The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

In this issue:

NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

NCCTM

Volume 46, Issue 2 • Spring 2021

The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

Subscribe by joining NCCTM. For more information go to http://www.ncctm.org.

Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

Guidelines for Authors
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Debbie Crocker, CrockerDA@appstate.edu. Persons who do not have access to email for submission should contact Dr. Crocker for further instructions at the Department of Mathematics at Appalachian State, 828-262-3050.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures, and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:
  - Bruner, J. S. (1977). *The process of education* (2nd ed.). Harvard University Press.
  - National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*.

  Journal articles:
  - Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.

  Chapters or sections of books:
  - Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). National Council of Teachers of Mathematics.

  Websites:
  - North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* Retrieved from http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

The Centroid

The Journal of the North Carolina Council of Teachers of Mathematics

Volume 46, Issue 2 – Spring 2021

About the Cover
The image on the cover depicts the centroid of a triangle.

Contact Information
The Centroid
c/o Dr. Deborah Crocker, Editor
Department of Mathematical Sciences
Appalachian State University
Boone, NC 28608
or send email to
CrockerDA@appstate.edu
Please include a return email address with all correspondence.

Table of Contents

NCCTM Thanks its Sponsors

Thanks to the following organizations who sponsored our move to virtual activities last fall. Please be sure to check out their websites!

- Twin City Quarter and Benton Convention Center: https://twincityquarter.com/meetings/
- i-Ready: https://www.curriculumassociates.com/products/i-ready
- Imagine Learning: https://www.imaginelearning.com/
- Braining Camp: https://www.brainingcamp.com/
- Math Olympiads: http://moems.org/

NCCTM Annual Conference and Leadership Seminar

November 10, 11, 12, 2021

Plans are underway for the State Mathematics Conference and Leadership Seminar to be held this fall – hopefully in person – at the Benton Convention Center in Winston Salem. Watch the website for information coming soon!

President's Message

State President Kathy Jaqua
Western Carolina University, Cullowhee NC
kjaqua@email.wcu.edu

As I sit at home waiting for my next zoom meetings with my classes, my colleagues, my children, my grandchildren, my friends, (fill in the blank here with all of the other meetings you have had), I realize how much of my world has become remote by necessity. I know that I am looking forward to returning to my classes face-to-face in a couple of weeks (YEA!). I have, however, renewed my commitment to making every class purposeful and to inviting participation of all students regardless of the modality.

I have struggled with getting all of the technology to work together as I want, and I still believe in the power of direct interaction between a teacher and a student to help that student reach the aha! moment that we all love. I also believe that there are many other ways that I can reach that student, and zoom may be one of them. I plan to keep some of the zoom aspects as we return to class. In a recent calculus class, while the conversation about limits was flowing from one person to another, I found the chat box buzzing with additional comments and questions. Students who weren't confident enough to contribute directly to the conversation or who had more basic questions were willing to interact in the chat box. Students answered questions for their classmates, and even brought some of the ideas and questions expressed there into the oral conversation. The comfort of today's students with expressing themselves online was translated into including more students in the classroom conversation. It also gave me a stronger lens into students' mathematical understanding.

Like you, I have "teacher ears" and "teacher eyes," and I can pretty well see when a student is confused, but not always what the source of confusion is. Having a way that I could "listen" with both my ears and my eyes to gauge the understanding of my students was amazing. How much that ability to hear more students in real time helped me was very clear, because my other calculus class did not choose to use the chat box. Both classes were based on the same plan with the same activities, but the discussion in the class that relied only on oral conversation was not as robust, nor did it include as many students. I finished class less confident of the students' understanding. I am curious to see what questions and concerns from the assigned homework come from each class at the next meeting. Did the extra interaction in the chat box have a strong positive effect? If it did, how can I encourage my other class to develop that means of interaction? How can I provide an outlet similar to a chat box to engage more students in mathematical interactions in real time?

So the moral of the story is: Not everything about virtual class meetings has been bad. I look forward to a movement back to "normal," but I want my new normal to be better--better for student interaction and participation in class and better for my ability to assess my students' understanding and identify the sources of confusion as the class proceeds rather than afterwards.

Finally, as I prepare to pass leadership in NCCTM to the incoming President, Stephanie Buckner Hill, I want to thank you for allowing me to serve in this role. Throughout all of the adjustments to schedules for the Board of Directors' Meetings, the time and form of the Annual Meeting, and all of the other changes that were required, I have had the steadfast support of all members of the leadership team. I am grateful for their willingness to work with me and to find creative ways to continue the work of NCCTM. Their willingness to do whatever tasks were required, in spite of the huge increase in work needed to successfully continue teaching mathematics, has been greatly appreciated. To all of the members who agreed to run for one of the Vice-President positions in NCCTM, I thank you. You are all outstanding educators who were recognized as such by your colleagues who nominated you. The newly elected officers will bring new ideas and new voices to the Board of Directors, and I look forward to watching our organization grow and change as we continue in our mission to improve mathematics education for every student in North Carolina.

Mathematics as a Metaphor

Dr. Cacey L. Wells, Appalachian State University, Boone, NC

The author describes how to use a metaphor assignment to encourage students to think more abstractly about mathematics concepts.

I began my mathematics teaching career in a social studies magnet school in South Texas. Early on, I noticed how my colleagues were very interested in collaborating cross-curricularly in order to create grade-level projects and learning experiences for our students that would be unique, culturally relevant, and fit for 21st century learning. This contagious mindset gave me space in my practice to begin thinking about the interdisciplinary nature of my own classroom. One day I was casually speaking with one of my English Language Arts (ELA) colleagues and was hoping to run an idea by him that I hoped to try in class. My idea was to take the concept of metaphor and apply it to some of the content my AP calculus class was investigating. At the time, we were wrapping up exploring the concept of a derivative and techniques for finding them. I noticed my students were having a difficult time understanding the behavior of a function and how its derivative can be used to describe its behavior. Rather than trying to create contrived, “real-world” scenarios, I decided to use what students already connected with through their ELA classes. In this case, the idea of a metaphor.

In many cases, mathematics teachers have alluded to the notion that students are not able to use “school-learned methods and rules because they do not fully understand them” (Boaler, 1998, p. 42). This leads to teachers being asked to create more open-ended tasks that allow space for students to engage in mathematics exploration rather than simply copying rules or formulas (Boaler, 2002). When students are able to write about mathematics, they have the opportunity to explore deeper understandings and find joy in the fascination of mathematical concepts they are exploring (Sanders, 2009). Metaphors are often associated with flowery and figurative language that usually do not find their way into mathematics courses. As is the case with many ELA teachers, Moe (2011) found that metaphors are essential to processes of forming arguments in students’ writing. This idea can and should be carried over into the realm of teaching and learning mathematics. Taking often abstract concepts in math classes and connecting them to our students’ experiences and personal lives allows students opportunities to form arguments, justify their reasoning, and better understand content.

Traditionally speaking, metaphors have provided platforms for teachers allow students to relate experiences they have had in their lives to what they are learning in class (Presmeg, 1997). As mathematics teachers, we routinely use metaphors in our pedagogies to make connections between mathematical concepts and with students’ shared experiences. For example, a teacher may connect an idea like understanding end behavior of rational functions and horizontal asymptotes to the magnetic force. In students’ experiences, they most likely have experienced magnetic pull, so using that shared experience may help teachers liken the nature of horizontal asymptotes in rational functions to a strong magnet that pulls functions closer to them as they tend towards infinity.

In this case, most students are familiar with magnetic pull, so connecting this experience to the new or abstract idea of horizontal asymptotes helps students make connections. However, what I am proposing in this article is actually the opposite of what we typically find in classrooms across the US. In this case my hope is to show how I was able to use mathematical concepts as metaphors to help students explain phenomena and characteristics of their lives and lived experiences. In other words, the shared experience of a mathematical concept is the metaphor used to connect to different aspects of students' lives.

Mathematics as a Metaphor

As I mentioned earlier, students in my AP calculus class were wrapping up exploring the concept of a derivative and understanding basic derivative rules. My students were able to find derivatives pretty well using the various techniques we had explored. So, any passerby probably would not think twice about whether or not my students were learning differential calculus. While they seemed proficient with the mechanics of the concept, I still felt like something was missing in their conceptual understanding. What my students were struggling with was not how to find derivatives, but why the functions and their derivatives behaved the way they did. This is where the idea of using mathematics as a metaphor was implemented in order to help students really grasp the concept at hand.

After speaking with my ELA colleague across the hall, I came up with an idea for having students begin thinking more abstractly about derivatives. I asked my students to take the concept of a derivative and use it as a metaphor to describe who they were. Essentially students were to pick a function and its derivative and explain how its behavior represented them as a person in one to two pages of text. The task was incredibly open-ended and the submissions from students were quite varied; however, in every case students were able to dig into the behavior of functions and derivatives by connecting them to their lives. Below are three examples from students in my AP calculus class.

Example 1

Throughout my studies of functions in calculus, I have found that the cosine function and its derivative represent me as a person. The cosine function is one of the six trigonometric functions that we have studied and it can be expressed as $y = cos(x)$. The derivative of a function, is a function that expresses the slope of the original function. The derivative for the cosine function can be expressed as $y = -sin(x)$.

Firstly, the graph of the cosine function looks somewhat like a constant wave with many peaks and troughs. These peaks and troughs are also known as maximums and minimums, which are very important because they help us understand where the slope of the graph changes in the means of being either increasing or decreasing. I see these points as representing multiple unique aspects of my personality. I possess many different hobbies and interests with just a few being: my interest in medicine, running, reading and helping others. These interests are all very important to defining who I am, just as the maximums and minimums help define the way the graph looks.

Secondly, the cosine graph creates a simple, predictable pattern. For the most part, we know what pattern it will follow as it starts at zero and only fluctuates between $y = 1$ and $y = -1$. This aspect of the function represents me in that I always try my very best to have integrity. Having integrity can be described as being the same person in all of the varying aspects of your life and staying true to your values no matter the situation or environment. You can say that the cosine graph is integrous because it follows the same structure and pattern as you move towards infinity.

Lastly the derivative, $y = -sin(x)$, graphs the slope of the cosine function. With the derivative, we can directly see when the slope is negative or positive. I see the derivative as representing me because no matter what negative or bad events occur in life, I believe there is always something else positive to focus on. Despite bad things that can happen in life, we can still look to see the positive side of things and be grateful for something else amazing in our lives. This is like the derivative because it can recognize negative and positive slopes in the function.

In this case, the student connected the sinusoidal nature of the cosine function to the ups and downs of everyday life. As the derivative of the graph measures its slope, it is simply the sine function only reflected about the x-axis. In this case, she seems to see this reflection over the axis to illustrate that when there are negative situations in life there can be positives that come out of it.

Example 2

Looking at who I am as a person, I like to see my life as the derivative of a function using the product rule. I am my own person, and I will always be me. Things will change overtime but Mariana Rodriguez will remain Mariana Rodriguez. The derivative of the function is like growing up. You could find the derivative of me which would be Mariana multiplied by anything new that shapes my life. Through life people are always learning new things and constantly facing change, finding derivatives in their lives. The rule of finding the derivative of any function with a product function is: $f\,'(x)$ multiplied by $g(x)$ plus $g'(x)$ multiplied by $f(x)$. So the derivative looks like $f'(x) \times g(x) + g\,'(x) \times f(x)$. I know life can throw obstacles in your way and things are always changing, but you stay the same. My life is f(x), the things I learn throughout life is $g(x)$, when you find the change of my life you see my original self with the addition of different views on life. I see my life as $f(x)$ multiplied to a bunch of other functions, just waiting to get the derivative. Each $g(x)$ is something new I will learn as I live my life. Each new thing adds change to my derivative, each time I find a new derivative the function increase, my knowledge on life increases. The more times you continue to search for the derivative, the more you will add on to your life. I am the $f(x)$, the base of the functions, I will always be carried along in each derivative, but my knowledge will continuously expand my view on life. With each derivative my function begins to form into one giant $f(x)$, each other functions will just be pulled into the large mass of numbers, because as I age there will less and less new things to learn, since I had already done the product rule on them.

In this second example, the student sees the product rule for finding derivatives as a meaningful metaphor for her life. Here she sees herself as a function comprised of two entities being multiplied together, a product $f(x) \times g(x)$. In this case her life is $f(x)$ and her learned life experiences are $g(x)$. When she finds her derivative, she sees how her life is intertwined with her lived experience. Due to the nature of how the derivatives of product functions behave, the result larger and more powerful: $f'(x) \times g(x) + g'(x) \times f(x)$.

Example 3

If a single function and its derivative could represent my entire being, that function would have to be $f(x) = x$. It is noticeable that this derivative is not very complex, in fact its simple. I am not complex in the sense that I am honest. I say what I am thinking, I mean what I say, and I do not often lie. I have found many people to be indecisive, they will say one thing but think something completely different. Like $f(x) = x$, I do not make anything over-complex.

Much like this function, I am not extravagant, loud, or attention grabbing. Above all, I'm not obnoxious perhaps like the function $f(x) = 4x + 9$. I do not seek approval, nor am I overcome with the need to impress. $f(x) = x$ is symbolic of myself not perceiving myself as flashy or at all showy.

The function graphed looks like a diagonal line across the cartesian plane, increasing in height. The graph of this parent function increases, as does my constant thirst for knowledge. As long as I'm alive I want to learn new things, create and explore with that knowledge. The function never ends in the x or y direction. As so long as time is increasing (x-axis), knowledge thirst is increasing as well (y-axis).

The derivative of $f(x) = x$ is equal to 1: $f'(x) = 1$. Beneath my outer shelling I am actually one of a kind. I am an individual, semi-independent, with my own thoughts and my own motivations. The graphed version of this derivative is a solid, horizontal line that rests above the origin. This is symbolic of the positive and negative things that affect my life; I'm always resting in the positive side of the graph.

$f(x) = x$ is not over-complicated, nor flashy, and the graphed function symbolizes my infatuation with learning. The derivative of this wonderful function, $f'(x) = 1$, represents my

unique individuality, and when it is graphed it reflects my positive attitude. Both of these functions represent a metaphor of myself.

In this final example, we see how the work of a student who tended to be less engaged in the day-to-day activities of my class throughout the year. His metaphor for how the derivative is used to explain his identity is really quite powerful. You see in his writing that he sees himself very simply—linear. His derivative metaphor is quite powerful in that it represents his uniqueness as an individual.

In these three examples you can see how different students, with different experiences, writing abilities, and perceptions took the assignment and ran with it in different directions. Some students were more ambitious and took on more difficult derivative concepts, while others went a simpler route. In any case, the purpose of the assignment was fulfilled. I set out to have students use mathematics as a metaphor for understanding how derivatives can represent themselves as human beings. The task had a high ceiling in that students could creatively take the concepts we were covering and construct very elaborate metaphors that connected the behavior of a function and its derivative to their individual lives. On the other hand, this task had an incredibly low floor in that students who were really struggling in class were able to easily grasp the ideas we were discussing in the assignment. In the case of a struggling mathematics student, they could take a less-complicated derivative and still manage to create a powerful metaphor.

Conclusion

For my students, using mathematics as a metaphor helped to solidify conceptual understanding in my mathematics classes. By connecting concepts that we were learning in class to facets of their everyday lives, students were able to construct for themselves instances to which they could relate. Further, as a teacher, I was able to read students' creative metaphors to better understand for myself how they were processing information. Whether it was the nature of graphs, the behavior of functions, or the process of finding a derivative, I could formatively assess their conceptual understanding of the topic at hand.

It should be noted that I have used this concept in other classes, besides AP calculus. When I share this example with other math teachers, they often think my students are gifted or because they are labeled "AP" they can better engage in tasks like this. This simply is not true. My precalculus and algebra I students were able to also came up with powerful metaphors for topics like convergent and divergent series. The concept, whether found in calculus, precalculus, algebra, or geometry works exactly the same. Take a topic you are covering in class and have students make connections to their lived experiences, their character traits, or their identity.

A few years ago, the now famous Dan Meyer was delivering a Ted Talk in which he spoke about teaching mathematics in today's age of high-stakes testing and accountability. In his talk, he said something quite profound that caught my attention as a new mathematics teacher and continues to inspire me to think outside-the-box about mathematics today. Meyer says that our job as math teachers is to "sell a product to a market that does not want it, but is forced by law to buy it" (Meyer, 2010). When we hear statements like this we can go one of two ways. We can think how messed up the work of education is in its current state, give into the status quo, and trudge forward in a survivalist's mode of operation. Or, we can take Meyer's words as a call to action, make it our personal mission to engage our students, and create creative spaces for them to explore mathematics in fun and interesting ways. What better way to connect students to mathematics than to have them use mathematics as a metaphor for themselves.

Here are a few more examples I have generated over the years in varying mathematical subjects:

Geometry	This week we have been exploring similar figures. How do similar figures represent the different relationships in your life?
Algebra	We have been talking about the nature of functions. Take one of the functions we have been studying and use its characteristics to describe some of your characteristics.
Trigonometry	Sine and Cosine are complements of one another. How does the nature of sine and cosine resemble human behavior?

Calculus	Take the three outcomes of finding limits towards infinity in rational functions: Convergence to 0, Convergence to a non-zero limit, and Divergence. What characteristics of rational functions are in place for each scenario? Take these three instances and the corresponding characteristics to describe different outlooks one might have on life.

References

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings, *Journal for Research in Mathematics Education*, *29*(1), 41-62.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33,* 239-258.

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. Jossey Bass.

Meyer, D. (2010, May). *Ted: Ideas Worth Spreading* [Web log message]. Retrieved from http://www.ted.com/

Moe. P. W. (2011). Rethinking Metaphor: Figurative language and first-year composition. *Teaching English in the Two-Year College, 38*(3), 282-290.

Presmeg, N. C. (1997). Reasoning with metaphors and metonymies in mathematics learning. In L. D. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images* (pp. 267–280). L. Erlbaum.

Sanders, C. Exploring writing geometry. *Mathematics Teacher, 102*(6), 432-439.

Wanko, J.J. (2017). Teaching inductive reasoning with puzzles. *Mathematics Teacher*, 110(7), 514-519.

NCCTM Elects New Officers

On March 27, 2021, the following new state and regional officers will assume their duties.

State
President Stefanie Buckner, Past President Kathy Jaqua
Vice President for Elementary Meredith Stanley
Vice President for Middle Grades Allison Landry
Vice President for Secondary Emily Bryant Hare
Vice President for Colleges & Universities Vincent Snipes
State Secretary Katie Mawhinney

Eastern Region
President Kaneka Turner, Past President Ginger Rhodes
Vice President for Elementary Leigh Belford
Vice President for Middle Grades Katie Martin
Vice President for Secondary Heather Davis
Vice President for Colleges & Universities Kayla Chandler

Central Region
President Sara Vaughn, Past President Denise Johnson
Vice President for Elementary Sha Mosley
Vice President for Middle Grades Anthony Finlen
Vice President for Secondary Ebony Jason
Vice President for Colleges & Universities Perry Gillespie

Western Region
President Kwaku Adu-Gyamfi, Past President Sheila Brookshire
Vice President for Elementary Amanda Thompson-Rice
Vice President for Middle Grades Charlcy Carpenter
Vice President for Secondary Jennifer Reed
Vice President for Colleges & Universities Erica Slate Young

2020 NCCTM Award Cycle

This year has been challenging for everyone, including those individuals who donate their time to make NCCTM work for all math teachers in North Carolina. During the height of the pandemic over the late spring, summer, and fall, we made the decision not to pursue selecting the following award winners. We look forward to resuming recognizing these outstanding individuals in 2021.

Outstanding Mathematics Education Students
Outstanding Mathematics Teachers
Math Contest Coaches Awardee
Innovator Awardees
Rankin Awardee

Please be sure to nominate your hard-working, deserving colleagues for these awards! We will include their names in the Spring 2022 issue. Check out https://ncctm.org for details.

Outstanding Mathematics Education Students

In an effort to encourage student enjoyment of mathematics and mathematics education, NCCTM recognizes outstanding mathematics education students at colleges and universities across the state. Each college and university with a teacher education program may nominate one rising junior or senior working towards elementary, middle school, or secondary certification. Nominations are due on May 30.

Outstanding Mathematics Teachers

Each year, school principals in North Carolina schools are encouraged to nominate the teacher they believe does the most effective job teaching mathematics in their school. From those nominated, each LEA selected one teacher to represent the best in mathematics teaching from the entire system. The mathematics consultants at NC DPI handle the nomination process.

Math Contest Coaches Award

The North Carolina Council of Teachers of Mathematics accepts nominations for the State Math Contest Coach's Award each year. The purpose of this award is to recognize and reward an individual who has made an outstanding and noteworthy contribution to mathematics education and NCCTM by having formed, coached, or sponsored teams or groups of students in mathematics competitions. Any NCCTM member may submit nominations by August 1 of each year.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The Circle Game

Dr. Gail Kaplan, Towson University, Towson, MD

And the seasons they go round and round
And the painted ponies go up and down
We're captive on the carousel of time
We can't return we can only look
Behind from where we came
And go round and round and round
In the circle game

- Joni Mitchell (1966)

The author presents an activity that leads students to the discovery of the geometric definition of an ellipse.

Children learn about circles at a very young age. In nursery school teachers often ask the students to sit in a circle. The Common Core math standards for kindergarten state students should be able to “identify, name, and describe two-dimensional shapes, such as squares, circles. . .” (Common Core State Standards Initiative [CCSSI], 2010a). For high school, the Algebra Standards include “Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically" (CCSSI, 2010b). For example, find the point of intersection between the line $y = -3x$ and $x^2 + y^2 = 3$. The Geometry Standards include “Translate between the geometric description and the equation for a conic section” (CCSSI, 2010c) and “Know precise definitions of angle, circle, . . . based on the undefined notions of point, line, distance along a line . . .” (CCSSI, 2010d).

In this discovery experience, students actively explore why the geometric definition of an ellipse makes sense, by applying their prior knowledge of properties of circles. The exploration assumes that students already know the definition of a circle as the set of all points equidistant from a fixed point called the center. The initial challenges not only engage students, but also refreshes their memory of the definition of a circle.

The Activity

First, form groups of students and provide each group with a piece of yarn or string and small circular stickers (Fig.1). Ideally, all groups have different string lengths. Instruct the groups to do the following:

Figure 1. Red stickers to use for the activity

I. Place a red sticker on the floor or a desk and label it the center. Use the string to find 5 points that are exactly the same distance from the fixed point and place a sticker at the location of each point. How many points are there that are exactly that distance from the fixed point? What is the name of the shape formed by all of these points?

II. Is it possible for 2 circles to have no points in common? If so, draw a diagram. Is it possible for 2 circles to have exactly 1 point in common? If so, draw a diagram. Is it possible for 2 circles to have exactly 2 points in common? If so, draw a diagram. What is the largest possible number of points 2 different circles can have in common?

The goal of the student exploration is to use two sets of concentric circles to find points **P** so that the distance from point **P** to point **A** plus the distance from point **P** to point **B** is exactly 10 units. On the graph shown in Figure 2, point **A** and point **B** are the center points for the two sets of concentric circles shown. The two smallest circles have a radius of one unit and each circle has a radius that is exactly 1 unit larger than the radius of the previous circle. Note: On the student handout (included at the end of this article) all circles are blue and the ellipse is not shown.

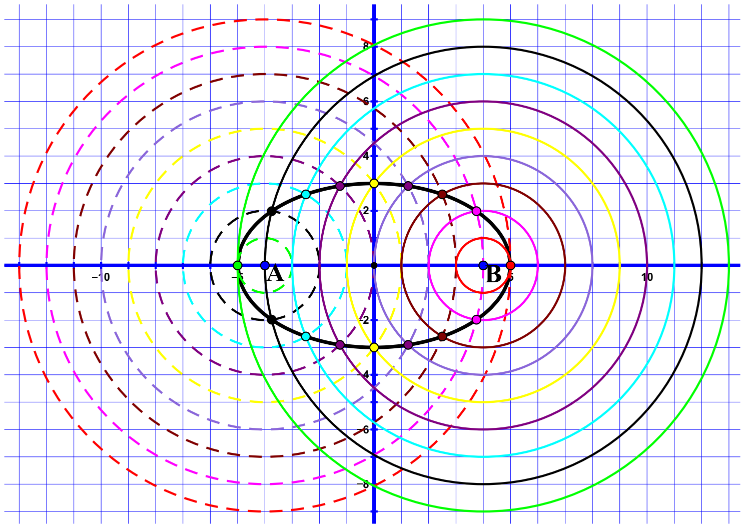


Figure 2. Diagram illustrating the final step in the development of the ellipse

Stage 1: Students are given a grid of two sets of concentric circles and asked to highlight all points that are exactly 9 units from point **A** (see the red dashed circle) and all points that are exactly 1 unit from point **B** (the solid red circle). The next prompt is to draw a solid dot at all points that are both 9 units from point **A** and 1 unit from point **B**. There is only one point satisfying this condition, as shown by the red dot.

Students repeat the same process using a different color to draw a solid dot at the point that is exactly 1 unit from Point **A** and 9 units from Point **B**, illustrated by the green circles and dot. We now have 2 points, each of which verifies the sum of the distance to point **A** plus the distance to point **B** is exactly 10 units.

Stage 2: Students are prompted to use a different color and highlight all points that are exactly 8 units from point **A** and 2 units from point B. Once again, students need to recognize these points are the intersections of the two pink circles, shown by the two pink dots. Students repeat the process and identify all points on the concentric that are exactly 2 units from Point **A** and 8 units from Point **B**, as shown by the two aqua dots.

Stage 3: Students continue in this manner, finding 4 points in each subsequent stage until there are 16 points highlighted and for each point the sum of the distance to point **A** plus the distance to point **B** is exactly 10 units. The next prompt is to connect the dots with a smooth curve. What do you notice? What do you wonder? An elliptical shape appears! It is truly an aha! moment.

This process enables students to gain an in depth understanding of the precise geometric definition of an ellipse: An ellipse is the set of all points in a plane so that the sum of the distance to two given points is constant. For the previous student exploration, the constant is 10.

The next part of the exploration provides an opportunity to determine the equation of this ellipse. Points **A** and **B** are called the two foci of an ellipse. Students are told to assume the coordinates of point **A** are (-3, 0) and point **B** are (3, 0). Using this information, students are prompted to appropriately draw the x and y axis on the given graph. Next, students find the coordinates of the points already found with the largest and smallest x-values: (5, 0) and (-5, 0), and the largest and smallest y-values: (0, 3) and (0, -3). Students are then prompted to choose which of the equations below is satisfied by all 4 points.

$$\frac{x^2}{25} + y^2 = 1, \quad x^2 + \frac{y^2}{9} = 1, \quad \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Through this exploration students gain an in depth understanding of the formal definition of an ellipse.

Fascinating Properties of Ellipses

Students may find the following applications of elliptical shapes quite interesting. The reflective property of an ellipse states that if a light source is located at one of the foci of an ellipse and the ray of light is aimed at the boundary of the ellipse, the ray reflects to the other foci, as shown in Figure 3.

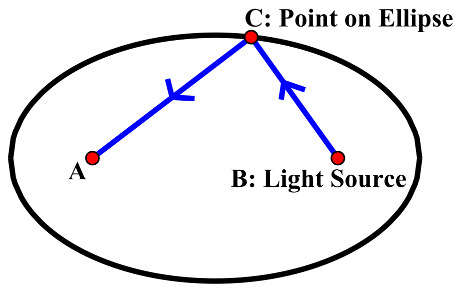


Figure 3. Illustration of the reflective property

The reflective property of ellipses works for sound rays as well. Children often make tin can phones by taking two metal cans, removing one end from each can, and making a small hole in the end remaining. Next, take a long string, thread it through the hole in each can, and tie a knot so the string cannot come out of the hole. If each child holds one can, moves away until the string is taut, and talks, the cans provide a tin can phone! Why does this work? There is actually a simple explanation. Hold the palm of your hand in front of your mouth and speak. Can you feel the vibrations in the air from your mouth? The tin phone works because the vibrations from your voice inside of the cans force the metal end to vibrate which then makes the string vibrate and move to the other can. Imagine a space where you and a friend can stand more than 30 feet apart in a huge room filled with hundreds of people and have a private conversation. Sound impossible? It's not! This type of space is called a whispering gallery and is the result of properties of the elliptical shape explored in this article. Sound waves which start at one of the foci of an ellipse travel along the boundary of the ellipse to the other foci. Grand Central Terminal in New York City is an architectural marvel (Fig. 4). Over three quarters of a million people pass through each day. It is a VERY, VERY noisy and busy place! However, in the whispering gallery, two people can stand far apart and have a conversation while whispering because the ceiling is an elliptical shape.

Figure 4. The whispering gallery in Grand Central Station

Another use of elliptical shapes is in modern medicine. There are many options for getting rid of kidney stones. For many, many years removing a kidney stone (they are very painful) required major surgery. But researchers were able to use the reflective property of an ellipse by generating shockwaves from one foci so that they reflect off the boundary of an ellipse and move to the other focus. The waves target the stone and break it up into smaller pieces that can be released through urine. A patient is put on a table at a location so that the kidney stone is exactly at the focus. Not only is this procedure, called Shock wave lithotripsy, almost painless, it is often done as an outpatient procedure. The mathematical properties of the ellipse make it possible!

Abstract mathematics can often lead to significant advances in other fields!

References

Common Core State Standards Initiative. (2010a). *Kindergarten: introduction*. Retrieved from http://www.corestandards.org/Math/Content/K/introduction/

Common Core State Standards Initiative. (2010b). *High school: Algebra » Reasoning with equations & inequalities.* Retrieved from http://www.corestandards.org/Math/Content/HSA/REI/

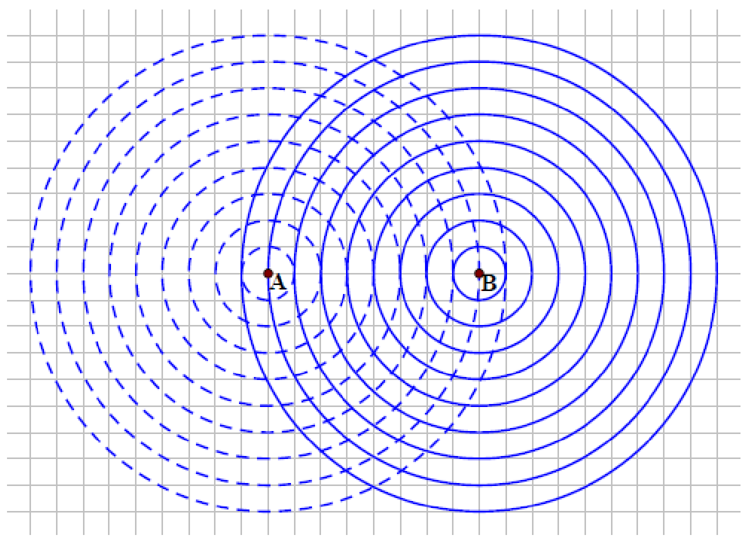
Common Core State Standards Initiative. (2010c). *High school: Geometry*. Retrieved from http://www.corestandards.org/Math/Content/HSG/

Common Core State Standards Initiative. (2010d). *High school: Geometry » Congruence*. Retrieved from http://www.corestandards.org/Math/Content/HSG/CO/

Mitchell, J. (1966). *The circle game* [song lyrics]. Retrieved from https://jonimitchell.com/music/song.cfm?id=39

Student Activity

Part I: Find points so that for each point the sum of the distance to point A plus the distance to point B is exactly 10 units.



Note: Ideally for each of the numbered steps use the same color to highlight the appropriate dashed and solid circles.

Stage 1:

- Highlight all points that are exactly 9 units from point A.
- Use the same color to highlight all points that are exactly 1 unit from point B.
- Draw a solid dot at each point that is exactly 9 units from point A <u>and</u> 1 unit from point B.
- How many points did you find?
- For each point you found, what is the sum of the distance from that point to point A plus the distance from that point to point B?
- Use a new color to highlight all points that are exactly 1 unit from point A.
- Use the same color to highlight all points that are exactly 9 units from point B.
- Draw a solid dot at each point that is exactly 1 unit from point A <u>and</u> 9 units from point B.
- How many points did you find?
- For each point you found, what is the sum of the distance from that point to point A and the distance from that point to point B?

Stage 2:

- Use a different highlighter color and repeat the procedure from Stage 1 to find all points that are exactly 2 units from point A and 8 units from point B. How many points did you find?
- Repeat the same procedure to find all points that are exactly 8 units from point A and 2 units from point B. How many points did you find?

Stage 3: Using the given circles, continue this process to find the points so that the sum of the distance from each point to the given points A and B is exactly 10 units. This should result in finding a total of **16 points**.

Stage 4: Connect the points found to create a smooth curve. What do you notice? What do you wonder? Does this curve look familiar?

Stage 5: Describe how you could find more points so that for each point the sum of the distances to the given points A and B is 10 units.

Stage 6: Theoretically, how many points exist so that for each point the sum of the distances to the given points A and B is 10 units? Justify your response.

<u>Part II Exploring this Shape</u>

Assume that the coordinates of Point A are (-3, 0) and the coordinates of Point B are (3, 0). Draw the x and y axes in the appropriate location using thick black lines.

Carefully look at all of the points found in **Part I** and answer the following questions

1. What are the coordinates of the point with the largest y-value?
2. What are the coordinates of the point with the smallest y-value?
3. What are the coordinates of the point with the largest x-value?
4. What are the coordinates of the point with the smallest x-value?
5. Determine which of the following equations satisfy all of the points above. Show all work.

$$\frac{x^2}{25}+y^2=1,\quad x^2+\frac{y^2}{9}=1,\quad \frac{x^2}{25}+\frac{y^2}{9}=1$$

Your answer is the equation of an ellipse, a shape defined by the set of all points so that the sum of the distance to two fixed points is constant. The two fixed points for this ellipse are A and B, and the sum of the distance to these two points is 10 for each point on the ellipse.

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800.

The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [jmccormick@rock.k12.nc.us] with questions.

Building Conceptual Understanding in the Age of Virtual Learning

Dr. Winn Crenshaw Wheeler, Bellarmine University, Louisville, KY
Dr. Jessica Ivy, Bellarmine University, Louisville, KY
Amy Cordrey, Oldham County Schools, Buckner, KY

The authors describe an activity for elementary level students that integrates a conceptual context, and include advice on implementing activities virtually.

Consider the comfort of a warm bowl of chili…a nourishing meal that fills not only the body but fills the home with savory warmth as ingredients (beef, beans, spices, tomatoes, onions) cook for hours to coalesce and become a synthesized whole. The individual cozying up with this warm delight would think very differently about a bowl filled with the raw ingredients—the whole (the bowl of chili) is certainly more than the sum of the parts (a tablespoon of chili powder, a finely diced onion, a pound of raw meat, a jar of canned tomatoes). Yet, in thinking about elementary mathematics, it is often the case that classroom practices focus on the development of individual skills without attention to the unifying, synthesizing, and essential role of conceptual understanding. Conceptual understanding (much like the cooking of chili) is the necessary frame for students to be able to use basic facts productively, efficiently, and effectively to solve real world problems. Equating the memorization of multiplication facts with conceptual understanding of multiplication is akin to suggesting that the individual ingredients of the chili recipe are the same thing as the cooked chili that has simmered all afternoon.

Although the importance of conceptual understanding is widely accepted, it is often abandoned for "fact fluency" when challenges emerge, such as student's struggling with grade-level tasks, inflexible curriculum maps, parent's expectations, or even global pandemics. The rapid shift to online learning during the COVID-19 pandemic is an extreme example of the kind of challenge that can tempt many to shift to procedural tasks; however, less significant complications are often blamed for shifting the classroom focus away from conceptual contexts, "so our students don't fall further behind." We must remind ourselves, that isolated attention to individual skills will not support or scaffold long term learning. We must understand that the development of fact fluency is one ingredient in the context of a larger recipe.

In this paper, we describe the work of one teacher's integration of a conceptual context into a virtual lesson. In April 2020, two weeks into the shift to non-traditional instruction, Ms. C engaged a small group of her third-grade students in the Lemonade Dilemma (Figure 1). Each group member was provided with a clue for solving the problem. She used Google Classroom to foster interaction with and between students and facilitated instruction using the same questioning strategies she had used in their classroom.

The Lesson

Students were provided with the clues shown in Figure 1 and asked, "What do you notice?" Despite challenges of collaborating remotely, students engaged in the task. Some of the observations involved a literal reporting of the clues, "they have 3, 32-ounce [containers] of lemon juice,” and “Every clue is asking the same question, ‘how much lemonade can they make and how much of the ingredients will be left over?’” In other instances, students made inferences about the context of the problem, “they have a kitchen to mix the lemonade”; this inference helped students realize that the children in the problem would have access to a sink and water in making the lemonade.

Student 1 THE LEMONADE DILEMMA Sam and his friends are planning a lemonade stand. Molly brings a 5-pound bag of sugar she found in her pantry. How much lemonade can they make? What ingredients will they have left over?	Student 2 THE LEMONADE DILEMMA Sam and his friends are planning a lemonade stand. Sam’s dad gives him a half-full bag of sugar. The bag weighed 5 pounds when it was full. How much lemonade can they make? What ingredients will they have left over?
Student 3 THE LEMONADE DILEMMA Sam and his friends are planning a lemonade stand. Nelson brings three 32-ounce bottles of lemon juice. How much lemonade can they make? What ingredients will they have left over?	Student 4 THE LEMONADE DILEMMA Sam and his friends are planning a lemonade stand. Lily’s family has volunteered to set the lemonade stand up in their front yard and allow the children to mix the lemonade in their kitchen. How much lemonade can they make? What ingredients will they have left over?

Figure 1. The Lemonade Dilemma clues

Other observations focused on mathematical content as illustrated by this exchange:

> “It says the sugar weighs five pounds when its full, but it is only half full so that would be half of five.”
> Another student replied, “That’s 2.5.”

Finally, students used the invitation for dialogue to determine problem solving needs. Ultimately, this dialogue led students to the conclusion there were some key questions:

- What are the ingredients needed to make lemonade?
- How much lemon juice does it take to make lemonade?
- How much sugar does it take to make lemonade?

After three minutes making observations, Ms. C provided the remaining two clues (Figure 2). Students worked diligently, with one parent noting, “That’s the most excited I’ve seen him.” Ms. C provided support through questioning, “Where would be a good place to start?” Students were unable to see others’ representations clearly, but they persevered. Engagement was evident as students talked about Molly, a character in the problem, as if she was their neighbor. Rian[1] recognized, “We can get water out of her sink! We can have as much water as we need—that is not the problem.”

[1] *Student names are pseudonyms.*

Additional Clue 1 THE LEMONADE DILEMMA	Additional Clue 2 THE LEMONADE DILEMMA
Ten pounds of sugar will fill 22 and a half cups! A cup of liquid is equivalent to eight ounces.	The Best Lemonade Ever Recipe 8 cups of water cups lemon juice cups sugar Combine all ingredients. Mix well and serve over ice.

Figure 2. The remaining Lemonade Dilemma clues

Once students understood one clue conceptually, their new knowledge gave way to other pieces, and they determined they could break the problem into parts. They needed to determine how much sugar and lemon juice they would use, then figure out what was left over. The most intimidating aspect was the “half-full bag of sugar” Ms. C reflected, “the half was throwing them off; they needed to see the information from the problem. They were looking at me to see it all, and they couldn’t because they only had it on Google Classroom. They couldn’t see all the data.” After they understood the problem, the group wrestled with the challenge of agreeing on a strategy. Sam wanted to use mental math, while Ani and Natali wanted to draw. They agreed that in their classroom they would have drawn a model as they reasoned through the problem.

While communicating online, Ani, Natali, Sam, and Rian’s work was less organized than in the classroom. They created individual models but could not collaboratively create representations. Despite this challenge, they worked together and critically considered each other’s work. Student-created models included repeated addition, symbolic work, and pictorial descriptions. At one point, Natali questioned Sam, “How did you know this?” As Sam explained he found a computational error, “Oh, no, no, I’m wrong, miscalculated” (Figure 3).

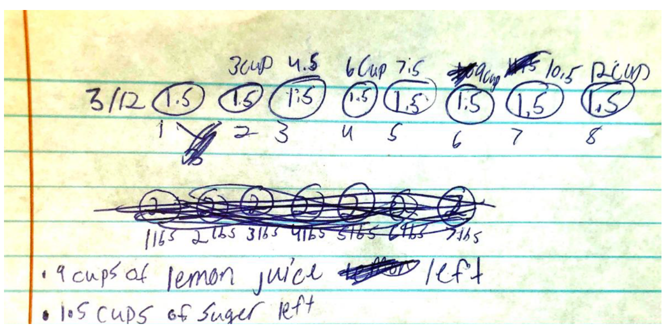


Figure 3. Students sharing strategies

Fact fluency alone—like a single ingredient of chili recipe fails to nourish like the richness of a hot bowl of chili—does not equate to the larger whole of mathematical proficiency. These students demonstrate an understanding fed through a rich recipe of ingredients including engagement through meaningful tasks, dialogue, and application of skills rather than memorized procedures. They were equipped with a combination of conceptual understanding, representational modeling, and fact fluency empowering flexibility and sophisticated problem solving. How might their interaction with this dilemma have been different if their experience with multiplication was limited to drill and practice? Their reality would be more representative of one described by Van de Walle (2007), “[drill] has been an ever-present component of mathematics classes for decades, and yet the adult population is replete with those who proclaim, ‘I was never any good at mathematics’ and who understand little more about the subject than arithmetic” (p. 67). These students' mathematical proficiency was cultivated though they were not occupying the same room or even able to see each other’s work clearly, yet they knew they were mathematicians and problem solvers. The situation described presents a desirable alternative: development of a multiplication understanding that will transfer to the development of strong mathematical understanding. They worked on the problem because they had a strong foundation. For third graders, operational concept expectations pivot from addition and subtraction to multiplication. Educators are left at a crossroads: Our students have developed fluency with addition. They understand that mathematics is a language with conventions, concepts and skills. Is this foundation enough to introduce multiplication as a procedural extension of addition?

The short answer is no. This quick transition is like thinking a bowl of raw, uncooked, unmixed, unprepared ingredients is akin to the steamy bowl of chili at the dinner table – as the hard work of seasoning, dicing, preparing, and cooking take time, there is need to develop the conceptual understanding of multiplication more fully. Teachers must engage students in the essential work of building a framework of understanding around multiplication through investing time in concept development and concept understanding. Specifically, “effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems” (NCTM, 2014, p. 10).

The learning trajectory is kaleidoscopic, not linear. It is not that the focus of instruction will shift from conceptual understanding to procedural fluency over time. Instead, as students have the conceptual understanding to develop procedural fluency with more basic tasks, they are then able to engage more complex conceptual tasks using the benefit of procedural fluency. Even as students develop automaticity with standard algorithms, for example, teachers can attend to extending meaning making opportunities to apply the algorithm and other concepts increasingly complex problems. Ms. C implemented a contextual, authentic problem rather than providing drill and memorization sheets.

Looking Back

The virtual environment posed a variety of challenges. Ms. C adapted by sharing clues through Google Classroom, ensuring that students only received their assigned clue. She also used breakout rooms to allow students to interact within their groups; however, students noted that not being able to see one another's work throughout the process was a disadvantage. Reflecting on this challenge, the use of a common workspace, such as the whiteboard tool or Jamboard, would provide a collaborative space for students to synchronously share their thinking as they progress through the problem. In future implementations of this task, Ms. C plans to provide the common workspace students are seeking and to monitor their work using browser tabs connected to their shared workspaces. This strategy will help monitoring, selecting, and sequencing, thus producing a more robust and purposeful discussion.

Looking Ahead

Purposeful teaching practices can empower the development of students’ capacity with multiplication and the larger whole of mathematics as evidenced by this lesson from Ms. C’s third grade class. Her students demonstrate an understanding of multiplication indicative of rich representations. This process started earlier in the year when students engaged a multiday inquiry using tools such as tiles, counters, and paper/pencil diagrams and drawing to explore real world problems. These problems were designed to help students see the operation of multiplication as combining equal groups and to extend this notion into an array model. Problems that students engaged ranged from the straightforward to the complex and offered connection and relevance to different kinds of models. For example, “The students in Ms. West’s class are seated in 4 rows. There are 6 students in each row. How many students are in Ms. West’s class?” relates well to an array model; whereas other problems helped students conceptualize multiplication over time, “Jerry walks 3 miles each day. How many total miles will he have walked after seven days?” More complex problems required students to perform multiple steps. For instance, “The Rodriguez family is attending a matinee movie. Tickets are $5 each during matinees. If Mom, Dad, and the 4 kids all go, how much will it cost?” Eventually, this work led to contexts which allowed students develop flexibility by using more than one strategy and to connect multiplication and division through the different models that they had explored. Understanding possessed by these students is not afforded to students whose procedural development was obtained through rote memorization.

Mrs. C. engaged her students in a process of conceptual development that grew across the year. Students first learned about multiplication conceptually, once this was firmly in place, experiences to develop automaticity with basic facts were employed. Ultimately, these skills were used as a tool in solving problems which called for a more sophisticated application of multiplication concepts. To design instruction focused on conceptual understanding, questions posed by Smith, Steele, and Raith (2017) are an empowering reflection and planning tool. These questions provide a starting point for reflection and planning (Table 1).

Table 1: Questions adapted from Smith, Steele, and Raith (2017; p. 76)

Topics and Themes related to Planning Instruction for Conceptual Understanding and Procedural Fluency	**Questions to ask when planning for Conceptual Understanding and Procedural Fluency in Multiplication**
Conceptual Understanding	• What are the key conceptual understandings students should develop for multiplication? • What are the opportunities that have been offered to build understanding? • How have students had the opportunity to build understanding over time?
Meaningful Tasks	• What task might I use to build different facets of understanding? • How might I follow-up on conceptual understanding task with opportunities to develop procedural fluency? • When students are asked to engage in tasks promoting procedural fluency, how have conceptual understanding contributed to their background knowledge and experiences?
Reflecting on Practice	• Considering that the development of conceptual understanding is an investment, how might I allocate instructional time to make an investment in student understanding? • How might tasks with procedural fluency be incorporated as a finishing touch to the transformative work of building conceptual understanding?

In the end, a hearty bowl of chili is the synthesis of ingredients, preparation, and time. In the mathematics classroom, supporting students to use strategy, tools, and dialogue in the service of mathematical development yields understanding that is substantive and builds capacity for their further mathematical growth. The engagement with the task (sharing in the group, working independently, sharing again) the resilience to talk through it with peers and find ways of demonstrating thinking (drawing on paper and showing the image, asking questions, clarifying processes), the capacity to determine needed information and find answers (how many ounces and cups in a pound), and the ability to apply different strategies (skip counting, creating a diagram) speaks to the power of investing in conceptual understanding as a sustaining ingredient to the long term development of mathematical thinking.

References

National Council of Teachers of Mathematics. (2014). *Principles to actions*. National Council of Teachers of Mathematics.

National Research Council. (2001). *Adding it up*. The National Academies Press. https://doi.org/10.17226/9822.

Smith, M., Steele, M., & Raith, M. L. (2017). *Taking action: Implementing effective mathematics teaching practices 6-8.* National Council of Teachers of Mathematics.

Van de Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally, 6th ed.* Pearson.

NCTM Annual Conferences

NCTM is hosting its spring national meeting virtually in April. Featuring 600+ education sessions, the NCTM 2021 Virtual Annual Meeting will provide the full range of program content, learning opportunities, and collaboration typical of major NCTM events. The event will be held from Wednesday-Saturday across a two-week period - April 21-24 and April 28-May 1.

NCTM is also planning to be back in person for its Annual Meeting and Exposition in Atlanta, September 22 – 25, 2021. The Strands include Broadening the Purposes of Learning and Teaching Mathematics; Advocacy to Make an Impact in Mathematics Education; Equitable Mathematics Through Agency, Identity, and Access; Building and Fostering a Sense of Belonging in the Mathematics Community; and Effective Mathematics Teaching Practices. For more information: https://www.nctm.org/annual/

Problems2Ponder

Holly Hirst, Appalachian State University, Boone, NC

In each issue of **The Centroid**, *Problems2Ponder* presents problems similar to those students might encounter during elementary and middle school Olympiad contests. Student solution submissions are welcome as are problem submissions from teachers. Please consider submitting a problem or a solution. Enjoy!

Problem submissions: If you have an idea for a problem to publish, please email Holly Hirst [hirsthp@appstate.edu] a clear photo or PDF document of a typed or neatly written problem statement, along with a solution. Include your name and school affiliation so that we can credit you with the submission.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst [hirsthp@appstate.edu] a clear image or PDF document of the correct solution, with the name of the school, the grade level of the student(s), the name of the student(s) if permission is given to publish the students' names, and the name of the teacher.

Deadline for publication of problems or solutions in the Fall 2021 Centroid: June 30, 2021.

Spring 2021 P2P Problems

Problem A: Let $\boxed{X}$ represent the sum of whole numbers less than *x* that are not factors of *x*. For example, $\boxed{6}$ = 9, because 4 and 5 are the only whole numbers less than 6 that are not factors of 6, and 4 + 5 = 9. What is the value of $\boxed{12} - \boxed{10}$?

Problem B: One hundred students were asked their opinion on three ice cream flavors. Sixty-five said they liked rocky road, 75 said they liked chocolate, and 85 said they liked butter pecan. What is the smallest number of students who could have said they liked all three of these flavors?

Fall 2020 P2P Problems and Solutions.

Problem A: A train is exactly 12 miles from Greensboro at 7:00 PM. It is traveling at a constant speed of 45 mph. At what time will the train reach the terminal in Greensboro?

Solution: Let's employ the notion that $distance = rate \times time$. We are given the distance (12 miles) and the rate (45 miles per hour). Keeping track of units (always a good idea!):

$$12\,miles = 45\,mph \times time = \frac{45\,miles}{1\,hour} \times time$$

Solving for time:

$$time = \frac{12\,\cancel{miles}}{45\,\frac{\cancel{miles}}{hour}} = \frac{12}{45}hour = \frac{4}{15}\,hour$$

What if we want to know the time in minutes? There are 60 minutes in an hour, so:

$$\frac{4}{15}\,hour = \frac{4}{15}\,\cancel{hour} \times \frac{60\,minutes}{1\,\cancel{hour}} = 16\,minutes$$

Problem B: The Math Olympiads began in the prime year 1979. Find the product of the fractions below in simplest form.

$$\left(1-\frac{1}{1980}\right)\left(1-\frac{1}{1981}\right)\left(1-\frac{1}{1982}\right)\cdots\left(1-\frac{1}{2004}\right)\left(1-\frac{1}{2005}\right)$$

Solution: Let's look at each of the parenthesized expressions and complete the subtraction! The first one gives:

$$\left(1-\frac{1}{1980}\right)=\frac{1980}{1980}-\frac{1}{1980}=\frac{1979}{1980}$$

Completing the same process for each parenthesized factor gives the following pattern of factors.

$$\frac{1979}{1980}\times\frac{1980}{1981}\times\frac{1981}{1982}\times\frac{1982}{1983}\times\ldots\times\frac{2002}{2003}\times\frac{2003}{2004}\times\frac{2004}{2005}$$

Note that the denominator of one is the same as the numerator of the next, so we could rearrange the order of the multiplications to cancel like factors.

$$\frac{1979\times\cancel{1980}\times\cancel{1981}\times\cancel{1982}\times\ldots\times 2002\times\cancel{2003}\times\cancel{2004}}{\cancel{1980}\times\cancel{1981}\times\cancel{1982}\times 1983\times\ldots\times\cancel{2003}\times\cancel{2004}\times 2005}$$

Notice that the 1983 and 2002 factors would also cancel out using fractions not shown explicitly, leaving the final very simple answer:

$$\frac{1979}{2005}$$

Trust Fund Scholarships

Scholarships of $1000 are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants & scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website. More information can be obtained from Janice Richardson Plumblee [richards@elon.edu].

Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher? Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

NCCTM Board

contact information can be found at ncctm.org

Officers

	State	Eastern Region	Central Region	Western Region
President	Kathy Jaqua	Ginger Rhodes	Denise Johnson	Sheila Brookshire
President Elect	Stefanie Buckner	Kaneka Turner	Sara Vaughn	Kwaku Adu-Gyamfi
Elementary Vice President	Ana Floyd	Karen Walker	Michelle Harger	Meredith Stanley
Middle Grades Vice President	Kim McCuiston	Allison Landry	Tanya Ford	Sumer Williams
Secondary Vice President	Lauren Baucom	Bill Worley	Wendy Bartlett	Emily Bryant
College Vice President	Charity Cayton	Maureen Grady	Adam Pennell	Julia Webster
Other State Officers	Secretary Emily Elrod	Parliamentarian Wendy Rich		

Committee Chairs

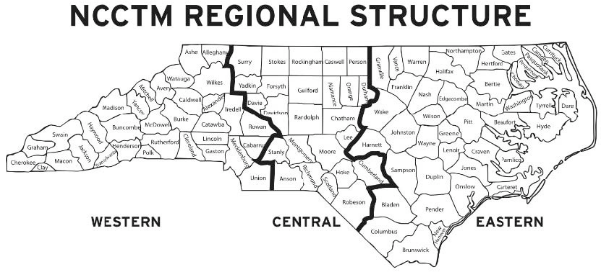
Conference and Exhibit Services, Kay Swofford
Centroid, Holly Hirst and Debbie Crocker
Convention Services, Marilyn Preddy
Financial Chair, Ray Jernigan
Handbook, Ray Jernigan
Historian, Kathryn Hill
Inclusion and Diversity, Denise Johnson
Leadership Conference, Kathy Jaqua
Management Services, Joette Midgett
Math Celebrations, Anthony Finlen and Pamela Parham
Math Contest, James Beuerle and Phillip Rash
Math Counts, Harold Reiter
Math Fair, Betty Long
Mini-grants, Joy McCormick
NCDPI Representative, Lisa Ashe
NCSSM Representative, Ryan Pietropaolo
NCTM Representative, Debbie Crocker
NCSM Representative, Debbie Crocker
NCMATYC Representative, Luke Walsh
Nominations, Julie Kolb
Rankin Award, Lee Stiff
Special Awards, Ana Floyd
Student Affiliates, Holt Wilson
Trust Fund, Catharina Middleton
Webpages, Holly Hirst

Becoming a Member

Follow the “Membership Information” link on the ncctm.org website, or go directly to:
http://www.ncctm.org/members/register.cfm

NORTH CAROLINA COUNCIL OF TEACHERS OF MATHEMATICS

NCCTM REGIONAL STRUCTURE



North Carolina Council of
Teachers of Mathematics
PO Box 33313
Raleigh, NC 27636