The Centroid

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The Centroid is the official journal of the North Carolina Council of Teachers of Mathematics (NCCTM). Its aim is to provide information and ideas for teachers of mathematics—pre-kindergarten through college levels. *The Centroid* is published each year with issues in Fall and Spring.

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Submission of News and Announcements
We invite the submission of news and announcements of interest to school mathematics teachers or mathematics teacher educators. For inclusion in the Fall issue, submit by August 1. For inclusion in the Spring issue, submit by January 1.

Submission of Manuscripts
We invite submission of articles useful to school mathematics teachers or mathematics teacher educators. In particular, K-12 teachers are encouraged to submit articles describing teaching mathematical content in innovative ways. Articles may be submitted at any time; date of publication will depend on the length of time needed for peer review.

General articles and teacher activities are welcome, as are the following special categories of articles:

- *A Teacher's Story*,
- *History Corner*,
- *Teaching with Technology,*
- *It's Elementary!*
- *Math in the Middle*, and
- *Algebra for Everyone*.

<u>Guidelines for Authors</u>
Articles that have not been published before and are not under review elsewhere may be submitted at any time to Dr. Holly Hirst, hirsthp@appstate.edu. Persons who do not have access to email for submission should contact Dr. Hirst for further instructions at the Department of Mathematics at Appalachian State, 828-262-2869.

Submit one electronic copy via e-mail attachment in *Microsoft Word* or rich text file format. To allow for blind review, the author's name and contact information should appear *only* on a separate title page.

Formatting Requirements

- Manuscripts should be double-spaced with one-inch margins and should not exceed 10 pages.
- Tables, figures, and other pictures should be included in the document in line with the text (not as floating objects).
- Photos are acceptable and should be minimum 300 dpi tiff, png, or jpg files emailed to the editor. Proof of the photographer's permission is required. For photos of students, parent or guardian permission is required.
- Manuscripts should follow APA style guidelines from the most recent edition of the *Publication Manual of the American Psychological Association*.
- All sources should be cited and references should be listed in alphabetical order in a section entitled "References" at the end of the article following APA style. Examples:

  Books and reports:
  Bruner, J. S. (1977). *The process of education* (2nd ed.). Harvard University Press.
  National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.*
  Journal articles:
  Perry, B. K. (2000). Patterns for giving change and using mental mathematics. *Teaching Children Mathematics*, *7*, 196–199.
  Chapters or sections of books:
  Ron, P. (1998). My family taught me this way. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics: 1998 yearbook* (pp. 115–119). National Council of Teachers of Mathematics.
  Websites:
  North Carolina Department of Public Instruction. (1999). *North Carolina standard course of study: Mathematics, grade 3.* http://www.ncpublicschools.org/curriculum/mathematics/grade_3.html

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About the Cover
The image on the cover depicts the centroid of a triangle.

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A New Look!

Check out the new look for the NCCTM logos, website, social media, and the Centroid. We hope you enjoy it!

Fall 2023 State Math Conference And Leadership Seminar

Seminar: November 8, 2023
Conference: November 9-10, 2023

Benton Convention Center
Winston-Salem

Submit Speaker Proposals coming soon at https://ncctm.org

President's Message

Stefanie Buckner Hill
State President
stefanie.hill@bcsemail.org

If I had to guess, and maybe it would be better if I had some background music, most of us in an academic role could complete the phrase "It's a beautiful day in the neighborhood, a …."

(beautiful day in the neighborhood. Would you be mine? Could you be mine….Won't you be my neighbor.")

There's lots of reasons why and perhaps some of those reasons surface some fun childhood memories—for me it's snow days! Regardless of the reasons I can complete the theme song, I also have a strong memory of the community Mr. Rogers built. From daily mail delivery from Mr. McFeely, a helping hand from Handyman Negri and a daily trip to see King Friday and the original Daniel Tiger, Mr. Rogers had a community of friends that he relied on.

My "why" for running for NCCTM President three years ago was centered on community. In fact, in my bio from 2019 (thank you google-email search features) I wrote:

Stefanie sees first hand, from her work with NC^{2}ML and the Annual Conference, the power of collective effervescence. Recent trends in powerful and meaningful collaboration in North Carolina have led to novel levels of collective efficacy (NC^{2}ML and NCTools4Teachers) directly impacting the mathematical power of all students across our state. Stefanie's vision for NCCTM is to capitalize and connect with recent statewide collaborations to drive mathematics teaching and learning to new and exciting areas that we cannot achieve in isolation. To borrow from Dr. Valerie Faulkner's concept – Stefanie wants to explore how 1+1 can be 4 instead of 2. (Disclaimer: Stefanie does know basic facts and if this statement of 1+1 is 4 is bothering you then make sure you check out Dr. Faulkner's sessions at NCCTM on Friday!)

If you were at Leadership this fall you may recall the term collective effervescence and collective efficacy—this time with an Encanto reference, as I closed out our Leadership conference. Mr. Rogers or Encanto, you can pick but both have a theme of "we need each other." To go further—we all have our unique talents, we have our gifts and by coming together as a community we can bubble up ideas greater than we could alone (collective effervescence). Quite simply, we're better together and I truly believe this (collective efficacy).

This spring wraps up my tenure as NCCTM President, which made me reflect on how this even happened. Those that came before me built an organization centered on regional and statewide communities for which I am grateful. I feel so fortunate to have stepped into this community even before I started teaching! Dr. Crocker saw to it that her math ed students were well aware of NCCTM—and even would bring us back NCTM and NCCTM swag. My first tenure on the NCCTM Board of Directors was on Dr. Crocker's board. And then there's Randy! If you don't know, I oftentimes describe my current position as "Randy's job" and many know exactly where I work and what I do. (Although I have been in it for 10 years now, it will always be "Randy's job.") Randy was instrumental in developing a strong community of math teachers in Buncombe County and North Carolina and welcomed me into this community. Dr. Crocker and Randy Harter are two standouts for me as I think about how collective efficacy made me better; they shared this community and made an impact on my professional trajectory. But what really matters is the larger community that they introduced me to and brought me into that matters. It's all of NCCTM! It's our collective effervescence benign together and our collective efficacy in a shared responsibility for powerful mathematics teaching and learning.

Specific to my tenure, reading my vision for NCCTM made me smile! I am happy to report that NC^{2}ML (North Carolina Collaborative for Mathematics Learning) and NCMTC (North Carolina Math Teachers Circles) are now official, board approved, "friends" of NCCTM. Thank you to the 2021-2023 NCCTM Board of Directors for formalizing these partnerships. This allows all of us to move forward with intentional collaborations and support of each other. It broadens and bolsters our communities. Thank you to all of my Board of Directors for helping me in achieving my vision of NCCTM!

Oftentimes, you don't know what you don't know. I had no idea how much collective effervescence and collective efficacy our annual state conference contained until we didn't have it. But wow was it fun, rejuvenating, exciting and engaging to be back together again—for real—in person this fall. What a wonderful opportunity to come together as a community to celebrate mathematics, to share and extend our pedagogical knowledge, to learn from each other but above all to just be together. Yes, we figured out how to communicate and connect in the interim but it was a highlight of my time with NCCTM and my year to be with my statewide math community in November. If you were there, I hope you felt the same. If you weren't there we hope you can join us in 2023. In fact, mark your calendars - NCCTM 2023 Fall Conference is November 9 and 10 in Winston Salem, NC.

And so I sign off as president by saying, could you be, would you be my neighbor. We have a lot of work to do, I'm not going anywhere, but I hope you too, if you are reading this, will continue to think of the joy that this community, the community of NCCTM, brings to you and your work. It has been a pleasure to serve as President and to be able to add my name to a list of distinguished mathematicians from North Carolina. Thank You!

Applying for NCCTM Mini-grants

NCCTM provides funding for North Carolina teachers as they develop activities to enhance mathematics education. This program will provide funds for special projects and research that enhances the teaching, learning, and enjoyment of mathematics. There is no preconceived criterion for projects except that students should receive an on-going benefit from the grant. In recent years, grants averaged just less than $800. The application is available on the NCCTM website [ncctm.org]. Proposals must be postmarked or emailed by September 15, and proposals selected for funding will receive funds in early November. Be sure that your NCCTM membership is current and active for the upcoming year! Each year we have applications that cannot be considered because of the membership requirement. Email Joy McCormick [jmccormick@rock.k12.nc.us] with questions.

Rankin Award Nominations

The Rankin Award is designed to recognize and honor individuals for their outstanding contributions to NCCTM and to mathematics education in North Carolina. Presented in the fall at the State Mathematics Conference, the award, named in memory of W. W. Rankin, Professor of Mathematics at Duke University, is the highest honor NCCTM can bestow upon an individual.

The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from Emogene Kernodle, nekernodle@yahoo.com.

Innovator Award Nominations

The North Carolina Council of Teachers of Mathematics accepts nominations for the Innovator Award at any time. The Committee encourages the nomination of organizations as well as individuals. Any NCCTM member may submit nominations. The nomination form can be obtained from the "awards" area of the NCCTM Website, www.ncctm.org. More information can be obtained from: Dr Ana Floyd, afloyd@randolph.k12.nc.us.

Reflections on Co-teaching with Interns in High School

Mara Moye, Renea Baker, D.H. Conley High School, Greenville NC
Anita Koen, South Central High School, Winterville NC
Charity Cayton, Maureen Grady, East Carolina U, Greenville NC

The authors share reflections of using co-teaching strategies with interns during their culminating clinical experience, sometimes referred to as student teaching. Six co-teaching strategies and specific examples of their use in face-to-face instruction are described.

The use of co-teaching in a variety of settings has increased rapidly in recent years. The purpose of this article is to share the co-teaching strategies as they have been incorporated in one secondary mathematics education program since 2014. The perspectives shared incorporate the experiences of three clinical teachers and two university faculty who serve as university supervisors. The clinical teachers have more than 70 years of combined teaching experience, have hosted a total of 14 interns using a co-teaching model, and have hosted more than a dozen interns prior to the use of co-teaching. The faculty/university supervisors have supervised more than 30 interns in co-teaching settings, and additional interns in non-co-teaching settings prior to 2014. These experiences provide a unique perspective to compare and contrast the former, traditional model and co-teaching.

We have two main priorities when hosting or supervising an intern: 1) ensure that students receive the best instruction and 2) ensure that interns are prepared to teach in their own classroom upon completion of the program. When we implemented co-teaching in 2014, the first reaction by clinical teachers was that it was one more thing coming from the university that would not work in the high school math classroom. The major concern was that using a co-teaching model would not provide interns with sufficient independence during the internship to prepare them to teach on their own; however, additional information and conversations about co-teaching with clinical teachers revealed that the teachers had been using many of the strategies, but they did not necessarily have a name for them or a way to talk about what they were doing. Contrary to our initial concerns, we have also found that the use of these strategies provides increased teaching opportunities and independence while providing real-time support, benefitting both interns and high school students. As co-teaching became a norm in our clinical experiences, including internships, a shift from a sink-or-swim internship model, towards a model that supported intern growth without compromising learning opportunities for our high school students, emerged.

Background

Co-teaching has been used successfully in inclusion classrooms with subject-area teachers and special educators for a number of years (Friend et al. 2010; Scruggs et al., 2007). In reviewing the literature on co-teaching in inclusion settings, we found consensus that co-teaching provided students with models of collaboration, led to increased teacher attention for students, and improved student achievement. Dieker et al. (2012) and Hunt (2010) both provide examples of the successful implementation of co-teaching in middle grades mathematics inclusion classrooms. Given its success in inclusion settings, co-teaching began to be used in internship settings, again, with generally positive results. Interns placed in co-teaching settings developed better classroom management skills (Bacharach, 2007; Heck et al., 2008), a deeper understanding of the curriculum (Bacharach, 2007; Heck et al., 2010), and enhanced collaboration skills (Bacharach, 2007; Darragh et al., 2011; Heck et al., 2010). Interns in these settings also spent more time involved in classroom instruction than interns in non-co-teaching settings (Bacharach, 2007).

When we began to explore co-teaching in internship settings at our university, we found that most of the literature, examples, and resources focused on elementary education or the use of co-teaching in inclusion settings. Our secondary mathematics clinical teachers had difficulty envisioning how it might be used in their classrooms and with their interns. In this article, we will provide a brief introduction to the co-teaching strategies and share stories and experiences of implementing these strategies with interns.

Context and Co-teaching Strategies

Our final field placement is a year-long internship experience. During the fall semester (Senior 1) interns are in their placements one day per week. During the spring semester (Senior 2) the interns are in the same placement, with the same clinical teacher, full time. The Senior 1 field experience is anchored by a methods course taught by one of the faculty authors. During Senior 2, each intern is assigned a supervisor, most often one of the faculty authors.

During the year-long internship experience, we rely on six specific co-teaching strategies (Table 1), adapted from Murawski et al. (2011) and Bacharach et al. (2010). Table 1 contains a description for each of the co-teaching strategies we use and some benefits and concerns that have emerged throughout our use of these strategies. Table 1 provides an overview, but the usefulness and successful implementation of the strategies is best captured through the lens of an actual classroom. The next section includes examples of and reflections on how we have used each co-teaching strategy during the internship experience.

Table 1. Co-teaching Strategies (adapted from Grady et al., 2016)

Strategy	Description	Benefits	Concerns
One Teach, One Observe	One teacher leads instruction; the other teacher gathers specific information.	Extra set of eyes; provides data about instruction or student learning; easy to implement.	Easy to become a habit; must agree in advance what is to be observed.
One Teach, One Assist	One teacher works with the whole class; the other assists individual students or small groups.	Provides assistance to individual students; easy to implement; may provide a "voice" to share student concerns.	Easy to become a habit and for one teacher to always feel like an assistant; changing roles is essential.
Station Teaching	Teachers create three or more "stations", each with a clearly defined learning goal. Small groups circulate through the stations with teachers facilitating learning at individual stations or monitoring circulation.	Smaller groups for better instruction, assessment, and class management; allows for differentiation, movement, and hands-on activity.	Need to use space differently; class management and transition needs to be structured; requires careful planning.
Parallel Teaching	Each teacher takes about half of the students in the class and instructs their small group.	Smaller groups for better instruction, assessment, and classroom management; interns teaching the same lesson can mirror teacher.	Need to use space differently; both teachers need to plan for their group; volume control is important.
Alternative Teaching	One teacher works with most of the students; the other works with an individual or small group that may need remediation, enrichment, or alternative instruction.	Good for smaller and more specific group work; good for addressing IEP/504 goals; teachers can plan separately.	DO NOT always pull the same kids; need a place for group to meet; watch noise levels; must plan how to re-integrate group.
Team Teaching	Two teachers fluidly work together to facilitate the lesson.	Demonstrates parity and collaboration between teachers; good for modeling; fun for role-playing.	Requires planning and good relationship.

Examples and Reflections

One Teach, One Observe: In this strategy, the observer is asked to attend to a specific aspect of instruction, rather than general observation of everything going on in the classroom. Providing a focus for an observation helps the observer move beyond the overall classroom results to focus on the components that help create the result. The teacher candidate as an observer is utilized extensively in our methods courses with clinical experiences prior to the internship experience. For example, during a junior methods course, they are asked to conduct an observation focused on student understanding. In this observation, they document evidence of how two students are processing and coming to understand a new mathematical concept being taught by the clinical teacher. Focusing on observable evidence of student thinking provides the pre-service teacher with an opportunity to reflect on what individual students understand and to brainstorm potential instructional decisions to build on this understanding. During the internship, a common use of this strategy is to have interns observe the questions that the clinical teacher poses to promote mathematical reasoning and support productive struggle. The use of this strategy prior to the year-long internship experience allows clinical teachers and interns to move quickly to the *one teach, one assist* strategy during the Senior 1 internship semester. Hence, during the internship, it is most often the clinical teacher, not the intern, observing with a specific focus to reflect on instruction and support intern growth.

***One Teach, One Assist**:* This is one of the most important strategies that we use from the very beginning of the internship. This is a good strategy to get the intern started with classroom involvement and building rapport with students. *One teach, one assist* gives the intern the opportunity to begin the Senior 2 experience in a role that will show the students that they are a co-teacher in the classroom. One clinical teacher reported:

> *On day 1 of the internship, I was the lead teacher for a lesson on Representations of Quadratic Functions. As I launched the lesson, the intern was helping our students as they were trying to discover ways to go between the different representations. The intern was facilitating the discussion among groups of students so that they could discover how to change a quadratic function into any type of representation (algebraic, graphical, tabular, or verbal). This does require that you have gone through the lesson well with your intern before they get to your classes.*

As the Senior 2 experience progresses, the clinical teacher becomes the assistant while the intern is teaching, creating a full circle at the end of the internship experience. This results in an environment in which students view the intern as a co-teacher, capable and equally responsible, for classroom instruction.

***Station Teaching**:* Station Teaching requires co-planning and lots of discussion between the co-teachers but gives the intern a chance to shine. Interns are often knowledgeable about new technology and creative activities that benefit our students. As the clinical teacher and intern plan, the intern has the opportunity to bring their ideas to the table. Every year clinical teachers report learning something new from their interns. During implementation of the lesson, interns have the opportunity to see and experience facilitation in action. One clinical teacher reported:

> *In the classroom we did a station activity to review for a test on Inverse Functions, Transformations, and Piecewise Functions. Station 1 was a technology station on Desmos called Polygraph: Transformations. Station 2 was a graphing activity where students sketched transformations on individual whiteboards. For station 3, students created their own function, using the transformations that were assigned, and graphed their transformation. Students then exchanged functions with their partner and graphed their partner's function. They compared answers and discussed discrepancies. Station 4 was matching piecewise functions to their graphs. At Station 5, students proved that two functions are inverses of each other. Each station included vocabulary and discussion between partners. We each facilitated specific groups. The intern facilitated Station 1, the technology station, since this was one of their strengths. They also facilitated Station 2, the graphing activity. I, as the clinical teacher, facilitated Station 4, the matching activity and Station 5, the proof station. We both facilitated Station 3. While facilitating we simply moved from one station to the next, answering clarifying questions and spurring on the discussion. Facilitating different stations allowed us to touch base, in real time, as we were noticing common student misconceptions, novel solution strategies, and ideas to re-emphasize at the end of class.*

This strategy supports interns in learning how to facilitate collaboration in the classroom through student-led learning by observing and incorporating instructional strategies used by the clinical teacher.

Parallel Teaching: Parallel teaching is a strategy that typically splits the class into two, equally-sized groups within the same space, where the clinical teacher and intern are teaching the same lesson to each group. This decreases the teacher to student ratio and allows for the clinical teacher and intern to be able to listen to and draw from each other without the students even realizing it. This can help the intern with time management of the lesson and allows them to hear the clinical teacher's instruction and mirror this, if needed, for their own group. Parallel teaching in the same room may reduce discipline issues as both the clinical teacher and intern are present. In a small group setting, students may be more confident and more likely to be engaged; they don't get lost in the whole class setting. Parallel Teaching is not limited to only two groups. One of our clinical teachers had the unique experience of co-teaching with three pre-service teachers for a lesson:

> *We all met together to co-plan a lesson on properties of quadrilaterals, and then split the class into four groups based on each student's mastery of the material. This allowed us to focus on individual student learning needs in the small groups and draw on the strengths of each co-teacher as to which group they would work best with. With the four of us sharing a space, we were able to quickly consult on any questions that might have come up from students, helping the pre-service teachers check in with me without anyone's authority or ability being questioned. For example, after one of the pre-service teachers had responded to a student's question, "is a square always a rectangle or is a rectangle always a square," the student still seemed confused. The pre-service teacher asked me if there was a different way to help the student; I was able to pause all groups to discuss the student's question and clarify it for the whole class. We all then resumed leading our individual groups.*

There can be some drawbacks to *parallel teaching*. Sometimes classes are large and classrooms small, leaving little room between the groups and, therefore, noise can be an issue. It takes good management by the co-teachers to ensure the students are engaged with their group and not other groups. Despite these potential challenges, *parallel teaching* provides individualized attention for students and support for intern development.

Alternative Teaching: This is a strategy that is not often used for an entire class period. It can be a very helpful strategy when a student or small group of students has missed or not fully mastered previous lessons, and it allows for either the clinical teacher or intern, whichever one is not leading the instruction for that day, to work with students to help them catch up. It is critical to plan carefully so students do not fall further behind, missing new instruction while catching up. Good time management is crucial for this strategy to be effective. One clinical teacher reported:

> *I had a Math II student that was absent for over a week because of a medical procedure. This student missed the introduction to Trigonometric Ratios, Solving Triangles using Trigonometry, and Angle of Elevation and Depression Trigonometry problems. When the student returned to school, it was during a time of review of these concepts for the class; therefore, the intern worked with this student for a couple of days to teach these concepts that the student had missed.*

With this type of instruction, the student(s) can often master the material in a shorter amount of time, allowing them to fill the gaps in their learning.

Team Teaching: Team teaching is the strategy that takes the most preparation. To successfully implement team teaching, both teachers must be flexible and have respect for what each brings to the lesson. The clinical teacher and the intern need a good working relationship, which is something that takes time and effort to build. There have been situations where team teaching went seamlessly and others where it was not a good experience for either the clinical teacher or the intern. When using this strategy, it is best if the teachers rehearse the lesson a couple of times before implementation. Interns have expressed a feeling of comfort during team teaching because the students treated both intern and clinical teacher as lead teachers. When students see the clinical teacher leading the classroom with the intern as an equal, it gives the students more confidence in the intern's ability. One of the most successful experiences shared by one of our clinical teachers:

I had the opportunity for two interns to team teach in my classroom, while I served as an observer for the lesson. They co-planned a lesson about the key features of quadratic functions, then used team teaching in both of their Honors Math 2 classes. The two interns split up the key features of quadratic functions; one went through vertex, axis of symmetry, y-intercept, domain, and range, while the other went through zeros, max/min, interval of increase, and interval of decrease. Their finished lesson plan looked like a script for a play, but it was so well rehearsed that the lesson went beautifully. The experience of co-planning and co-teaching this lesson provided some needed support for one of the interns who had been struggling. It also provided students with the opportunity to engage with multiple teaching styles.

Team teaching may require time and effort on the part of the clinical teacher and intern, but successful use of this strategy allows both teachers to bring their unique voices to a lesson and to provide enhanced learning opportunities for students.

Final Thoughts

We are committed to a co-teaching model for our interns; we will not return to a traditional model. This commitment transferred well into our unique teaching structure during the COVID-19 pandemic when our interns and clinical teachers were teaching in virtual and/or hybrid teaching settings. We have been able to adapt and implement the co-teachings strategies successfully regardless of the instructional context. While not discussed in this paper, co-planning serves as a critical support for negotiating roles and effectively implementing the co-teaching strategies (in person or virtual). We rely on specific co-planning strategies (Grady et al., 2019) to facilitate co-planning.

As clinical teachers and university supervisors, we have found that co-teaching is a huge improvement over the traditional student teaching experience. Our experiences are consistent with research findings that co-teaching can be beneficial in clinical experiences (Bacharach, 2007; Darragh et al., 2011; Heck et al., 2008). Based on our experiences, the process of negotiating the roles of the clinical teacher and intern using the co-teaching strategies, and switching these roles, increases interns' confidence and supports growth throughout the internship experience. Another major benefit of co-teaching is that we have not experienced the level of parental and student pushback that we often saw when hosting interns before we began co-teaching. A parent can no longer "blame" the intern's inexperience for their student's lack of success, as all lessons are co-planned and co-taught. Our concerns about interns not being prepared for the demands of their own classroom due to co-teaching have dissipated. In talking with our former co-teaching interns, some of whom are now colleagues and serving as clinical teachers, we see an increased confidence in their beginning years when compared with interns who experienced a traditional internship. We have seen first-hand that the level of collaboration they experienced through a co-teaching internship better prepares them for collaborating with colleagues as they begin their teaching careers.

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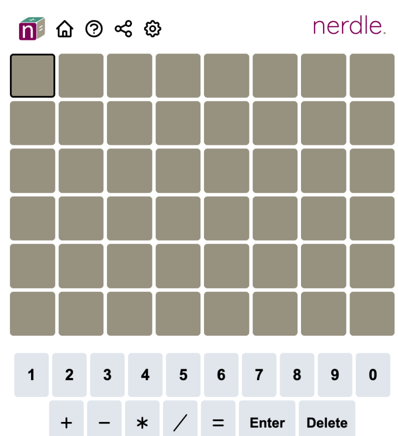
Have You Seen Nerdle?

In the last year or so several guessing games have taken the internet by storm.

Wordle (https://www.nytimes.com/games/wordle) asks users to guess a five-letter word. Once the user inputs their guess and presses enter, the game shows which letters are used in the word with a yellow highlight and which letters are used and *also* in the right position with a green highlight. Users have six chances to correctly determine the word.

Worldle (https://worldle.teuteuf.fr/) asks users to guess the country or territory given the image of its outline. If they guess incorrectly, the distance from guess to the intended country or territory is provided. As with Wordle, users have six chances to guess correctly.



There is an arithmetic oriented game that some teachers have used as a pre-class warmup called Nerdle (https://nerdlegame.com/). The rules are simple:

Each of eight spaces holds a digit, an operation, or an equal sign. The goal is to guess the calculation knowing that one of the spaces is an equal sign.

Once a guess is entered, the space will be highlighted with purple if the symbol is used in the puzzle, but in the wrong position, and green if the symbol is used and in the correct position. Here is a guess that is not completely correct, showing that 4, *, and – are in the correct positions in the calculation and that 5, 1, and = are used but need to be moved.

As with the other games, users have six chances to get the calculation correct. There are several variations of the game available. Click on the house icon at the top of the game board to see the other options.

Off the Beaten Path: Estimation in a 3rd Grade Classroom

Ashley Whitehead, Appalachian State University, Boone NC

The author describes a series of estimation activities that help students to gain comfort and accuracy with estimation questions by using a common problem: estimate the number of objects in a container.

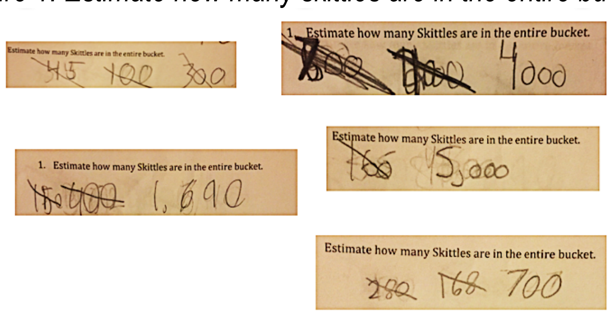
One afternoon, in the midst of a school-wide contest that required students to estimate the amount of candy in a jar, our third-grade students were completing an activity focused on measurement and data. The activity initially asked them to determine the number of Skittles in a bucket by grabbing a handful and plotting their total number of Skittles on a line plot at the front of the room. Our goal was to discuss "the typical number of Skittles a 3rd grader could grab," and make a hypothesis around "how many Skittles another 3rd grader might pull from the bucket"; however, we soon realized our third graders had no concept of the size of numbers, nor did they understand what it meant to estimate a value. Their estimations were wildly off—either too small or too large for the total number of Skittles in the bucket. This led us off the beaten path by adapting our Skittles lesson to help our students become better estimators, all while learning the magnitude of numbers larger than 1,000.

Estimation takes practice and is considered a higher-level thinking skill (Van de Walle, 2006). It allows students to make educated guesses, check the reasonableness of their answers (NCTM, 2000), and "show an understanding of the value of numbers and of the operations used" (Berry et al., 2014, p. 142). The Common Core State Standards begin discussing estimation in second and third grade in the Measurement & Data Standards when students learn to "measure and estimate lengths in standard units." Then in fourth grade, within Operations and Algebraic Thinking students learn to "assess the reasonableness of answers using mental computation and estimation strategies including rounding" (CCSSM, 2010); however, many students feel hesitant to estimate because it does not give an exact answer or they do not understand number size and magnitude just yet.

Task 1: The Skittles Task

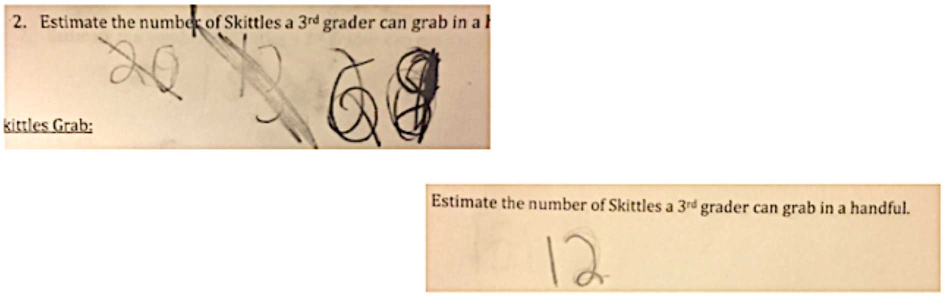
Over the course of several months, we enacted The Skittles Task using a variety of objects. As described in the introduction, we first began by showing the students a clear bucket filled with Skittles. We asked them to make an initial estimate, and as we continued the lesson we allowed them to revise their estimates if they wished (hence why some are crossed out); however, most estimates ranged from 45 to 600 Skittles in the bucket (see Figure 1).

Figure 1. Estimate how many skittles are in the entire bucket.



Next, we asked students to estimate the number of Skittles a 3rd grader could grab in a handful (see Figure 2). Once they made their estimates, students washed their hands and then pulled a handful of Skittles from the bucket. They were asked to count the number of Skittles they grabbed, write it on a sticky note, and place the sticky note on a number line drawn at the front of the room.

Figure 2. Estimate the number of Skittles a 3rd grader can grab in a handful.



Finally, we wrapped up the lesson by discussing "what is the *typical* number of Skittles a 3rd grader could grab," and turned our attention to the original estimates our students made. We noticed that some students pulled 50 Skittles from the bucket, but estimated there were 100 in the bucket to begin with. We discussed the reasonableness of this and asked students to revise their estimates. As a class, we determined that a 3rd grader could pull 42 to 58 Skittles in a handful; therefore, each table group (with 4 students) pulled about 200 Skittles. Since there were four table groups, as a class they removed about 800 Skittles.

We then asked the students to take a look back at the original bucket and notice about half of the original number of Skittles in the bucket remained. Before continuing we asked students to revise their estimate from the original question of "how many Skittles were in the bucket?" Many students offered up 1000, 2000, 3000, and even 4000 as possible amounts. However, over half of the class did not believe there were more than 1000 Skittles originally in the bucket! At this point we were stumped—how could we help students estimate if they didn't believe there were greater than 1000 Skittles in the bucket to begin with? We knew that our students had not been exposed to numbers that large before and started collecting soda tabs to show a value of 1000 in our classroom. We decided some students might have only focused on the number of Skittles removed and not considered what was left in the bucket. And finally, some students were so focused on getting the right answer that they didn't want to estimate too high.

Task 2: Estimating Golf Balls

Similar to the Skittles Task, we began this activity by asking students to estimate how many golf balls were in a bucket. We chose golf balls because we could do a similar process to the Skittles Task, without students being able to grab as many which would force them to work with smaller numbers. Their estimates ranged from 30 to 88 golf balls in the bucket.

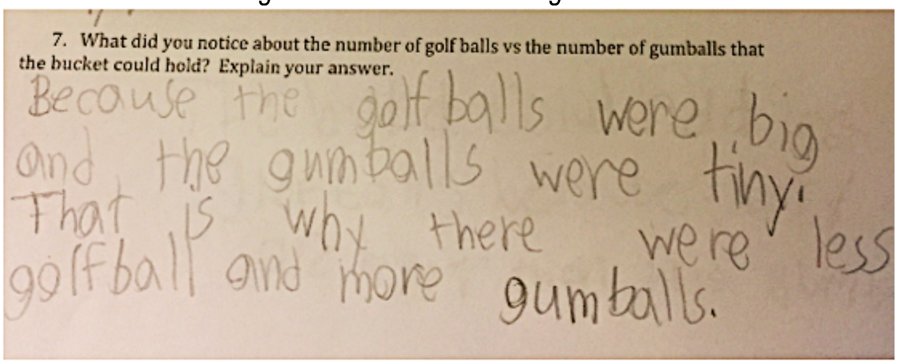
Once the students had a chance to pull the golf balls from the bucket, they determined that each person could pull 2-4 golf balls in a single handful, with a total of 48 golf balls removed from the bucket as a class. Similar to the Skittles problem, we had roughly half of the bucket remaining. We then asked students to revise their original estimates. Many students changed their responses to 60-70 golf balls in the bucket initially since they knew 48 had been removed, so it must be more than that. This helped us to see that the students' reasoning around the quantity in the bucket were getting closer to sound estimates.

Task 3: Gumball Estimation

Two months after the Skittles task we tried a similar estimation task with our students, this time with gumballs. We chose gumballs because they were slightly smaller than the golf balls, but larger than Skittles. This would force students to work with slightly larger numbers than they did with the golf-ball activity, but not as large as the numbers were with the Skittles. We began, like before, by asking them to estimate how many gumballs were in the bucket. Many of the students knew there should be more gumballs than the amount of golf balls we had, and they knew it should be similar or less than the amount of Skittles we had. However, many forgot the original Skittles activity and estimated the amount of gumballs to be over 3000.

Once each student took their turn pulling a handful of gumballs, the students noticed that the typical amount that a 3rd grader could grab was about 20 gumballs, and the sum of all gumballs in the class was 362. This time, however, the amount of gumballs pulled from the bucket was only about ¼ of the original amount. This required the students to reason about the original amount a bit differently, and discuss that if 362 gumballs were approximately ¼ of the bucket, then 362 x 4 = 1,448 would be a good estimate for all the gumballs in the bucket. Once students finished their revised estimations, we discussed what they noticed about the number of golf balls versus the number of gumballs that the bucket could hold (see Figure 3). Several students talked about how they could fit gumballs inside of a golf ball, so that meant we should have more gumballs in the bucket because of their size.

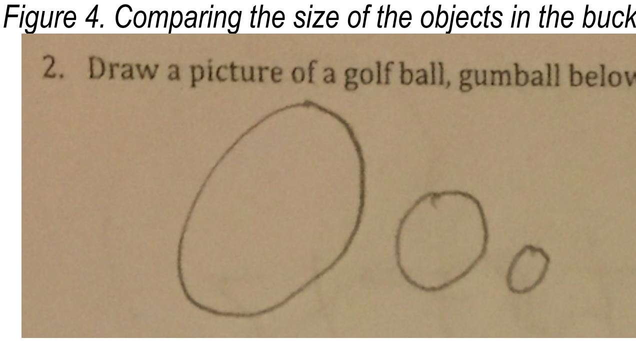
Figure 3. Golf balls versus gumballs.



Task 4: M&M Estimation

The final task, which we did almost three months after the initial Skittles task, was with a similar sized candy: M&Ms. We wanted to see if students could use their estimation skills along with their developing knowledge of numbers over 1000 to determine the amount of M&Ms in the bucket. We began as before by asking students to estimate the total number of M&Ms in the bucket. Some students were still over or under estimating, but several knew there were more than 1400 M&Ms, since that was their estimate for the gumballs in the bucket, and M&Ms are smaller than gumballs (see Figure 4).

Figure 4. Comparing the size of the objects in the bucket.



After each student pulled a handful of candies they counted the total at their tables, and then calculated the total for the class. Altogether they had 1,014 M&Ms which was approximately half of the original amount in the bucket. Next, we asked students to revise their original estimation, this time with less help from us in guiding their thinking. Several students realized that if half of the bucket was removed, and that was equal to approximately 1000 M&Ms, then there should be around 2000 M&Ms total in the bucket to start.

Once the students completed the final task, we asked them to reflect on whether they had become a better estimator after working through the estimation activities over the past few months. Every student felt they had improved because it helped them learn to round, or they learned how to get closer to an actual approximation rather than choosing a number that was too large or small.

Reflection & Tips for Teaching Estimation

Although we did not originally intend to launch into a series of estimation tasks with our students, we found they related quite well to the *Standards for Mathematical Practice*. Through the use of real-world contexts (e.g., I had a bucket of golf balls and needed to know approximately how many I had to play with at the driving range) students *persevered* in solving problems. They chose appropriate *tools* such as a hundred's chart or base 10 blocks to help them add quantities of objects, and they *modeled* the bucket by drawing an area model to represent the objects removed and the objects remaining. While *making sense* of the problems, students *reasoned quantitatively* by using fractions, such as halves, thirds, and fourths, to represent how many objects had been taken out of the bucket. However, we were careful to *attend to precision* and not over or under estimate the original amount of objects in the bucket by helping students understand the value of numbers, especially those that were new to them (e.g., those greater than 1000). Through the series of tasks, students were able to *make sense of structure* by recognizing patterns in how we were estimating (e.g., pulling a handful, calculating the total amount we pulled from the bucket, deciding how many objects remained in the bucket…), and used *repeated reasoning* to understand how each object related to the next due to its size. For example, as the object size decreased, the amount of objects in the bucket increased. Finally, students used the tasks as a way to work collaboratively and *critique the reasoning* of their peers by discussing their estimates and whether they were (or were not) reasonable.

Overall, we learned many things as we engaged our students in learning about estimation. We discovered that students did not like to be wrong with their estimations. In order to help students with this "right or wrong" mentality we allowed them to revise their original estimates based upon new information (e.g., how many Skittles were pulled by a group or the entire class). We allowed students to collaborate with their small groups and also discussed the various strategies as a class that were used to determine their estimate. This helped students who might have over or under estimated to see a more reasonable estimation. In order to help students learn magnitude, we provided students the opportunity to see 1000 objects by asking them to bring in soda tabs and collecting them into bags of 10 which we then grouped into bags of 100 until we had 1000 total soda tabs. Finally, and probably the most helpful, was providing students multiple opportunities to make estimates in our classroom. This included estimating various objects (e.g., Skittles, golf balls, gumballs, and M&Ms) and comparing the size of the objects to the number in the original container.

This activity can be adapted and extended to fit your students and their level of experience with estimation. You can also use this activity to discuss statistical ideas such as "typical" and "range," create a line plot of the number of Skittles removed from the bucket by each student, sort the Skittles and create a chart using tally marks, or even create a bar graph and/or pictograph of the number of Skittles removed from the bucket. Regardless of what activity you choose to use, practice estimation strategies frequently and expose students to a variety of large numbers through the use of everyday objects (soda tabs, M&Ms, gumballs) in order for students to become more experienced with the magnitude and value of such numbers.

Acknowledgements

A special thanks to Robin Hyatt who allowed me into her classroom and helped with the planning and implementation of these activities.

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A Math III Teacher's Story

Rebecca Joy Martin Philyaw, NBCT, Caldwell Early College High School, Lenoir NC

The author describes their journey through rethinking the organization of the Math III curriculum to allow students to gain the skills they need and better experience the "big picture" of interconnections between the topics.

Over the course of my 21-year career teaching high school mathematics, I have been able to teach nearly every course offered regardless of students' ability or grade level. In the past few years, I have settled into a Math III position teaching juniors at an early college high school. Being in this position has allowed me more curricular freedom than my former traditional high school setting because students are not moving in or out of the school. Our rosters are set when students enter as freshmen. At the traditional high school where I taught previously, I taught semester-long courses. At my current school, I am on an A-day/B-day schedule, meaning that I see each group of students two days per week for the entire school year.

While taking a class last spring, I was introduced to Understanding by Design (UBD), a curricular program created by Wiggins and McTighe (1998). The goal of the program is true student understanding with clear goals and assessments (Authentic Education, 2015). The planning process is described as "backward design" and is presented as a comprehensive training package that transforms curriculum planning (Authentic Education, 2015). Admittedly, I have not fully immersed myself in the process. I did not fully follow the backward design procedures (a goal for another time), but it did serve as a trigger that forced me to look at my teaching practices through a different lens. I started asking the question, what do I really want my students to understand? If you have ever read the North Carolina Standard Course of Study (NCSCOS) for Math III, you know that this can be a daunting question. The breadth or depth dilemma is a struggle we face every year. So, I stopped looking at that NCSCOS document for a moment and started looking at the "big picture." I had to consider what I really wanted my students to learn and understand. I began to look at the themes that ran through the topics and started to group some ideas.

I decided that I wanted my students to realize that when we talk about graphs, the language and features are common to all function families. I wanted my students to think of equations and graphs as different representations of the same information. I also did not want my students to get caught up in the computational skills that were required to make those other connections. It's not that I had not tried to teach these topics before, but I had taught them in neat little "boxed" lessons. I previously taught lessons on domain and range, on finding zeros of absolute value functions, on finding the zeros of polynomials, on parent functions and transformations, and so on; however, I did not feel that most students ever made those big connections because instead of looking at the similarities, I kept everything separated into units based on function type. I taught everything about absolute value together. Similarly, I taught everything about polynomials together. I basically followed this type of grouping for each type of function in the NCSCOS for Math III. How could I expect students to make connections when I was separating it for them? I decided that I had to throw those old "boxes" (units) away. I did not literally toss and delete all of the resources that I had compiled over the years because I knew I had some good stuff to build on, but I did reorganize everything with my goals of understanding in mind. I knew that I would have to revamp some things and would have to make some new resources to convey my connective points.

First, I decided that I didn't want all of the computational skills to get in the way of the main ideas I wanted to get across. I knew that these were foundational for many of the concepts that I wanted students to deeply understand. This meant that a computational unit would have to be where I started. I put things like synthetic division, evaluating piecewise functions, composition of functions, operations with rational expressions, and operations with complex numbers in this unit. I also included a few things I was a little unsure about in terms of placement, like learning the unit circle, finding algebraic inverses, and converting between logarithmic and exponential forms. In teaching this unit for the first time, I was flexible in my timing, allowing my students to move at a pace that fit the needs of the class. It seemed to go on forever and I had that looming feeling that I was never going to get to the big ideas. During this time, I also questioned if I was doing the right thing. Should I be teaching these things without the surrounding context? The unit finally ended, having taken up about one-third of the course time.

I started my second unit on graphs with the parent functions. This opened a door to talk about inverses as graphs (log/exponential, cube/cube root, quadratic/square root). I taught graph features and applied them to all the functions we were learning. Then we learned features that were specific to polynomials, rational functions, absolute value, and trigonometric functions. Surprisingly, this unit seemed to go much faster, taking only a few weeks. Entering the third unit, Equations, the pace had picked up. In this unit, the benefits really came through. For example, when reviewing the quadratic formula, I did not have to stop and teach or reteach complex numbers. When teaching students how to find all the zeros of a polynomial, they already were competent and confident with using synthetic division, the quadratic formula, and complex numbers. They did not get caught up in the details of the steps because they knew the ultimate goal, which was finding the correct number of zeros. When teaching how to solve rational equations, students already knew about domain restrictions because they understood vertical asymptotes and points of discontinuity. The missing context I was unsure about in the first unit came into play and everything started to fully develop. Students were frequently commenting that new topics seemed familiar. Instinct for considering the graph seemed to be built into their routine. Conversations about appropriate domains were easily initiated. In the past, students had compartmentalized domains of graphs separately from domains of equations. With the changes I had made, students were able to realize that these were not two different concepts and seemed to gain a true understanding of domain.

One of the best things about revamping was that skill practice kept resurfacing. For example, teaching operations with rational expressions early on allowed me to point out those same fraction operations when using the unit circle, solving rational equations, and any time that a fraction popped up in other problems. Knowing the unit circle opened up better conversations about the graphs of sine and cosine and some trig equations became much easier. Sometimes student questions even brought in those computational skills when I was not expecting such insight. I used each opportunity to connect that smaller version to those starting operations from the first unit. I would also strategically plan my warm-ups to revisit a skill that we would use in that day's lesson. At the time of writing this article, my students had not yet taken the NC End-of-Course exam. I have no quantitative results to compare to previous years. However, I know the discussions students have in my classroom and how they are doing on my assessments. Every time one of my students makes a comment that something is familiar or easy, I get excited. I am confident that they will do well on the EOC, but more importantly I am confident that they understand and will retain what I have taught.

As I reflect on my work, I feel that I have made big gains in the understanding of the algebra components of the course. The next challenge lies in considering the geometry components of the course, which I expect to be a more difficult task. It takes more work to truly integrate the geometry and algebra topics when there are few resources available. As math teachers know, most textbooks and content websites, even if the title says integrated, keep geometry chapters separate from the algebra chapters. They throw in one or two questions here and there that have a polynomial expression as a side length and ask for the area or volume of a figure as an algebraic expression. True integrated content is usually developed by a teacher who sits down and creates their own content to meet this objective. My goal for next year is to figure out a better way to truly integrate the geometry components of Math III into the course in a way that works for my students and helps them to gain insight into the connectivity found in mathematics. I know that there are many excellent teachers out there already making these, but I want to challenge all teachers to find new ways to bring in connections. We can always learn, grow, and work on perfecting our craft. Ask yourself what YOU want your students to understand and then figure out how to make that happen!

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Outstanding Teachers 2021 and 2022

Reported by Denise Schulz, NCDPI

Each year, principals are encouraged to nominate the teacher who does the most effective job teaching mathematics in their school. From those nominated, each LEA selects one teacher to represent the best in mathematics teaching from the entire system. The teacher receives a membership in NCCTM, recognition at the State Conference, and a special memento of the occasion. The grade level cycles, and the teachers from the last two years were honored in November 2022: Outstanding Elementary Teachers and Outstanding Secondary Teachers.

Congratulations to these outstanding teachers!

Outstanding Elementary Teachers who were present at the Awards Ceremony in November

Outstanding Secondary Teachers who were present at the Awards Ceremony in November

District	Elementary	Secondary
Alamance-Burlington	Kristen Puckett	Amanda Downs
Alexander County	Christy Crouse	Melissa Sharpe
Ashe County	Shera Ashley	Erika Donahue
Asheboro City	Melissa Belote	Bridgette Kiser
Avery County	Jade Evaul	Shelby Barrier
Beaufort County	Autumn Wallace	Miranda Whitley
Brunswick County	Cassandra Anderson	
Buncombe County	Paula Ottinger	Gillian Tart-Snyder
Burke County		Katie Gordon
Cabarrus County	Carla Looney	
Caldwell County	Katie Bast	Kimberley Hindman

District	Elementary	Secondary
Camden County		Denise Thomas
Carteret County		Mikaela Edge Worsinger
Catawba County	Amy Reep	Nicki Brittain
Chapel Hill-Carrboro	Molly Kearns	Kimberly Sholler
Charlotte-Mecklenburg		Kimberli Darling
Craven County	Taylor Whitford	
Cumberland County		Cynthia Rios
Currituck County		Christian Lowe
Davidson County	Patricia Burkhart	Samantha Murchison
Edenton-Chowan	Amy Sasscer	Gwen Hughson
Elizabeth City-Pasquotank	Jennifer Hilliard	Liz Wooten
Franklin County	Lindsay Duncan	Kendall Doyle
Gaston County	Allie Kleinheinz	
Greene County	Griffin Gurkin	
Harnett County	Brandy Martinez	
Henderson County	Cara Brock	
Iredell-Statesville	Kelly Carney	Sarah Johnson
Jackson County		Barb Sink
Johnston County		Wendy Srinivasan
Kannapolis City	Alexandra Medley	Lisa Emerson
Lee County		Brandi Johnston
Lenoir County		Belinda McGinnis
McDowell County		Andrea Hardy
Montgomery County	Audrea Samuels	Kymberlie Hare
Nash County	Amy Worrell	
New Hanover County		Jacqueline Anderson
Newton-Conover	Leonor Portilla	Ashley Bandy
Onslow County	Rochelle Curlee	
Orange County	Taryn Stock	Claire Capps
Pender County	April Kinlaw	
Perquimans County	Courtney	Baccus
Person County	LaJeanne Ashley	Laura Jennings
Polk County	Stephanie Uhrich	Jamie Graber
Randolph County	Ashley Cheek	
Roanoke Rapids		Kay Allen
Rockingham County	Kelsey Heiney	Ramona Bankston
Rowan-Salisbury		Taren Davis
Rutherford County	Dana Haire	Amy Owens
Sampson County	Elizabeth Dailey	
Stanly County	Nancy Barbee	
Stokes County	Shannon Rose	
Surry County	Eden Phillips	Tina VanHoy
Wake County	Tiffany Varnell	Tamara Ormandy
Watauga County	Sarah Holt	Sumer Williams
Wayne County	Veronica McCoy	Robin Bryant
Wilkes County	Heather Holloway	
Wilson County	Tereasa Newton	Teresa Dantzler
Winston-Salem/Forsyth County	Lisa Putnam	
Yadkin County	Kari Koons	Ashley Atkins

Outstanding Mathematics Education Students

Reported by Ana Floyd, Randolph County School System, Asheboro NC

After an awards ceremony hiatus due to COVID, NCCTM is pleased to announce the selection of Outstanding Mathematics Education Students for the years 2020, 2021, and 2022.

2020 Outstanding Student: Abigail Brown from East Carolina University in the Eastern Region of NCCTM

Abigail "Abby" Brown was a Noyce Scholar at East Carolina University with a double major in Mathematics and Mathematics Education. She was described as a dedicated student that demonstrates a passion for teaching and persistence in solving ill-defined and complex problems. While Abby's academic accomplishments were certainly noteworthy, her contributions and leadership extended beyond the classroom. Abby was a member of the ECU Student Chapter of the NC Association of Educators. She served as President of ECU's Gamma Student Chapter of NCCTM for two years. During this time, she helped the organization through a pivotal transition period and was a strong leader that was organized, proactive, and well respected by her peers. One professor wrote, "Never outspoken, she has risen to a leadership role among her fellow students with her example, work ethic, understanding of mathematics, and willingness to help others."

Abby's service-oriented accomplishments extended to students both on and off campus. She worked in the ECU Math Lab for 12-15 hours per week where she provided one-to-one assistance, formative and summative feedback, and whole group instruction for both remedial and college algebra courses. Additionally, she tutored students from local high schools across a variety of courses including Math 1. One professor described Abby as being a quiet leader and excellent student. Another professor was impressed by her character, work ethic, exceptional professionalism, and dedication to becoming an outstanding mathematics teacher.

Abby is currently pursuing a Master's degree in Mathematics at East Carolina University.

2021 Outstanding Students: Elizabeth Rodgers from North Carolina State University in the Eastern Region of NCCTM and Maia Tice from Elon University in the Central Region of NCCTM

Elizabeth "Beth" Rodgers was a Park Scholar and a double major in Mathematics and Mathematics Education. Described as an excellent and conscientious student, Beth is recognized for such scholarship skills as growth mindset, personal motivation, natural inquisitiveness, and perseverance. In service to NCCTM, Beth served on the executive board of NC State's Kappa Student Chapter of NCCTM for three years—serving as vice-president and then president of the organization. As president, she hosted virtual meetings and participated in book clubs. She was instrumental in inviting other NCCTM organizations across the state to attend these virtual events.

Beth prioritized service to her community as well. She helped lead a mental health identity group dedicated to building safe spaces for high achieving students who struggle with mental health, worked as a camp counselor for the YMCA's Camp Hanes, volunteered in organizations such as Junior Achievement and Service Raleigh, and even traveled to Guatemala to learn about gender issues and violence in Guatemala and Latin America. One professor described Beth's exceptional leadership skills, and especially noted her communication, flexibility, vision, and dependability. She looked forward to Beth becoming a well-rounded teacher that provides excellent mathematics experiences for her students.

Beth graduated from NC State University and is currently teaching Honors Pre-calculus and Honors Math 2 at Green Level High School (home of the Green Level Gators) in Cary, NC.

Maia Tice was a Teaching Fellow who completed a degree in mathematics with secondary teacher licensure. Maia maintained a stellar academic record. She was described as being well regarded by faculty, engaged in class, and deeply interested in understanding math concepts. Maia demonstrated outstanding commitment and contributions to mathematics education. She conducted research on student learning of number sense in high school, taught as part of a study abroad program in New Zealand, served as a teaching assistant and tutor for the Department of

Mathematics and Statistics, and completed an intercultural learning certificate. Additionally, Maia served as an education coordinator for Habitat for Humanity and also President of the Elon University chapter of Pi Mu Epsilon, the national mathematics honorary society. She also tutored local high school students in such courses as Calculus I, Multivariable Calculus, and Introduction to Statistics. One professor noted, “Maia goes beyond seeking research-based strategies for making math meaningful to students and applies these strategies to her teaching practice. She was active participant and interested in learning how teachers meet the many challenges they face.”

2022 Outstanding Student: Avery Bowman from Meredith College in the Eastern Region of NCCTM

Avery Bowman has certainly demonstrated that she is an outstanding candidate, representing the goals and ideals of NCCTM. Originally declaring a dual major in mathematics and engineering, Avery came to the realization that she wanted to be in the classroom, working with students and teaching mathematics. Described as a “top student” that is prepared and conscientious, Avery was labeled as “one of the strongest mathematics majors and licensure students at Meredith College.” One professor noted, “Not only did she exhibit the characteristics of an outstanding student—responsible, prepared, engaged in class, participatory, collaborative—but also she demonstrated a high level of understanding of the content, and great insight as we developed new ideas in number theory in class. Her work was outstanding!”

Avery achieved her outstanding academic achievements while competing in NCAA Division III volleyball for four years and serving as the captain of the team during her junior and senior years. She was also an assistant volleyball coach at a local volleyball club and was involved in local volleyball camps as an instructor and coach. Her experiences coaching and teaching volleyball have positively influenced her as a teacher. She is described as having a strong classroom presence, demonstrating an intuitive sense of teacher awareness and “withitness,” that allows her to trouble shoot the nuances and sense the places where students have difficulties. Her leadership activities extended beyond the volleyball court. She was President of the mathematics honor society, Pi Mu Epsilon, and inducted into several honor societies at Meredith College. She tutored at the Meredith College Learning Center and assisted in high school and middle school math classrooms in Wake County.

Avery is currently teaching mathematics in her hometown at Person County High School.

Presidential Awards for Excellence in Mathematics and Science Teaching

Reported by Joseph Reaper, NCDPI

2019 Presidential Awardee in Mathematics

- Tyler Erb; Community House MS; Charlotte-Mecklenburg Schools

2020 Presidential Finalists in Mathematics

- Rebecca Criste; Stedman Elementary School; Cumberland County Schools
- Sara Lilley; Clark Elementary School; Vance County Schools

2020 Presidential Awardee in Mathematics

- Sarah Remery; Trindale Elementary School; Randolph County Schools

2021 Presidential Finalists in Mathematics

- Chasity Bolch; Newton-Conover Middle School; Newton-Conover City Schools
- Margaret McDade; East Henderson High School; Henderson County Schools
- James O’Neal; Piedmont IB Middle School; Charlotte Mecklenburg Schools

2022 Presidential Finalists in Mathematics

- Rebecca Criste; Stedman Elementary School; Cumberland County Schools
- Kimberly Inman; Pinnacle Classical Academy; Shelby, NC
- Nardi Routten; Creekside Elementary School; Craven County Schools

2022 Lee Stiff Instructional Leader Award

Laura Baker

Reported by Ana Floyd, Randolph County School System, Asheboro NC

Dr. Lee Vernon Stiff passed away in March 2021. He was a nationally-recognized expert and scholar on equity and mathematics education who served and led in NC State's College of Education for 37 years. He was past president of the National Council of Teachers of Mathematics and winner of the W.W. Rankin Award. Throughout his nearly 45-year career in education, Dr. Stiff worked to improve mathematics education at the state and national level for all students. He served our organization and was devoted to improve student success and teacher effectiveness. This award, the Lee Stiff Instructional Leader of Mathematics Teaching Award, was named in honor of him and his legacy to mathematics education in North Carolina.

The purpose of the award is to recognize an individual at the district or school level who has made an outstanding and noteworthy contribution to mathematics education and NCCTM by working directly with classroom teachers to improve student learning of mathematics. This individual may support classroom teachers through coaching, professional development, demonstration lessons, and the development of curriculum materials and resources.

Laura Baker (r) pictured with NCCTM President Stefanie Hill

Laura Baker has exemplified this role as a coach and instructional leader. She is currently the district-wide Elementary Mathematics Coach for Kannapolis City Schools. Laura supports classroom teachers in ways that directly improve student learning of mathematics. Not only does she lead curriculum development so that teachers have consistent, aligned instructional materials, but she also provides layers of ongoing support for teachers to deliver the instruction to students effectively. She models effective planning processes and provides feedback and instructional strategies to teachers. She often participates in PLC meetings across the district and unpacks data on a variety of assessments. Her work also includes regular classroom visits to observe instruction and give teacher feedback.

Laura is known for being dependable and hard-working, and she strives to help others. Laura extends her support to provide additional one-on-one coaching support for individual teachers, especially beginning teachers, through co-planning, co-teaching, or modeling instruction. She works with teachers to learn how to teach mathematics from an inquiry-based approach, implement number talks, move towards a student-centered model, and build classroom community. She provides tools and strategies to use in the classroom and models how to facilitate such strategies. In addition, Laura plans and delivers professional development in varying models for schools and the district. This includes both school and district administrators to help them understand major underlying concepts as well as identify specific look-fors in the mathematics classroom based on the content standards as well as the Standards for Mathematical Practice.

Laura not only serves her district. She also seeks to improve mathematics education in our state. She has worked with the NC Department of Public Instruction to expand the library of K-2 assessment tasks and items. Laura was a lead writer and professional development facilitator in theTools4NCTeachers grant, and she also served as a member of the NC2ML Instructional Framework group providing feedback and helping to determine which standards would be grouped into clusters. One administrator in her district praised Laura's knowledge, understanding, and dedication to mathematics: "Her knowledge of content standards for kindergarten through 5th grade is profound. She is able to assist teachers with aligning standards while providing rigor to students. Further, she has been willing and able to support teachers in their classroom to support their mathematics teaching." Another school administrator stated, "Laura was instrumental in opening our teachers up to the idea that moving the work and discussion to a student centered model would increase student understanding while building classroom community. As a result, teacher confidence grew and classroom conversations became more evident and frequent in math classes. She was so successful the teachers asked for an extension of the professional development for the next year."

2022 Innovator Award Winner

Dawne Coker

Reported by Ana Floyd, Randolph County Schools, Asheboro NC

The purpose of the Innovator award is to recognize and reward individuals or groups who have made an outstanding and noteworthy contribution to mathematics education and NCCTM by having founded, initiated, pioneered, or developed some program in mathematics education of service to a geographic region of the state or the entire state.

Dawne Coker (l) with NCCTM President Stefanie Hill

NCCTM proudly recognizes **Dawne Coker** as the 2022 NCCTM Innovator for the establishment and continued commitment to developing mathematics resources for elementary teachers, students, and families. Dawne has made outstanding and noteworthy contributions to mathematics education and served the state of North Carolina for over ten years.

Elementary teachers in North Carolina are fortunate to have a tremendous amount of high-quality mathematics resources to support classroom instruction and assessment. Many of these resources were developed or revised by Dawne Coker. Dawne has partnered with the North Carolina Department of Public Instruction on many occasions to create and improve resources for elementary mathematics. Dawne served on the standard revision team for kindergarten through second grade prior to the adoption of the 2018 Standard Course of Study. She co-authored, reviewed, and provided feedback for the K-5 Mathematics Unpacking Documents. She was part of a writing group that created sets of Formative and Instructional Assessment Tasks (that are now a part of the Tools4NCTeachers website). She also consistently provides feedback on the K-2 formative and end-of-year assessments.

Dawne has contributed to state-wide grants and initiatives including the North Carolina Collaborative for Mathematics Learning (NC2ML) and Tools4NCTeachers. Dawne served as part of the team that provided feedback and input into the NC2ML Instructional Frameworks. She attended meetings and provided feedback about how the new standards should be organized and grouped into clusters. On the Tools4NCTeachers grant, Dawne served as a lead author and professional development facilitator for the kindergarten resources and materials. In that role, Dawne was one of the leaders of a team who wrote lessons, tasks, parent letters, and other supplemental resources to support teachers. She also edited and assisted with the creation of the Next Steps documents for several grades on the Tools4NCTeachers project.

In the last few years, Dawne has created instructional materials that she has shared for free. During the 2019-20 year, Dawne created free homework and activity sheets with mathematical tasks designed for third grade, and helped edit and provide feedback to the activity sheets created for grades 4 and 5. Dawne also developed a YouTube channel that includes a wealth of resources for elementary teachers --- videos on number talks, how to use manipulatives and technology resources, mathematical tasks, content support, and tips for the end-of-grade assessments. Dawne's YouTube channel has over 3,000 subscribers. During the Spring of 2020 when COVID-19 required remote learning, Dawne added videos to her YouTube library by conducting live video lessons which used inquiry and the exploration of mathematical tasks. The city of Fayetteville's local television station aired her videos during July 2020 as a resource for parents and students. In addition to the videos, Dawne also created and shared end-of-year tasks and activities that parents and teachers could utilize to gather information on students' mathematical understanding. One person described Dawne as, "An incredible individual who gives freely of her time to provide feedback, support, and resources to teachers and educational leaders across the state."

2022 Rankin Award Winners

Dr. Vincent Snipes and Dr. Holt Wilson

Reported by Emogene Kernodle, Elon NC

After a hiatus due to COVID, NCCTM is pleased to announce recipients of the prestigious W. W. Rankin Memorial Award for Excellence in Mathematics Education, the highest honor that NCCTM can bestow upon an individual. At the 2022 State Mathematics Conference in November, NCCTM honored two dedicated individuals: **Vincent Snipes and Holt Wilson**.

Dr. Vincent Snipes is Director of the Center for Mathematics, Science, and Technology Education and a Professor of Mathematics Education at Winston-Salem State University. Vincent is regarded as a dedicated mathematics educator who is devoted to the teaching of mathematics and the "advancement of the teaching profession." He has worked tirelessly over his career to facilitate the success of students in classrooms as well as the teachers who serve these students. He has demonstrated strong leadership in a variety of roles and situations, all of which promoted the enhancement of mathematics education in North Carolina and beyond. Vincent has provided dedicated service and leadership through his friendly personality and innate ability to the teachers of North Carolina and the nation for many years. For example, he has:

- presented at multiple conferences including NCCTM regional and state conferences, NCTM, the Triangle High Five/Math Summit, as well as conferences for other organizations,
- demonstrated service to the greater mathematics education community through multiple publications, including being a co-author of a NCTM book.
- served multiple times as a member of the NCCTM Board of Directors including service as regional president and vice president for colleges and universities at the regional and state level, and
- helped coordinate a Summer Math Camp for underprivileged and underrepresented students including those from homeless families.

Dr. Holt Wilson, Associate Professor of Mathematics Education at UNC Greensboro, is honored in recognition of his professional commitment to the teachers and students of North Carolina. As an undergraduate, Holt was a member of the NCCTM student affiliate group and attended regional and state meetings and even an NCTM conference. During these years, North Carolina established new and rigorous standards for a North Carolina teaching license. Amidst the panic of meeting high PRAXIS scores, Holt calmly and quietly led peers in preparing for the test. As a high school teacher of mathematics, Holt saw a need to strengthen the quality of instruction for students. He organized a cohort to pursue a master's degree and was dubbed the Student Graduate Program Director. In graduate school, he saw a need for support and encouragement and met the need with the offering of friendship. Fellow students became life-long friends and created a powerful network of talented, dedicated mathematics educators.

Holt has served as chair of many mathematics education doctoral committees. These students are award-winning mathematics teacher leaders at schools, education offices, and universities across the state. He has authored or co-authored at least 20 journal articles, two books, seven book chapters, and 36 research reports in national or international conference proceedings. Additionally, he has:

- taught several different high school mathematics and statistics courses, five undergraduate, and 20 graduate courses in mathematics education.
- presented many sessions at NCCTM conferences, has served on many NCCTM conference committees, and has served as a judge at regional and state Math Fairs, and
- served multiple times as a Member of the NCCTM Board of Directors and its Executive Board.

Problems2Ponder

Holly Hirst, Appalachian State University, Boone, NC

In each issue of The Centroid, Problems2Ponder presents problems similar to those students might encounter during elementary and middle school Olympiad contests. Student solution submissions are welcome as are problem submissions from teachers. Please consider submitting a problem or a solution. Enjoy!

Problem submissions: If you have an idea for a problem, email Holly Hirst (hirsthp@appstate.edu) a typed or neatly written problem statement, along with a solution. Include your name and school so that we can credit you.

Solution submissions: If teachers have an exceptionally well written and clearly explained correct solution from a student or group of students, we will publish it in the next edition of The Centroid. Please email Holly Hirst (hirsthp@appstate.edu) a clear image or PDF document of the correct solution, with the name of the school, the grade level of the student(s), the name of the student(s) if permission is given to publish the students' names, and the name of the teacher.

Deadline for publication of problems or solutions in the Fall 2023 Centroid: July 30, 2023.

Spring 2023 P2P Problems

Problem A: The digits 1, 3, 6, and 8 are arranged to form two 2-digit numbers. Each digit is used exactly once. The two 2-digit numbers are multiplied. What are the smallest and greatest products that can be obtained?

Problem B: In an "up-and-down" counting number, the digits increase to a maximum digit and then decrease; the maximum digit must be internal to the number, i.e., the maximum is not the first or last digit. (For example, 256950 is an up-and-down number that increases to 9 and then decreases to 0, but 255690 is not, because 2-5-5-6-9 does not strictly increase with each digit.) How many different 5-digit up-and-down numbers are there with 6 as the maximum number?

Fall 2022 P2P Problems

Problem A: The arithmetic mean of the numbers denoted by A and B is 18, and the arithmetic mean of the numbers denoted by C, D, and E is 43. What is the average of all five numbers A, B, C, D, and E?

Solution: We know two equations based on the information given about A, B, C, D, and E:

$$\frac{A+B}{2} = 18 \qquad \frac{C+D+E}{3} = 43$$

Therefore

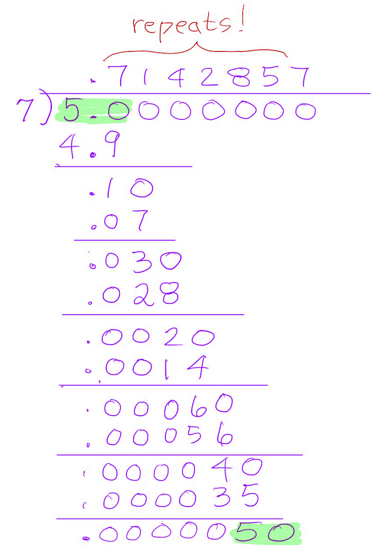
$$A+B = 36 \qquad C+D+E = 129$$

And using these two equations, we know the sum of all five numbers is 165, and the average is $\frac{165}{5} = 33$

Problem B: The fraction $\frac{5}{7}$ can be approximated by a decimal. What is the digit the lies in the 2022 place in its decimal expansion?

Solution: Because rational numbers have decimal expansions that either terminate or repeat, let's use long division to see if we can determine what happens with $\frac{5}{7}$. From the work we can see that the decimal expansion repeats .714285.

Since this string is 6 digits long, we know that: 7 will occur in the 1st, 8th, 15th, 22nd, … place in the decimal expansion. These positions are all 1 more than a multiple of 7. What about the other numbers?



- 1 will occur in the 2nd, 9th, 16th, 23rd, place…. 2 more than a multiple of 7.
- 4 will occur in the 3rd, 10th, 17th, 24th, …. 3 more than a multiple of 7.
- 2 will occur in the 4th, 11th, 18th, 25th, …. 4 more than a multiple of 7
- 8 will occur in the 5th, 12th, 18th, 26th, …. 5 more than a multiple of 7.
- 5 will occur in the 6th, 13th, 19th, 27th, …. 6 more than a multiple of 7.

So what digit must be in the 2022nd position?

$2022 = 7 \times 288 + 6$ (can you explain why? Long division again!)

So The 2022nd place is 6 more than a multiple of 7 and will contain a 5!

Trust Fund Scholarships: $1000

Scholarships are available from NCCTM to financially support North Carolina teachers who are enrolled in graduate degree programs to enhance mathematics instruction. Applicants must be:

- Currently employed as a pre-K-12 teacher in North Carolina;
- Currently an NCCTM member (for at least one year) at the time of submitting the application;
- Currently enrolled in an accredited graduate program in North Carolina;
- Seeking support for a mathematics or mathematics education course in which they are currently enrolled or have completed within the previous four months of the application deadline.

Applications will be reviewed biannually, and the deadlines for applications are March 1 and October 1. The application can be downloaded from the NCCTM website under the "grants & scholarships" link. The nomination form can be obtained from the grants and scholarships page on the NCCTM Website (ncctm.org). More information can be obtained from: Janice Richardson Plumblee, richards@elon.edu.

Donating to the NCCTM Trust Fund

Did you receive a Trust Fund Scholarship that helped you to complete your graduate coursework and you want to show appreciation? Do you wish to memorialize or honor someone important to you and your career as a math teacher?

Consider making a donation to the NCCTM Trust Fund, please send your donation, payable to Pershing LLC for the NCCTM Trust Fund, to:

Joette Midgett
North Carolina Council of Teachers of Mathematics
P. O. Box 33313
Raleigh, NC 27636

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Follow the “Membership Information” link on the ncctm.org website, or go directly to:
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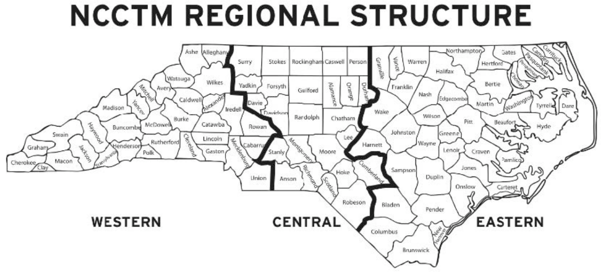

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