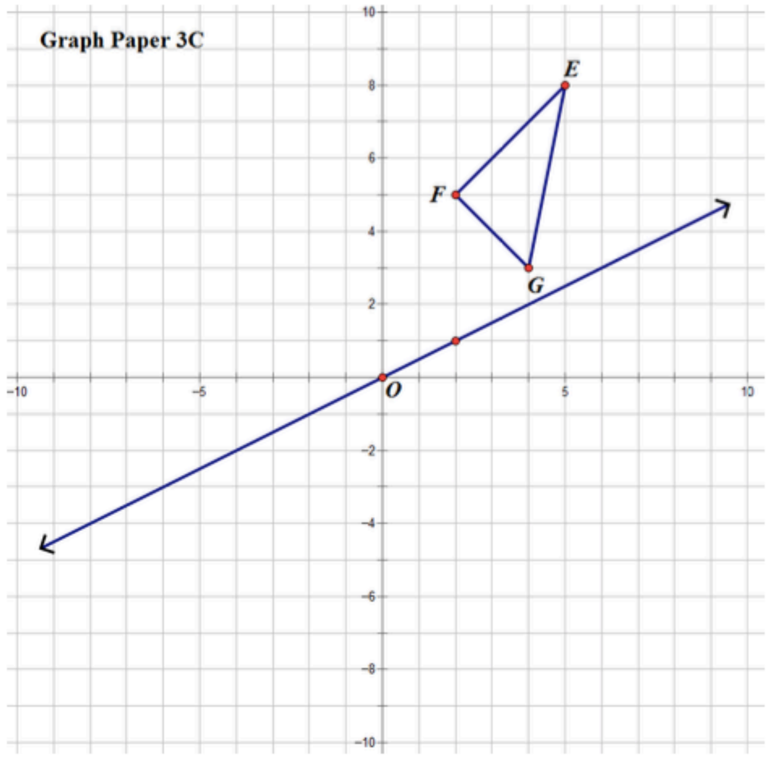


Figure 6. Graphs for activity 3 problem 3.

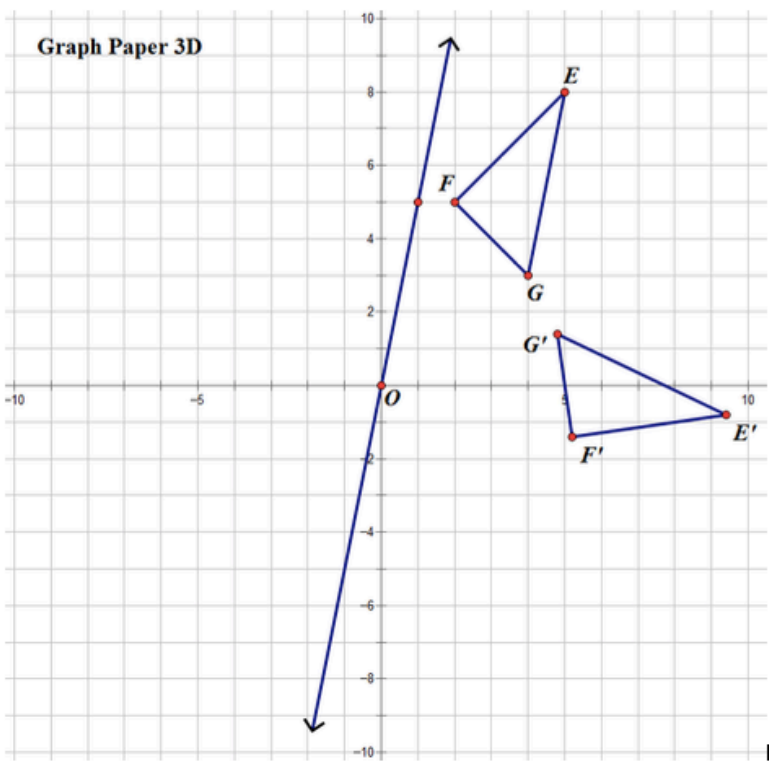


Figure 7. Graphs for activity 3 problem 4.

Activity Four: Translations

1) Using tracing paper or patty paper and Graph Paper 4A (Fig. 8), slide the plane along the line segment OB (from O to B) so that the coordinates of point B are now (0,0). Provide the coordinates of the images of these points:

(0,0):________ (-5,-5):_______ (1,1):________

(3,4):________ (-1,1):________ (x,y):________

2) Compute the length of line segment OB. Using the points in the previous problem and Graph Paper 4A (Fig. 8), compute the distance between each point and its image under the translation. What can you say about the distance that each point in the plane is moved by the given translation?

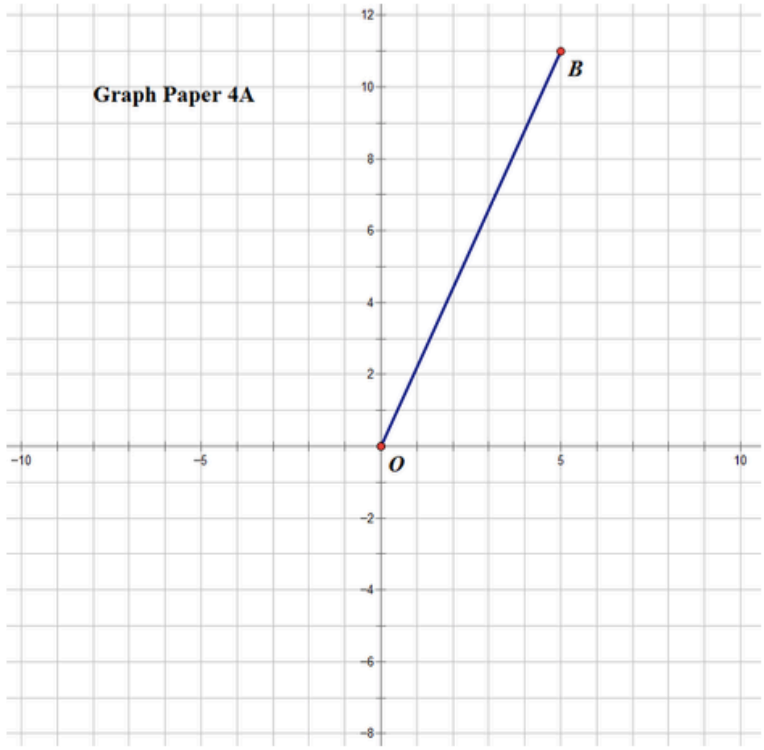


Figure 8. Graphs for activity 4 problems 1 and 2.

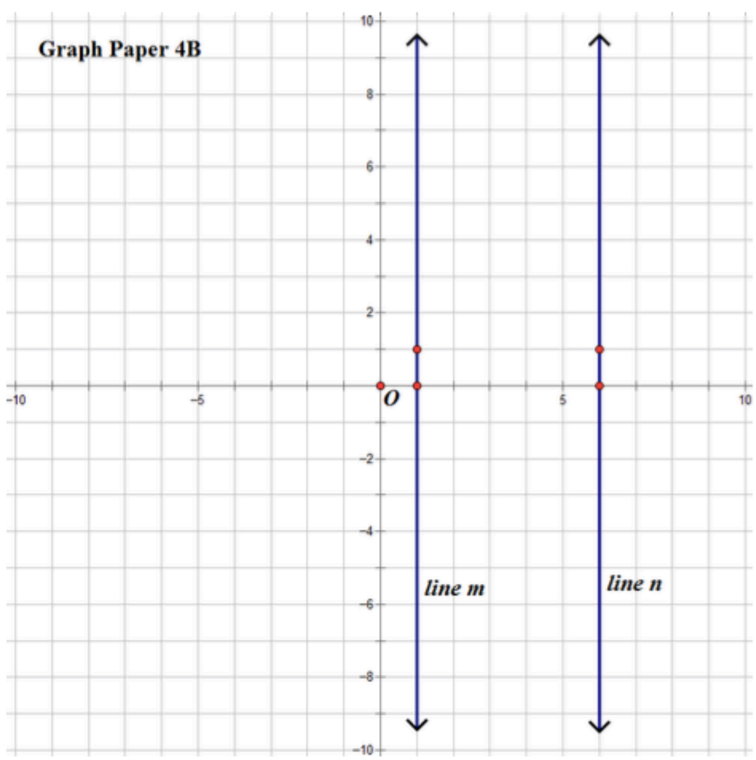


Figure 9. Graphs for activity 4 problems 3 and 4.

3) Using Graph Paper 4B (Fig. 9), draw a line *m* containing points (1,1) and (1,0) and the line *n* containing the points (6,1) and (6,0). Graph and label the following points: *X* (-5,2), *Y* (-2,6), and *Z* (-2,2). Draw the image of ΔXYZ

under a reflection over *m*. Label this image $\Delta X'Y'Z'$. Now reflect $\Delta X'Y'Z'$ over *n*. Label this image $\Delta X''Y''Z''$. Carefully describe a translation that has $\Delta X''Y''Z''$ as the image of the original ΔXYZ. Be specific and thorough in your discussion.

4) List any conclusions or generalizations that you made after completing each of the previous four activities. Share them with the teacher and your classmates.

2019 Outstanding Mathematics Education Student

James Austin Callahan

Reported by Ana Floyd, Randolph County School System, Asheboro NC

Each Fall, NCCTM sponsors the selection of Outstanding Mathematics Education Students, one from each region of NCCTM. This year's recipient is Eastern Region winner **James Austin Callahan**, from East Carolina University.

A double major in mathematics and mathematics education, Austin is a member of East Carolina's Honors College and is currently completing his honor's thesis on the impact of technology in secondary mathematics classrooms. He will graduate with over 150 semester hours of credit, and is described as an *impressive, conscientious, and exceptional* student.

While Austin's academic accomplishments are certainly noteworthy, his contributions and leadership extend beyond the classroom. Austin has been an active member of the ECU's Gamma Student Chapter of NCCTM -- volunteering at Pi Day Celebrations, assisting with annual mathematics competitions, tutoring student athletes, and both attending and speaking at NCCTM conferences. He has been recognized as a leader, encourager, and source of support among his peers – often heading up informal study groups.

Austin's service-oriented accomplishments extend beyond ECU. He has tutored middle school students in math and science, assisted with middle and high school AVID days, volunteered in mathematics and history classrooms at local high schools, and served as the lead volunteer at a local high-needs middle school as part of Americorps. One professor describes Austin as having a *servant's heart*. Another professor wrote: *Austin is not a typical mathematics education student at ECU, or at any campus in the Eastern Region of North Carolina. He demonstrates excellence in all facets of his academic, campus, and community roles. Above all else, he is a respectful, service-minded young man that possesses an unparalleled level of maturity and forward thinking*.

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [jmccormick@rock.k12.nc.us] with questions.

Eliciting to Understand Unfamiliar Student Strategies

Monica Gonzalez & Carrie Lee, East Carolina University, Greenville, NC

The authors present a sequence of questioning strategies teachers can use to understand students' thinking when solving problems. An example illustrating the use of the "Five Eliciting Routines" is provided.

When teachers are confronted with unfamiliar problem-solving strategies, teachers have an opportunity to explore students' thinking in a way that could be beneficial to the whole class. Skillful eliciting provides teachers with evidence of student thinking that can be used to strengthen everyone's conceptual understanding and mathematical fluency (Shaughnessy & Boerst, 2017). We utilized simulation assessments (Gonzalez, 2017) and virtual simulations (Lee et al., 2018) to engage preservice teachers in aspects of eliciting student thinking. From these experiences and a synthesis of research focused on eliciting student thinking (Shaughnessy & Boerst, 2018; Teaching Works, 2011), with a specific focus on equitable teaching practices (Aguirre et al., 2013; Featherstone et al., 2011), we created the "Five Eliciting Routines" to help teachers engage in this crucial teaching practice. These routines further operationalize the decomposition of eliciting practices presented in the research while also explicitly positioning students' intellectual contributions as valuable and important.

Using the Five Eliciting Routines is one way for teachers to begin to capture student's thinking with the goal of making it accessible to everyone. Embedded within these routines are two foci: 1) the interrelationship between students' processes and the mathematical understanding behind those processes; and 2) positioning the students' ideas as valuable. These routines are:

Routine 1:	*Start by posing an open-ended question about the student's process to have them explain their thinking.*
Routine 2:	*Probe when students say something intriguing or significant about the process.*
Routine 3:	*Ask students to demonstrate their understanding with a different representation.*
Routine 4:	*Revoice the student's process to highlight their thinking.*
Routine 5:	*Make student thinking accessible to everyone while making explicit connections between ideas.*

In this article, we demonstrate the Five Eliciting Routines in action. Ms. Duncan, a second-grade teacher in a North Carolina school, has students from many different backgrounds and experiences in her class. Her student Mirza is bilingual and recently moved from Guatemala after her mother took a job to be closer to family in North Carolina. Ms. Duncan poses the following word problem to discover more about how her students are thinking about and solving multi-digit subtraction problems.

The school collected 84 cans of vegetables and fruit for the food drive. 37 of the cans were fruit. How many of the cans were vegetables?

As the students work in small groups, Ms. Duncan circulates around the room to note student strategies. She is puzzled as she looks at Mirza's work (Fig. 1). She observes that Mirza has arrived at the correct answer, but she does not understand how Mirza solved it based on the written work. Ms. Duncan wonders if Mirza made a mistake or was lucky arriving at the correct answer. As Ms. Duncan investigates Mirza's thinking, we will highlight and explain the Five Eliciting Routines.

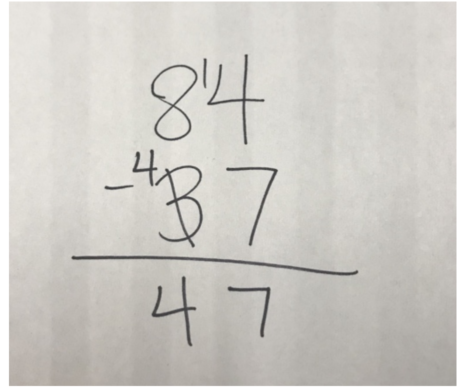


Figure 1. Mirza's work to solve the problem.

Routine 1: Start by posing an open-ended question about the student's process to have them explain their thinking. When faced with an unfamiliar mathematical strategy it is important to seek information from the student before making assumptions about the student's understanding. This line of questioning should begin with an open-ended prompt or question about their process such as, "Tell me more about how you solved this?" or "Can you walk me through how you solved this?" This open-ended nature of inquiry positions the student to lead the discussion while still allowing the teacher to follow-up on needed information.

> *Ms. Duncan: Mirza, I would like to know more about how you solved this problem. Can you please walk me through step by step how you solved this problem?*
> *Mirza: Sure. I knew I needed to subtract, so I did 84 minus 37.*
> *Ms. Duncan: How did you subtract 37 from 84?*
> *Mirza: I needed to add ten to each number.*

Ms. Duncan expresses her interest in learning from Mirza and prompts Mirza to share her process step-by-step. The opening prompt positions Mirza as knowledgeable about the mathematics being discussed. Ms. Duncan learns an interesting step that Mirza added ten to both numbers so she could subtract. Ms. Duncan wonders why Mirza thinks she has to do this, so Ms. Duncan moves on to Routine 2.

Routine 2: Probe when students say something intriguing or significant about the process. As students explain their process or rework a problem, they will recreate the part of the process that teachers may find unfamiliar. This is a pivotal point for teachers to begin to push students for the why behind the process. Pushing for the why helps make the students' conceptual understanding of the process clear to the teacher.

> *Ms. Duncan: Why did you have to add ten to each number if you are subtracting?*
> *Mirza: I know I need to add more to the four in order to subtract the seven.*
> *Ms. Duncan: Where did you get the ten from?*
> *Mirza: I just added it. I didn't take it from anywhere.*

The teacher was confused at this point. In the U.S. Standard Algorithm, the subtrahend is regrouped so that a ten is moved to the ones place. In Mirza's work, this was not the case.

> *Ms. Duncan: So, tell me more about what you meant when you said you added ten to each number.*
> *Mirza: I added ten to 84 and then I added ten to 37.*

This interaction helped Ms. Duncan realize that Mirza did not use the U.S. Standard Algorithm like she assumed; however, she still was not sure why Mirza would add ten to each number to subtract. She needed more clarification on Mirza's thinking, so Ms. Duncan moved to Routine 3.

Routine 3: Ask students to demonstrate their understanding with a different representation. Multiple representations, such as manipulatives or visuals, can help teachers and students have access to how someone else is thinking

(Fennell & Rowan, 2001). Representing a procedure in another way can also demonstrate a conceptual understanding of the process.

> *Ms. Duncan: Can you show me why it would work to add ten to each number when you subtract?*
> *Mirza: Ok. [Draws a number line to explain (Fig. 2).]*

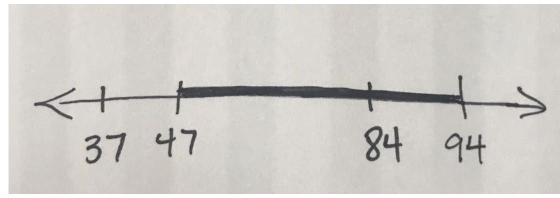


Figure 2. Mirza's number line explanation.

After seeing the number line representation, Ms. Duncan realizes that Mirza adjusted the subtrahend and minuend by the same amount and maintained the difference. She wanted to make sure that Mirza had a conceptual understanding of this algorithm, so she continued to ask questions.

> *Ms. Duncan: Tell me more about this number line.*
> *Mirza: When I added ten to each number the distance is the same. [She points to the space on the number line between 37 and 84 and then the space between 47 and 94 (Fig. 2).]*
> *Ms. Duncan: How is the 94 the same as adding ten to the 4 like you did here? [She points at Mirza's algorithm (Fig. 1).]*
> *Mirza: 80 and 14 is the same as 94.*
> *Ms. Duncan: Then what did you do?*
> *Mirza: Then I subtracted 80 minus 40.*

Ms. Duncan realizes that Mirza has a strong conceptual understanding of this adjusting strategy and place value. Mirza used the picture of the number line to demonstrate her understanding of maintaining the difference while adjusting the subtrahend and minuend by the same number. This process, called the Equal Additions Algorithm, is used in many Latin American countries. Ms. Duncan wants to ensure she and Mirza are on the same page, so she proceeds to utilize Routine 4 for her own clarification.

Routine 4: Revoice the student's process to highlight their thinking. Revoicing is a strategic discourse routine in which the teacher repeats back what a student has said in order to clarify and/or highlight a student's contribution (Chapin, O'Connor, & Anderson, 2009; Kazemi & Hintz, 2014). This communication exchange allows the student to address how their idea is being communicated and append the restatement as needed. In addition, revoicing positions the student's idea as valuable, and when done in a whole group it increases the access of the idea to all students.

> *Ms. Duncan: I think I understand what you did. Please let me know if I am correct. When you solved this problem, you knew you had to subtract 37 from 84. You first looked at the ones place and noticed that you needed to add ten to the four to make it fourteen in order to subtract 7. Since you added ten to the 87 you also needed to add ten to the 37 to change it to 47. Then you subtracted 40 from 80 to arrive at the answer of 47.*
> *Mirza: Yes. This is how I learned to subtract.*
> *Ms. Duncan: Thank you so much for teaching me a new way to subtract numbers. Would you be comfortable sharing your strategy with the whole class?*
> *Mirza: [Smiling ear to ear] Yes!*

Ms. Duncan positions Mirza as a knowledgeable mathematician who is teaching her a new way to think about subtraction. This positioning then leads Ms. Duncan to Routine 5 where other students can examine and connect Mirza's strategy to their own.

Routine 5: Make student thinking accessible to everyone while making explicit connections between ideas. Routine 5 is in place to extend the learning opportunities of all students in a way that promotes positive mathematics identities and leverages students' mathematical funds of knowledge (Turner & Drake, 2016). The student's thinking that was elicited is made accessible to the whole class through student-led strategy sharing. The goal of strategy sharing is not to just collect multiple ideas, but instead it is to make connections among different ways to think about the mathematics (Kazemi & Hintz, 2014). Making connections among students' mathematical ideas helps to build conceptual understanding and fluency.

During Ms. Duncan's whole class discussion, she calls on Mirza to explain the process to the class. The students in the class ask Mirza questions until they too understood the adjusting process she used. Ms. Duncan asks questions to have students compare the adjusting strategy to the regrouping strategy. Henry captures the excitement of making connections below.

> *Henry: Oh! I get it! In this one [points to his standard regrouping algorithm] the number is still 84 because it is 14 and 70. But in this one [points to Mirza's adjusting algorithm (Fig. 1)] the number is 94 because it is 14 and 80. You added a whole other 10 to 37 to make it 47. You just wrote it a different way instead of writing 94 minus 47. That's cool!*

Many students in Ms. Duncan's class used the U.S. Standard Algorithm to solve the subtraction problem, but Mirza's strategy provides the opportunity for the class to strengthen their conceptual understanding of place value and their flexibility with subtraction. Students are able to explain the different values used and how they relate to the original problem. Many students favor the efficiency of Mirza's strategy and try applying it on their own.

Discussion

Throughout using the Five Eliciting Routines the substance of the eliciting is not limited to the student's process but is instead continually extended to incorporate questions that attend to the student's conceptual understanding. One key way students are supported in sharing their understanding of the mathematics is through the use of multiple representations (Pape & Tchoshanov, 2001). We, as teachers, utilize multiple representations within instruction (Fennell & Rowan, 2001), but we also need to capitalize on how these multiple representations can be used by students to express their own understanding.

When students use strategies that we are unsure of, what if we approach these strategies with curiosity and adoration for how their minds are thinking? Embedded in the 5 Eliciting Routines is the belief that students' ideas are valuable and worth investigating. Students' are positioned to be heard and empowered to do the heavy lifting of explaining and justifying their own thinking. The teacher is positioned as the learner who is excited for this new knowledge to be shared with the rest of the class. A teacher's willingness to continue learning, especially through their students, is a way of improving instructional practice. The Five Eliciting Routines is one way to put this willingness into action.

References

Aguirre, J. M., Mayfield-Ingram, K. & Martin, D. B. (2013) *The impact of identity in K-8 mathematics: Rethinking equity-based practices*. Reston, VA: National Council of Teachers of Mathematics.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Sausalito, CA: Math Solutions.

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A., & Wood, M. (2011). *Smarter together! Collaboration and equity in the elementary math classroom*. Reston, VA: National Council of Teachers of Mathematics.

Fennell, F., & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7*(5), 288-292.

Gonzalez, M. (2017). *Applying designed based research in middle grades mathematics teacher preparation* (Unpublished doctoral dissertation). University of Houston, Houston, Texas.

Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Portland, ME: Stenhouse.

Lee, C., Lee, T., Castles, R., Dickerson, D., Fales, H., & Wilson, C. (2018). Implementation of immersive classroom simulation activities in a mathematics methods course and a life and environmental science course. *Journal of Interdisciplinary Teacher Leadership, 2*(1), 3-17.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice, 40*(2), 118-127.
Shaughnessy, M., & Boerst, T. A. (2017). Uncovering the skills that preservice teachers bring to teacher education: The practice of eliciting a student's thinking. *Journal of Teacher Education, 69*(1), 40-55.
TeachingWorks. (2011). *High leverage teaching practices.* Retrieved from http://www.teachingworks.org/work-of-teaching/high-leverage-practices
Turner, E. E., & Drake, C. (2016). A review of research on prospective teachers' learning about children's mathematical thinking and cultural funds of knowledge. *Journal of Teacher Education, 67*(1), 32-46.

Trust Fund Scholarships: Now $1000

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants & scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson Plumblee, richards@elon.edu.

Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

Preparing for the New 4th Math Courses

Have you looked at the revised courses yet? The NC DPI has the specific standards for the revised versions of Precalculus, Discrete Mathematics (now geared toward computer science), and the new "Math 4" replacing AFM. Read more at:

https://docs.google.com/document/d/1QkKQ_G743JBAaOthFo-k9vypqU3029WZzZ2EB3dH3tM/edit

2019 Outstanding Elementary Mathematics Teachers

Reported by Denise Schulz, North Carolina Department of Public Instruction, Raleigh, NC

Each year, principals are encouraged to nominate the teacher who does the most effective job teaching mathematics in their school. From those nominated, each LEA selects one teacher to represent the best in mathematics teaching from the entire system. The teacher receives a membership in NCCTM, recognition at the State Mathematics Conference, and a special memento of the occasion. The grade level cycles, and this year the teachers were chosen from among the best elementary mathematics teachers in North Carolina.

2019 Outstanding Teachers by School System

Christine Bertson, Alamance-Burlington
Katie Smith, Alexander
Jennifer Sheets, Alleghany
Laura Absher, Ashe
Melissa McKeown, Asheboro City
Jessica Jones, Avery
Katie Moore, Beaufort
Amy Mizell, Bertie
Carla Bethea, Bladen
Holly Needham, Brunswick
Amanda Fields, Buncombe
Nancy Mitchell, Caldwell
Laurie Fazio, Charlotte-Mecklenburg
Eva Membreno, Chatham
Anna Ross, Cleveland
Rhonda Wetherington, Craven
Rebecca Criste, Cumberland
Angela Hinkle, Davidson
Nell Mason, Edenton-Chowan
Shelley Greene, Elizabeth City - Pasquotank
Carah Reid, Franklin
Lovie Roscoe, Gates
Christopher Leas, Granville
Nadine DeLallo, Greene
Colby Williams, Guilford
Sherice Campbell-Ellison, Harnett
Cristin Ratkowski, Henderson
Chuck Chavka, Hickory City
Brittany Waguespack Hoganson, Hoke
Kenitra Waddell, Iredell-Statesville
Caroline Fongemy, Kannapolis City
Susan Jackson, Madison
Renee Robbins, Moore
Dana Samuel, Mooresville Graded
Melissa Martin, Mount Airy City
Christina Recker, Newton-Conover
Tracy Rettig, Orange
Beth Johnson, Pender
Rachel Benge, Perquimans
Carrie Briggs, Person
Ashlee Thomas, Pitt
Julia Tackett, Polk
Christy Hutchins, Randolph
Candi Horton, Roanoke Rapids
Signe Holley, Rockingham
Lindsey McCraw, Rutherford
Cynthia Lee, Sampson
Richard Ritchey, Scotland
Laura Williams, Stokes
Daniel Bowman, Surry
Nichole Hayes, Thomasville City
Stephanie Wacek, Wake
Debbie Glover, Watauga
Ashley Edwards, Wayne
Shannon Goss, Weldon City
Amber Faw, Wilkes
Gretchen Godinez, Wilson
Tiffani Richardson, Winston-Salem/Forsyth
Diane Reece, Yadkin

2019 Innovator Award Winners

Dr. Nathan Borchelt and Dr. Sloan Despeaux

Reported by Ana Floyd, Randolph County School System, Asheboro, NC

The purpose of the NCCTM Innovator Award is to recognize and reward individuals and/or groups who have made an outstanding and noteworthy contribution to mathematics education and/or NCCTM. The Recipients of this year's award are Western Carolina University faculty Dr. Nathan Borchelt and Dr. Sloan Despeaux for their establishment and continued commitment to Math Teachers' Circles.

Pictured: Nathan Borchelr and Sloan Despeaux (center), surrounded by many of their Math Teach Circle participants.

Professional development is intended to provide meaningful learning opportunities for teachers that potentially influence teacher practice and student achievement. An essential element of such professional development is that if offers opportunities for collective participation and professional inquiry within learning communities. The North Carolina Network of Math Teachers' Circles is a statewide network of mathematics teachers and faculty intended to do just that.

Math Teachers' Circles around the state provide an ongoing way for teachers and professors to stay connected around mathematics. Math Teachers' Circles (MTCs) are professional communities of K-12 teachers and higher education faculty who meet regularly to discuss and explore interesting and engaging mathematical problem solving. This community of mathematics professionals work on rich mathematics problems and informally discuss problems of practice. They are committed to fostering a love for and understanding of mathematics among all students.

Math Teachers' Circles started in 2006 at the American Institute of Mathematics. Since then, more than 80 MTCs in 37 states have been established. Nathan and Sloan have led the initiative in North Carolina. One participant describes Math Teachers' Circles as a unique and incredible experience for teachers in North Carolina, an opportunity for genuine and rich discourse about mathematics teaching and learning. This grant-funded project, which started as a small, local grassroots effort, has grown and expanded over the past 5 years and includes teachers from across the state. Additional MTCs have been created in our eastern, piedmont, and western regions of North Carolina. Our awardees are committed to growing this network, supporting existing learning circles, and providing teachers with enriching and meaningful experiences that ignite their passion for mathematics.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Rose Sinicrope, sinicroper@ecu.edu.

2019 Rankin Award Winner

Catherine "Katie" Stein Schwartz

Reported by Lee Stiff, North Carolina State University, Raleigh, NC

At the 2019 State Mathematics Conference, NCCTM presented **Catherine "Katie" Stein Schwartz**, with the W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual.

Catherine "Katie" Stein Schwartz, Associate Professor of Mathematics Education at East Carolina University, is honored in recognition of her professional commitment to the teachers and students of the State of North Carolina. Katie is regarded as a dedicated elementary mathematics teacher, one who possesses expertise in elementary mathematics and is always willing to share that knowledge and insight with students and colleagues alike. Indeed, it has been said that she is…"that rare gem of a mathematics educator with total focus and commitment to mathematics teaching and learning in the elementary school."

Katie has more than 20 years of classroom experience, and has excelled in the role of instructor over that entire period of service! She is a Chapel Hill/Carrboro City Schools Elementary Mathematics Teacher of Year, a Carrboro Elementary School Teacher of the Year, a UNC-Greensboro Graduate Student Teaching Awardee, a university Alumni Association Outstanding Teaching Awardee, and a NC Board of Governors Distinguished Professor of Teaching Awardee! She has authored numerous scholarly articles and presented workshops and sessions at numerous professional meetings including those of NCCTM, NCTM, American Education Research Association, Association of Mathematics Teacher Educators, and Research Council on Mathematics Learning;

Katie has provided leadership across the State by: serving as a curriculum developer and/or facilitator in grants such as Tools for Teachers, Partners, and Teachers and Administrators Partnering for Mathematics Learning (known as TAP Math); providing strong leadership as a founding member and secretary for the Association of Mathematics Teacher Educators–North Carolina; contributing "guidance on the revision of the NC Standard Course of Study for Mathematics,…and feedback for…the NC Unpacking Documents;" serving as a researcher with the North Carolina Collaborative for Mathematics Learning, a research-practice partnership focused on supporting mathematics education in North Carolina; served as NCCTM Eastern-Region Secretary and President, NCCTM State Secretary, Secretary of the NCCTM Trust Fund Committee; and volunteering at NCCTM State Conferences for the at least the last 18 years.

In support of this nomination to the Rankin Award Committee, one colleague wrote that Katie…"embodies a love for mathematics education and goes beyond…expectations…to help ensure students in NC have the best mathematics education possible…" Another supporter wrote that Katie…"is masterful in her ability to engage very diverse students of mathematics knowledge, ability, and attitude in mathematical problem solving. She challenges; she questions; and she transfers ownership of the problem-solving task to the participants and authority to the mathematical reasoning.…No one teaches teachers better."

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from Lee V. Stiff, lee_stiff@ncsu.edu.

Problems2Ponder

Holly Hirst, Appalachian State University, Boone, NC

In each issue of **The Centroid**, Problems2Ponder presents problems similar to those students might encounter during elementary and middle school Olympiad contests.

Student solution submissions are welcome as are problem submissions from teachers. Please consider submitting a problem or a solution. Enjoy!

Problem submissions: If you have an idea for a problem to publish, please email Holly Hirst (hirsthp@appstate.edu) a clear photo or PDF document of a typed or neatly written problem statement, along with a solution. Include your name and school affiliation so that we can credit you with the submission.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst (hirsthp@appstate.edu) a clear image or PDF document of the correct solution, with the name of the school, the grade level of the student(s), the name of the student(s) if permission is given to publish the students' names, and the name of the teacher.

Deadline for publication of problems or solutions in the Fall 2020 Centroid: June 30, 2020.

Spring 2020 P2P Problems

Problem A: A, B, and C represent digits in the following subtraction problem. Find the sum of A, B, and C.

$$\begin{array}{r} 8\,A\,4 \\ \underline{-3\,7\,B} \\ C\,7\,8 \end{array}$$

Problem B: the first four terms of the sequence 2, -3, -2, 3, 2, -3, -2, 3,… repeat endlessly. Find the sum of the first 2013 terms in this sequence.

Solutions will be posted in the next edition of The Centroid.

Fall 2019 P2P Problem Solutions

Problem A: Two consecutive positive integers are each less than 100. One integer is divisible by 17, and the other integer is divisible by 21. Find the two integers.

Solution: The simplest way to find the integers is to list multiples of 17 and 21, since there are just a few multiples less than 100:

17, 34, 51, 68, 85
21, 42, 63, 84

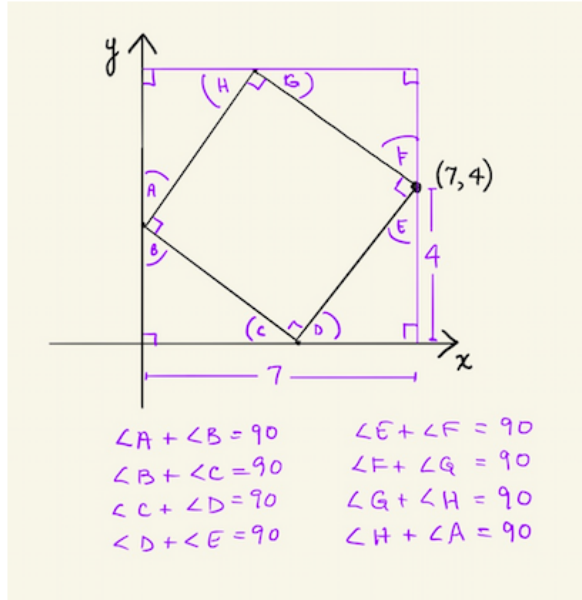
84 and 85 are the only two consecutive numbers in these lists!

Problem B: A square is positioned in quadrant I of the Cartesian Coordinate system so that one vertex lies on the x-axis, an adjacent vertex lies on the y-axis, and a third vertex lies at the point (7,4). Find the area of the square.

Solution: There are several ways to attack this problem. Here is one approach: Enclose the figure in a square as shown, noting all the known lengths and labeling all of the angles. Every adjacent pair of angles must add to 90° as listed, and from this we can deduce that the following angles are equal:

‹A = ‹C = ‹E = ‹G and ‹B = ‹D = ‹F = ‹H

Thus all the triangles are similar right triangles; in addition, all have the same length hypotenuse so all the triangles must be congruent!



One of the legs is length 4 and the sum of the two legs lengths must be 7, so the triangles all have legs of lengths 3 and 4. The hypotenuse must be 5 (Pythagoras!) and so the area of the inner square must be 5x5 = 25.

2019 Math Contest Coaches Award

John Noland

Reported by Phillip Rash, NC School of Science and Math, Durham NC

The purpose of this award is to recognize and reward an individual who has made an outstanding and noteworthy contribution to Mathematics Education and NCCTM by having formed, coached, and sponsored teams or groups of students in mathematics competitions. This year's recipient is John Noland, Cary Academy.

At Cary Academy John teaches Mathematics and Computer Science courses, and he has been involved with math contests for over 15 years. He consistently brings competitive teams (teams placing in the top 5) to the Wake Tech Regional Competition and to the State Math Contest. John also has helped out with the NC ARML program since 2002. For those of you not aware, ARML is a big contest. It's the only national level, team-based, on-site math contest in America. John has been recognized for his efforts by the national ARML committee with not one but two Samuel Greitzer Awards in 2012 and 2017. John continues as a Power Round Grader at the contest each year and actively contributes to the success of the NC ARML teams.

John's enthusiasm for math and his help in all the aspects of putting together a successful NC ARML team have been a wonderful constant over many years now. The other ARML coaches, students, and parents are all grateful for his dedication.

More information on the State Mathematics Contest, ARML, and other contests can be found at the State Math Contest website:

https://sites.google.com/site/statemathcontest/home

NCCTM Board

contact information can be found at ncctm.org

Officers

	State	Eastern Region	Central Region	Western Region
President	Kathy Jacqua	Ginger Rhodes	Denise Johnson	Sheila Brookshire
Past President	Julie Kolb	Tim Hendrix	Julie Riggins	Karen McPherson
Elementary Vice President	Ana Floyd	Karen Walker	Michelle Harger	Meredith Stanley
Middle Grades Vice President	Kim McCuiston	Allison Landry	Tanya Ford	Sumer Williams
Secondary Vice President	Lauren Baucom	Bill Worley	Wendy Bartlett	Emily Bryant
College Vice President	Charity Cayton	Maureen Grady	Adam Pennell	Julia Webster
Other State Officers	Secretary Emily Elrod	Parliamentarian Wendy Rich		

Committee Chairs

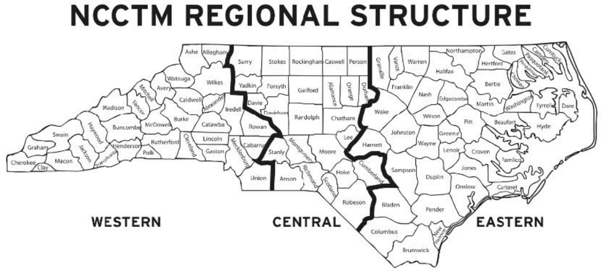
Conference and Exhibit Services, Kay Swofford
Centroid, Holly Hirst and Debbie Crocker
Convention Services, Marilyn Preddy
Financial Chair, Ray Jernigan
Handbook, Ray Jernigan
Historian, Kathryn Hill
Inclusion and Diversity, Denise Johnson
Leadership Conference, Kathy Jaqua
Management Services, Joette Midgett
Math Celebrations, Anthony Finlen and Pamela Parham
Math Contest, James Beuerle and Phillip Rash
Math Counts, Harold Reiter
Math Fair, Betty Long
Mini-grants, Joy McCormick
NCDPI Representative, Lisa Ashe
NCSSM Representative, Ryan Pietropaolo
NCTM Representative, Debbie Crocker
NCSM Representative, Debbie Crocker
NCMATYC Representative, Luke Walsh
Nominations, Julie Kolb
Rankin Award, Lee Stiff
Special Awards, Ana Floyd
Student Affiliates, Holt Wilson
Trust Fund, Catharina Middleton
Webpages, Holly Hirst

Becoming a Member

Follow the “Membership Information” link on the ncctm.org website, or go directly to:
http://www.ncctm.org/members/register.cfm

NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

NCCTM REGIONAL STRUCTURE



North Carolina Council of
Teachers of Mathematics
PO Box 33313
Raleigh, NC 27636